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A THEORY OF NEUTRON REACTIONS ON Li^6

by

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ABSTRACT

The predictions of a theory of neutron reactions on Li^6 are worked out and compared with experimental results with neutrons up to 14 Mev. The levels of the lithium nuclei involved are simulated by p^n-configurations of independent particle orbits with L-S coupling. Formation of compound nuclear states, direct knock-on, and pickup processes are considered. One of the purposes is to determine whether other mechanisms are required, such as would be possible in a cluster model, for example. The results are that formation of compound states accounts for about half of the magnitudes of observed cross sections. Direct collisions are of comparable importance to n - t and n - 2np reactions, and pickup appears to be adequate to account for the balance of the n - dn cross sections. The angular distribution in the latter was examined, but did not prove to be decisive. Additional mechanisms for the n - He^6 and the formation of the 2.2 Mev excited state of Li^6 appear to be required.

CONTENTS

A THEORY OF NEUTRON REACTIONS ON Li^6

The nuclei of Li^6 and Li^7 consist of tightly bound, alpha-particle cores to which two and three nucleons, respectively, are attached by relatively much weaker forces. Models of the configurations of these nuclei can be based upon arrangements of the loosely attached nucleons alone with some confidence so long as the excitation above the ground state does not exceed, say, 20 Mev. Several models are worthy of study. One of these conceives of the nuclei as composed primarily of two nuclear units, viz., the alpha-particle and a deuteron, or triton, bound together and forming a resonating group. Another considers individual particle orbitals in a central force field. In this model the alpha core is a completed 1 s shell and the remaining nucleons are in 1 p orbits. A specialization of the latter is the "shell model" in which the spins of the individual nucleons are strongly coupled to their orbits; but these forces are also weak in the lithium nuclei.

It is the purpose of this paper to investigate some predictions which can be made for neutron reactions on Li^6 and to do this on the basis of the configurations formed by nucleons in p-orbitals. The Russell-Saunders coupling is used. The comparative simplicity of these configurations makes it possible to describe the nuclear reactions in greater detail than one can hope to achieve among heavier target nuclei. In order to keep the excitation of the compound nucleus Li^7 below 20 Mev, we are limited to incident neutron energies below 14 Mev. Also, in this range of excitation the states of Li^7 should be predominantly p^3-configurations, so that the reactions which go through the compound nucleus are excited only in p-wave collisions.

The collision of a 14 Mev neutron with Li^6 (12 Mev, C.M. system) is characterized by a wavelength (divided by 2π) $\lambda\!\!\!^{-} = 1.44 \times 10^{-13}$ cm. A p-wave collision may be thought of as occurring within an annulus of outer radius $2\lambda\!\!\!^{-}$ and inner radius $\lambda\!\!\!^{-}$, hence a geometrical cross section of $3\pi\lambda\!\!\!^{-2}$. So to speak, collisions at distances greater than $2\lambda\!\!\!^{-}$ are not important to the formation of a compound nucleus. On the other hand, the total cross-section for 14 Mev neutrons on Li^6 is 1.40×10^{-24} cm^2 (barns), and if we set this equal to $2\pi q^2$ we get a "radius" $q = 4.7 \times 10^{-13}$ cm, indicating that more is taking place between a neutron and Li^6 than goes through the compound nucleus for which the cross-section must be less than $12\pi\lambda\!\!\!^{-2} = 0.78$ barns. Some of the difference is in the so-called "potential" scattering but a third possibility must be taken into account. This is a direct interaction between the incident neutron and either the neutron or the proton in Li^6, i.e., without going through three-body, compound configurations.

It has been mentioned that the nucleons in p-orbitals are lightly bound to the alpha-core. Hence the exponential decrease in their wave-functions is relatively slow and admits a noticeable probability that they be found outside the radius R of the potential to which they are bound. We estimate R from the well known formula derived from the beta-decay energies of mirror nuclei:

$$R = 1.465\, A^{\frac{1}{3}}\, 10^{-13} \text{ cm} = 2.662 \times 10^{-13} \text{cm} \tag{1}$$

That the value of R so obtained is reasonable is supported by the fact that the difference between binding energies of neutron and proton in Li^6 is 1.07 Mev, which would be just about the Coulomb energy at this radius. It follows from comparing R with $2\lambda\!\!\!^{-}$ at 14 Mev that direct interactions will be important to the study of reactions on Li^6.

In the following we take up the theory of reactions through the compound nucleus and through direct interaction, separately. For the former, we require still a fourth radius which we denote by d. This is the effective radius of interaction between a neutron and the Li^6 nucleus. Fortunately, the results are not at all sensitive to its exact value, so we estimate it to be R plus half the radius of a square well describing nucleon-nucleon interaction:

$$d = R + 1.42 \times 10^{-13} = 4.08 \times 10^{-13} \text{ cm}$$

For the direct interactions we use the impulse approximation and consider only those collisions which are orthogonal to the state in which the compound nucleus may be formed.

Reactions through the Compound Nucleus

When the collision between neutron and Li^6 has orbital angular momentum unity, an excited state of Li^7 may be formed. The probability of emission of a gamma-ray will be ignored in comparison with that of particle emission. The possible modes of the break-up of the compound nucleus are then:

(a) emission of a neutron and return of Li^6 to any of a number of states;

(b) emission of the proton, leaving He^6 in either its $^1S_0$ or $^1D_2$ state;

(c) emission of a deuteron and He^5 (in an s-wave) in one of four spin states, viz., singlet or triplet deuteron and doublet or quartet He^5;

(d) emission of the triton.

Some of the Li^6-states resulting from (a) can emit the second neutron by way of $He^5 + {}^1D$, He^5 + p, or Li^5 + n, but the triplet states have to compete with emission of the stable deuteron. The excited ($^1D_2$) state of He^6 emits two neutrons. Only the products p + He^6 (in the ground state) and He^4 + T do not lead to at least one neutron.

As suggested above, the intermediate states will be assumed to have the same quantum numbers as the p^3-configurations, which would be expected to be in the energy range of the excited nucleus. On the other hand, these states are very broad, hardly levels at all, and we do not know their exact location. Consequently, we select those p^3-configurations which can be excited by a neutron on the ground state of Li^6 and average the effect of each such state over a wide range of positions for the "resonant energy". The intensities resulting from the break-up of these states are then summed without regard to phase-relations among them.

For our purpose, we imagine a certain intermediate state ψ_i, where i stands for the quantum numbers: J, M, L, C, S, T. As is customary, J is the total angular momentum, M its Z-component, L the orbital angular momentum, S the true spin, and T the isotopic spin. The number C is the symmetry character of the p^3-state (without adding that created by the alpha-core). This is used instead of Y of Wigner's (STY) formulism because it gives a direct measure of the exchange character of the state and hence of its position relative to the others, all of which have essentially the same kinetic energy. Also, the numbers S and T used here have their direct physical meaning. The spacing in energy between levels of the same quantum numbers i will be denoted by D_i. This energy gap is very large for light nuclei, but its numerical value is not important to the theory.

Let k be the wavenumber outside d, the radius of collision ($k = \lambda\!\!\!^{-}{}^{-1}$), and K be the wavenumber inside d. The value of K is computed from the kinetic energy outside d plus the energy of binding of a neutron, 7.25 Mev. The cross-section for formation of the compound state ψ_i, averaged over the possible positions of the resonant energy in the Breit-Wigner formula, is (for $\ell = 1$ waves)

$$\sigma_i^* = 6\pi^2 \lambda\!\!\!^{-}{}^2 \frac{\Gamma_a^{\ i}}{D_i} \tag{2}$$

with

$$\Gamma_a^{\ i} = \frac{2}{\pi} \frac{k}{K} \frac{(kd)^2}{1 + (kd)^2} g_a^{\ i} D_i \tag{3}$$

In the usual parlance, $\Gamma_a^{\ i}$ is the neutron width, $(k\ d)^2 / [1 + (k\ d)^2]$ being the penetration factor for p-waves. The statistical factor $g_a^{\ i}$ is the square of the scalar product taken between the state ψ_i and that formed from the Li^6 ground state plus a plane neutron wave, normalized over angles. The subscript a denotes the combined spin state of the colliding system.

In order to compute the cross section for a given type of reaction we define the partial widths $\Gamma_f^{\ i}$ for emission from ψ_i into each final state ψ_f. The compound state ψ_i then has a total width

$$\Gamma^i = \sum_f \Gamma_f^{\ i} \tag{4}$$

Under our assumption that the probabilities of emission to the various ψ_f are independent of each other, the cross section for formation of a particular final state is

$$\sigma_f = \sum_i \sigma_i^{\ *} \frac{\Gamma_f^{\ i}}{\Gamma^i} \tag{5}$$

The $\Gamma_f^{\ i}/D_i$ are computed from eq.(3) (except for deuteron emission) but with k being the wavenumber of the emitted system and $g_f^{\ i}$ being the overlap between ψ_i and ψ_f

$$g_f^{\ i} = \left| < \psi_i \middle| \psi_f > \right|^2 \tag{6}$$

and with the Coulomb barrier factor

$$\frac{2\ \pi\ \eta}{e^{2\pi\eta} - 1} \qquad\qquad \eta = \frac{2\ e^2}{\hbar\ v} \tag{7}$$

when charged particles are emitted. The formula for the deuteron emission has unity for the non-Coulombic part of the penetration factor (s-waves).

The procedure adopted here differs from the usual statistical one in explicit calculation of the g-factors on the basis of a model, instead of estimating them, and in identifying the final states as nearly as possible from the known levels of the product nuclei.

Calculation of Statistical Factors

Among p-orbitals there are three choices of quantum number denoting space coordinates and two denoting spin direction for each nucleon. Li^6 has, therefore, thirty-six p^2-states. These are partially degenerate, of course, and actually lead to ten different energy levels. These ten levels have been identified, for the purpose of constructing a model, with the ten lowest known levels[1] in Li^6. The assignment agrees with the values of J and T, where these are known, and is shown in Table I. Also in the table we have given the symmetry character C. Each of the p^2-states is an eigenfunction of the operation of exchanging the position (m_ℓ-values) of the neutron and proton. Six of these states belong to the eigenvalue one and four to minus one. This helps to locate the states since the higher the value of C the greater the (negative) potential energy and the lower the state lies[2].

TABLE I

States in the Li^6 Model

C	T	Term	Energy (Mev)
- 1	1	$^3P_0$	9.3
- 1	1	$^3P_1$	8.37
- 1	0	$^1P_1$	7.40
- 1	1	$^3P_2$	6.63
1	0	$^3D_1$	5.5
1	1	$^1D_2$	5.35
1	0	$^3D_2$	4.52
1	1	$^1S_0$	3.56
1	0	$^3D_3$	2.18
1	0	$^3S_1$	0

The ground state of Li^6 is primarily a $^3S_1$-state with the substates M = 1, 0, -1. These states, taken with the incoming components of the neutron wave, which we denote by $n_0\uparrow$ and $n_0\downarrow$, give six independent states of collision. The overall cross section is the average of those computed for these states. Owing to the symmetry in positive and negative M-states, it is sufficient to average over three of them (three values of the subscript a).

In order to compute each $\Gamma_a^{\ i}$ from eq.(3) we need the overlap intensities $g_a^{\ i}$, and for these we need the p^3-configurations. These are classified in Table II. Multiple subscripts are used to indicate the various J-values. Primes denote $T = \frac{3}{2}$ states, which have symmetry character zero, and asterisks indicate the states of highest symmetry, C = 3. The ground state of Li^7 is the ${}^2P_{\frac{3}{2}}^{\ *}$ and the ${}^2P_{\frac{1}{2}}^{\ *}$ is close by. Hence they (and the F*-states) are not expected to play any role as compound states. On the other hand, the states with C = 0 should be in the correct range of excitation to be taken into account.

TABLE II

p^3 - States of Li^7

	$T = \frac{1}{2}$		$T = \frac{3}{2}$
C = -3	${}^2S_{\frac{1}{2}}$		${}^4S_{\frac{3}{2}}$
C = 0	${}^2P_{\frac{3}{2}\,\frac{1}{2}}$ ${}^2D_{\frac{5}{2}\,\frac{3}{2}}$	${}^4P_{\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}}$ ${}^4D_{\frac{7}{2}\,\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}}$	${}^2P'_{\frac{3}{2}\,\frac{1}{2}}$ ${}^2D'_{\frac{5}{2}\,\frac{3}{2}}$
C = 3	${}^2F^*_{\frac{7}{2}\,\frac{5}{2}}$	${}^2P^*_{\frac{3}{2}\,\frac{1}{2}}$	--

Tables of the p^3 and p^2-configurations in terms of particle orbitals were used to compute the $g_a{}^i$ and $g_f{}^i$. The tables are presented in the Appendix. Since we are not concerned with polarization we average the $g_a{}^i$ over M to obtain $\overline{g}_a{}^i$. Also, the compound Li^7 nucleus may emit a neutron, for example, into any one of the six neutron orbits: $n_1\uparrow$, $n_1\downarrow$, $n_o\uparrow$, $n_o\downarrow$, $n_{-1}\uparrow$, $n_{-1}\downarrow$. We sum over all these orbits and polarizations to compute $g_f{}^i$.

The resulting values of $g_f{}^i$ and $\overline{g}_a{}^i$ are given in Table III for the five states, ψ_i, of Li^7 which may be formed by adding a neutron in a p-wave to the ${}^3S_1$ ground state of Li^6. All these states have C = 0. Several features of the results may be noted. One is that the initial state, having L = 0 and T = 0 for the Li^6 and L = 1, $T = \frac{1}{2}$ for the neutron, leads only to L = 1, $T = \frac{1}{2}$ states of Li^7. Secondly, the factor $g_f{}^i$ for return to the ${}^3S_1$ state of Li^6 is larger than $\overline{g}_a{}^i$, the factor for formation from that state. The explanation is, of course, that the $g_f{}^i$ is a sum over all m_ℓ and spin values of the emitted neutron, whereas only one set is represented in the colliding system. Also, the sum of $g_f{}^i$ for emission of a neutron is two, corresponding to the fact that either neutron may be emitted from the Li^7 nucleus.

In Table IV we give the $g_f{}^i$-values for final states in which a deuteron is emitted. The deuteron states are analogous to the Li^6 states in Table III, but the sum over spins of the neutron must be taken differently, namely, to correspond to the ${}^2P_{\frac{3}{2}}$ and ${}^2P_{\frac{1}{2}}$ (He^{5*}) states of

TABLE III

Values of $g_f^{\,i}$ and $\bar{g}_a^{\,i}$

ψ_i =	$^4P_{\frac{5}{2}}$	$^4P_{\frac{3}{2}}$	$^4P_{\frac{1}{2}}$	$^2P_{\frac{3}{2}}$	$^2P_{\frac{1}{2}}$	Threshold (Mev)
			$n + Li^6$			
ψ_f						
$^3P_0$	0	5/36	1/18	1/72	1/18	9.3
$^3P_1$	1/8	1/8	3/8	1/16	1/8	8.37
$^1P_1$	0	0	0	3/4	3/4	7.40
$^3P_2$	3/8	17/72	5/72	25/144	5/72	6.63
$^3D_1$	1/120	7/40	5/8	1/48	5/24	5.5
$^1D_2$	0	0	0	5/36	5/36	5.35
$^3D_2$	1/8	13/24	5/24	5/48	5/24	4.52
$^1S_0$	0	0	0	1/9	1/9	3.56
$^3D_3$	7/10	7/60	0	7/24	0	2.18
$^3S_1$	2/3	2/3	2/3	1/3	1/3	0
			$p + He^6$			
$^1D_2$	0	0	0	5/18	4/9	11.70
$^1S_0$	0	0	0	2/9	5/9	9.99

TABLE IV

Values of g_f^i for Deuteron Formation

$\psi_i =$	$^4P_{\frac{5}{2}}$	$^4P_{\frac{3}{2}}$	$^4P_{\frac{1}{2}}$	$^2P_{\frac{3}{2}}$	$^2P_{\frac{1}{2}}$
$^1d(S) + He^{5*}$					1/9
$^1d(S) + He^5$				1/9	
$^3d(S) + He^{5*}$		10/27	16/27	4/27	1/27
$^3d(S) + He^5$	2/3	8/27	2/27	5/27	8/27
$^1d(D) + He^{5*}$				5/72	
$^1d(D) + He^5$				5/72	5/36
$^3d(D) + He^{5*}$	5/12	5/27	5/108	25/216	5/27
$^3d(D) + He^5$	5/12	35/54	85/108	65/216	25/108
$n^2(S) + Li^5$				2/9	

He^5. Also, since L is a good quantum number and since the states of He^5 have odd parity, the deuteron may be emitted only in S or D states which appear in parentheses. Triplet states represent the stable deuteron and singlets represent the singlet deuteron which subsequently breaks up. The possibility of emission of a di-neutron occurs and is allowed for, but the cross-section is very small.

The possible emission of a triton (n-α reaction) presents a problem. None of the intermediate states has the correct symmetry character to produce a triton because the latter has C = 3. On the other hand, a lot of kinetic energy is released when a triton is formed and the small departures from symmetry of the Hamiltonian may have an appreciable effect. This factor is unknown, therefore, and will be determined from the observed cross sections. An upper limit is given by applying the selection rule on spins so that only the doublet compound states may contribute. Moreover, only half the intensity in each doublet represents the two neutrons in a singlet state, so that the maximum value of g_f^i is 1/2 for the $^2P_{\frac{3}{2}}$ and $^2P_{\frac{1}{2}}$ states and zero for the quartet states.

The resulting values for cross sections are presented in a later section, where they are combined with the effects of direct interaction and, in the case of deuteron formation, with the pick-up process.

Direct Interactions

Direct interactions will be computed in the impulse approximation.

The energy of separation of the neutron from Li^6 is 5.663 Mev. That of the proton is 4.55 Mev but the proton has the additional Coulomb barrier of 1.07 Mev. Therefore, we shall assume that the distribution in momentum space is the same for neutron and proton in Li^6.

In our model, the neutron is bound to a potential which is constant within the radius R, eq.(1), and zero outside it. The wave-function is then determined by the values of R, the masses involved and the separation energy, plus the fact that it is a p-wave. As a function of radius we get:

$$\phi(r) = \frac{2.6446}{R^{3/2}} \frac{1}{\kappa r} \left(1 + \frac{1}{\kappa r}\right) e^{-\kappa r}$$

$$r \geq R \qquad \kappa = 4.790 \times 10^{12} \text{ cm}^{-1} \tag{8}$$

$$\phi(r) = \frac{3.7483}{R^{3/2}} \frac{1}{k_o r} \left(\frac{\sin k_o r}{k_o r} - \cos k_o r\right)$$

$$r < R \qquad k_o = 1.255 \times 10^{13} \text{ cm}^{-1}$$

The factors are chosen so as to normalize the complete radial intensity to unity.

The integrated intensity outside r = R is:

$$I(R) = \int_R^\infty \phi(r)^2 dr = \frac{6.994}{(\kappa R)^3} \left(\frac{1}{2} + \frac{1}{\kappa R}\right) e^{-2\kappa R} = 0.338 \tag{9}$$

The intensity inside R is therefore 0.662.

Let the axis of the p-orbital be the vector ($\cos\theta_o$, $\sin\theta_o$, 0). The complete spatial wave-function in the region $r > R$ is then

$$\psi(r,\theta,\phi) = \frac{2.6446}{R^{3/2}} \sqrt{\frac{3}{4\pi}} \; \frac{1}{\kappa r} \left| 1 + \frac{1}{\kappa r} \right| e^{-\kappa r}$$

$$\times (\cos\theta \cos\theta_o + \sin\theta \sin\theta_o \cos\phi) \tag{10}$$

The Fourier transform of $\psi(r, \theta, \phi)$, expressed as a function of the wave-number $\vec{k'}$ is,

$$\chi(\vec{k'}) = \frac{2.6446}{i\pi\kappa^2 k'} \left(\frac{3}{2R^3}\right)^{1/2} e^{-\kappa R} \cos\theta_1 \tag{11}$$

$$\times \left\{ \frac{\sin k'R}{k'R} + \frac{k'\kappa \sin k'R - \kappa^2 \cos k'R}{\kappa^2 + k'^2} \right\}$$

where θ_1 is the angle between k' and the axis of the p-orbital.

The intensity distribution of wavenumber $|k'|$ is the integral of $|\chi|^2$ over angles, viz.,

$$N_1(k')\, dk' = \frac{6.994}{(\kappa R)^3} \frac{e^{-2\kappa R}}{\kappa} \frac{2}{\pi}$$

$$\times \left\{ \frac{\sin k'R}{k'R} + \frac{k'\kappa \sin k'R - \kappa^2 \cos k'R}{\kappa^2 + k'^2} \right\}^2 dk' \tag{12}$$

An analogous transformation applies in the region $r < R$, where the kinetic energy is positive and rather large (38.86 Mev = E_o). Owing to the large kinetic energies involved in collisions taking place at

$r < R$, the cross sections are small and comparatively crude approximations may be tolerated. In particular, we shall say that the nucleons in p-orbitals always have the wave-number k_o, i.e., corresponding to eq. (12) we have, in the core,

$$N_2(k') = 0.662\,\delta(k' - k_o) \tag{13}$$

Under these assumptions, interference terms can be ignored between the two regions. The total cross section for direct interaction is then

$$\bar{\sigma} = \frac{1}{2}\int \left[N_1(k') + N_2(k')\right] \left[\sigma_{np}(k,k',\theta) + \sigma_{nn}(k,k',\theta)\right] dk'\, d\cos\theta \tag{14}$$

where θ is the angle between the incoming wave-number vector $\vec{k}$ and $\vec{k}'$. The factor $\frac{1}{2}$ forms the average over these angles.

Observed values of the neutron-proton cross section have been fitted to an empirical function of energy by J. Gammel (unpublished). In the region above $E_{lab} = 1$ Mev, the result, expressed in barns, is essentially

$$\sigma_{np} = 11.90 \left[\frac{1}{1.75 + E_{lab}} - \frac{1}{153 + E_{lab}}\right] \tag{15}$$

Gammel also deduced a similar expression for the singlet collisions alone. We assume the neutron-neutron collisions to follow the same law as singlet n - p collisions except that a factor <u>four</u> must be applied because the neutron wave is symmetric to exchange. This leads to

$$\sigma_{nn} = 9.58 \left[\frac{1}{0.136 + E_{lab}} - \frac{1}{77.7 + E_{lab}}\right] \tag{16}$$

In eqs. (15) and (16), E_{1ab} is the energy of the neutron beam when the target (proton) is at rest. We wish to apply the expressions to a target nucleon with wave-number $\vec{k}'$. This corresponds to a momentum in the lab system $M\,\vec{v}_1$. Let the momentum of the bombarding neutron be $M\,\vec{v}_o$. Then the "laboratory" energy corresponding to their collision is

$$E_{1ab} = \frac{1}{2M} \left| M\,\vec{v}_o - M\,\vec{v}_1 \right|^2 \tag{17}$$

Now $M\,\vec{v}_1 = \hbar\,\vec{k}'$, since M is 6/5 the reduced mass in the orbital and $\vec{v}_1$ is 5/6 the relative velocity. It is convenient, therefore, to define a new wave-number $\vec{p}$,

$$\hbar\,\vec{p} = M\,\vec{v}_o = \frac{7}{6}\hbar\,\vec{k} \tag{18}$$

where $\vec{k}$ is the reduced wave-number of the collision used hitherto. Then

$$E_{1ab} = \frac{\hbar^2}{2M}\ (p^2 + k'^2 - 2\,p\,k'\cos\theta) \tag{19}$$

Before substituting into eqs. (14), (15),(16) it is desirable to replace the variable k' by

$$x = k'\,R \tag{20}$$

and define

$$a \equiv p\,R \tag{21}$$

If E_n be the energy, in the lab, of the neutron bombarding Li^6,

$$E_n = \frac{\hbar^2}{2M} p^2 = \frac{\hbar^2}{2MR^2} a^2$$

$$a = 0.5846 \sqrt{E_n} \qquad (E_n \text{ in Mev})$$

Each cross section term in eqs. (15), (16) is of the form

$$\frac{\sigma}{\epsilon + E_{lab}}$$

Let $\epsilon = 2.926\, \alpha^2$; we then have the forms

$$\frac{\sigma}{2.926\left[\alpha^2 + a^2 + x^2 - 2\, a\, x \cos\theta\right]} = \frac{\sigma'}{\alpha^2 + a^2 + x^2 - 2\, a\, x \cos\theta}$$

Hence, in the new variables, a, x,

$$\sigma_{np} = 4.067 \left[\frac{1}{.598 + a^2 + x^2 - 2\, a\, x \cos\theta} - \frac{1}{52.3 + a^2 + x^2 - 2\, a\, x \cos\theta}\right] \qquad (22)$$

$$\sigma_{nn} = 3.274 \left[\frac{1}{.0465 + a^2 + x^2 - 2\, a\, x \cos\theta} - \frac{1}{26.5 + a^2 + x^2 - 2\, a\, x \cos\theta}\right]$$

Now, eq.(12) may be written

$$N_1\,(x)\,dx = \frac{6.994}{(\kappa R)^4}\,\frac{2}{\pi}\,e^{-2\kappa R}$$

$$\times \left\{\frac{\sin x}{x} + \frac{\kappa R\, x \sin x - (\kappa R)^2 \cos x}{x^2 + (\kappa R)^2}\right\}^2$$

or,

$$N_1(x) = \frac{6.994}{(\kappa R)^4} \frac{1}{\pi} e^{-2\kappa R} \tag{23}$$

$$\times \quad \mathrm{Re} \left\{ \frac{(\kappa R)^2 + 2\kappa R}{x^2 + (\kappa R)^2} + \frac{1}{x^2} - e^{2ix} \left(\frac{1}{x} + \frac{\kappa R}{x + i\kappa R} \right)^2 \right\}$$

The integral of $N_1(x)$ from $x = 0$ to ∞ can be written as $\frac{1}{2}$ the integral from $-\infty$ to $+\infty$ and evaluated by Cauchy's theorem. The only pole enclosed in the form, eq. (23), by completing the contour around the positive half plane is at $x = i\kappa R$, and it is readily seen that the result is the total intensity outside $r = R$, eq. (9), as is to be expected.

In order to apply the same method in evaluating eq. (14), we note that the average over angles of the cross sections, eq. (22), has the form

$$\bar{\sigma} = \frac{\sigma'}{4\,a\,x} \ln \frac{\alpha^2 + (a + x)^2}{\alpha^2 + (a - x)^2}$$

These are even functions of x, each of which have four branch points in the complex plane. In the positive half-plane one of the branch points comes from the numerator of the argument of the logarithm, the other from the denominator. These are connected by a cut, therefore, and there is no accumulation of phase on completing a contour which encloses the cut. The integral in eq. (14) which involves $N_1(x)$ can then be evaluated, as before, by taking the residue at $x = i\kappa R$. The result of the integration is then:

$$\bar{\bar{\sigma}} = \frac{687}{a\kappa R}\left[\tan^{-1}\frac{2\,a\,\kappa\,R}{.598 + a^2 - (\kappa R)^2} - \tan^{-1}\frac{2\,a\,\kappa\,R}{52.3 + a^2 - (\kappa R)^2}\right]$$

$$+ \frac{553}{a\kappa R}\left[\tan^{-1}\frac{2\,a\,\kappa\,R}{.0465 + a^2 - (\kappa R)^2} - \tan^{-1}\frac{2\,a\,\kappa\,R}{26.5 + a^2 - (\kappa R)^2}\right]$$

$$+ \frac{673}{a_o a}\ln\left[\frac{.598 + (a_o + a)^2}{.598 + (a_o - a)^2}\;\frac{52.3 + (a_o - a)^2}{52.3 + (a_o + a)^2}\right]$$

$$+ \frac{542}{a_o a}\ln\left[\frac{.0465 + (a_o + a)^2}{.0465 + (a_o - a)^2}\;\frac{26.5 + (a_o - a)^2}{26.5 + (a_o + a)^2}\right] \quad \text{millibarns.}$$

Here a_o is the parameter corresponding to E_o, $a_o = k_o R = 3.34$. The numerical results are shown in Table V.

The ratio of σ_{comp}, namely $\sum_i \sigma_i^*$, eq. (2), to σ_{dir} is seen to be quite constant. This is of considerable help in estimating the direct contributions to cross sections for events which may also go by way of the compound nucleus; for then we can determine a single factor to represent the fraction of overlapping. Assuming that fraction to be zero we can estimate the fraction of direct interactions which occur in p-wave collisions (with the Li^6) by multiplying $\sigma_{comp}/\sigma_{dir}$ with 9/5. The result appears as σ_p/σ_{dir} in the table. Since, in the energy range involved, collisions should be mostly in the S and P states we expect this number to be near 3/4. The table bears out this expectation, and this means that for reactions which can go both ways we must discount 5/9 of the p-wave part of the direct interaction as having already been accounted for.

TABLE V

Total Cross Sections for Direct Interaction (mb)

E_n (Mev)	5.67	6.80	9.07	11.34	13.61
			$r > R$		
σ_{np}	485	416	322	263	221
σ_{nn}	417	353	267	214	176
			$r < R$		
σ_{np}	198	201	206	210	215
σ_{nn}	150	153	160	167	173
σ_{dir}	1250	1123	955	854	785
σ_{comp}	508	465	395	343	303
$\sigma_{comp}/\sigma_{dir}$	.406	.414	.414	.402	.386
σ_p/σ_{dir}	.731	.745	.745	.724	.695
f_{sym}	.248	.250	.250	.247	.244

The last line in Table V, labelled f_{sym}, pertains to n,α reactions, which we now consider. Since kinetic energy is released in this process we assume that whenever statistical factors are favorable the n - α, T reaction takes place.

The first such factor to consider comes from the requirement that the collision be in a state with symmetry quantum numbers, C, S, T, which are proper for a triton plus an alpha-particle. These are the same as for the ground state of Li^7 ($C = 3$, $S = T = \frac{1}{2}$). We pointed out above that these states play no role in formation of the compound nucleus because they are too far from the region of excitation. We are not concerned, therefore, with orthogonality to the compound states (since none of these have the correct symmetry).

Calculations on the p^3-configurations show that 5/18 of all p-wave collisions have the symmetry required. A similar calculation for s-wave (and d-wave) collisions show that 1/6 are correct. The value of f_{sym} in Table V is then 5/18 of σ_p/σ_{dir} plus 1/6 $(1 - \sigma_p/\sigma_{dir})$. It is seen that about $\frac{1}{4}$ of the symmetry states are appropriate.

The second factor to be considered represents the chance that the third nucleon be close enough to the point of collision that a triton may be formed. We take this distance to be the radius of the nucleon intensity in H^3 (which is different for neutron and proton because their separation energies differ). If the separation energy be E_3, the extension of the wave-function beyond the range of the potential well is

$$R_3 = \hbar \Big/ \sqrt{\frac{4}{3} M E_3} \tag{24}$$

This has the value $R_3^n = 1.92$ fermi for the neutron and $R_3^p = 2.24$ fermi for the proton. Now, if we assume the effective radius for formation of a triton by a proton to be the "Coulomb" radius (see Blatt and Weisskopf, "Theoretical Nuclear Physics," Wiley, New York, 1952, p. 204):

$$R_c = 2.26 \times 10^{-13} \text{ cm} \tag{25}$$

then that for the neutron intensity should be less by $\frac{1}{2}$ $(R_3{}^p - R_3{}^n) =$ 0.16 fermi. Hence, when the neutron is the third particle, i.e., the collision takes place with the proton, we use

$$R_c{}^{(n)} = 2.10 \times 10^{-13} \text{ cm} \tag{26}$$

We estimate the probability that the third particle be within the required volume by multiplying the square of the nucleon density with the volume of the sphere, radius R_c or $R_c{}^{(n)}$, and integrating. Let us call the integral over the density squared $\overline{I^2}$. Since the p-orbitals of the two nucleons in Li^6 have the same dependence upon angle, inside $r = R$ we have

$$\overline{I_2{}^2} = \left(\frac{3 \times .662}{4\pi R^3}\right)^2 3 \int_0^R \cos^4\theta \, d\cos\theta \, r^2 \, d\, r \, d\, \phi$$

$$= .2628 \frac{3}{4\pi R^3} \tag{27}$$

Multiplying by the volumes of the spheres with radii given in eqs. (25) and (26) we get the respective geometrical factors. We wish to apply them, however, to cross sections given in Table V which contain the intensity factor, .662. Dividing by the latter we obtain

$$G_p = .2429 \tag{28}$$

$$G_n = .1953 \tag{29}$$

When r > R we have

$$\overline{I_1^{\,2}} = \left(\frac{3}{4\pi}\right)^2 \int_R^\infty \phi(r)^2 \, r^2 \, dr \, \cos^4\theta \, d\cos\theta \, d\phi \tag{30}$$

$$= \frac{4\pi}{5}\left(\frac{3}{4\pi}\right)^2 \frac{(6.994)^2}{\kappa^3 R^6} \int_{\kappa R}^\infty \left(1 + \frac{1}{x}\right)^4 e^{-4x} \frac{dx}{x^2}$$

$$\cong 0.2426 \; \frac{3}{4\pi R^3} \; I(R) \tag{31}$$

The corresponding geometrical factors will be written G' and are

$$\begin{aligned} G'_p &= 0.1485 \\ G'_n &= 0.1192 \end{aligned} \tag{32}$$

The final result for the n - α cross section by direct interaction is

$$\begin{aligned} \sigma_{n\alpha,\,dir} = f_{sym} \Big[& G_n \sigma_{np}(r < R) + G_p \sigma_{nn}(r < R) \\ & + G'_n \sigma_{np}(r > R) + G'_p \sigma_{nn}(r > R) \Big] \end{aligned} \tag{33}$$

where, as mentioned, the $\sigma(r\ldots)$ are the values given in Table V.

The final computation is presented in a later section, in which the contributions are summarized.

Direct Interactions Which Have Thresholds

The approximations used to estimate direct interactions will now be applied to the n - D and n- 2n reactions. These differ from an n - α process in several ways including having threshold energies. One of the other differences is that all collisions have an acceptable symmetry, since there is no symmetry problem for knocking out a neutron or proton singly and since the ground state of Li^6 has the same symmetry as the triplet deuteron. On the other hand, this fact requires that we exclude the effect of collisions which can form compound states, i.e., 5/9 of the p-waves. Also, the geometrical factor for the formation of the deuteron must be estimated differently because the radius of a deuteron is as large or larger than that of Li^6.

We consider first the threshold problem. For an n - 2 n, p reaction the energy transfer to the Li^6 neutron must be at least $E_1 = 5.66$ Mev, and considering the existence of the Coulomb barrier, essentially the same value applies when the collision is with the proton. If the energy transfer is sufficient to remove the nucleon, it is assumed it will do so, unless it is simultaneously possible to remove a deuteron or triton. In the latter event, it is assumed that the heavier particle may be formed.

Using the notation of eq. (17) *et seq.*, the velocity of the center of mass of the two nucleons which are making the direct collision is $\frac{1}{2}(\vec{v}_o + \vec{v}_1)$ and their relative velocity of collision, $\vec{v}_o - \vec{v}_1$. Assuming isotropic scattering in this system, the energy of a scattered nucleon in the laboratory is

$$E_{scat} = \frac{1}{8} M \left\{ \vec{v}_o + \vec{v}_1 + \vec{u}_o \left| \vec{v}_o - \vec{v}_1 \right| \right\}^2 \tag{34}$$

where $\vec{u}_o$ is any unit vector. This energy must be greater than E_1 plus the initial kinetic energy of the Li^6 nucleon, $\frac{1}{2} M v_1^2$, plus that of the recoiling mass of five nucleons, $\frac{1}{10} M v_1^2$.

$$E_{scat} \geq E_1 + \frac{3}{5} M v_1^2 \tag{35}$$

Expressed in ergs,

$$E_1 = \frac{\hbar^2 \kappa^2}{2\mu} = \frac{3}{5} \frac{\hbar^2 \kappa^2}{M}$$

and from $M v_1 = \hbar k'$ we get

$$E_{scat} \geqslant \frac{3 \hbar^2}{5M} (\kappa^2 + k'^2) \tag{36}$$

Hence,

$$\frac{1}{8}\left\{ \vec{p} + \vec{k'} + \vec{u}_o \left| \vec{p} - \vec{k'} \right| \right\}^2 \geqslant \frac{3}{5} (\kappa^2 + k'^2).$$

Let Θ be the angle between $\vec{u}_o$ and $\vec{p} + \vec{k'}$. We then find

$$\cos \Theta \geqslant \frac{\frac{12}{5} \kappa^2 + \frac{7}{5} k'^2 - p^2}{\sqrt{(p^2 + k'^2)^2 - 4 p^2 k'^2 \cos^2 \theta}} \tag{37}$$

where θ is the angle between $\vec{p}$ and $\vec{k'}$, as above.

In the notation of eqs. (20), (21) and defining

$$\beta = \kappa R = 1.275$$

the lower limit of $\cos \Theta$ is given by

$$\cos \Theta_l = \frac{12\beta^2 + 7 x^2 - 5 a^2}{5\sqrt{(a^2 + x^2)^2 - 4 a^2 x^2 \cos^2 \theta}} \tag{38}$$

if this be less than unity.

One can readily check that the right hand side of eq.(38) is greater than unity when

$$x^2 > 5 a^2 - 6\beta^2 \equiv 6 c^2 \tag{39}$$

which defines the quantity $c = c(a, \beta)$.

When $x^2 < 6\ c^2$ the $\cos^2\theta$ in eq.(38) is restricted by $\cos\theta < 1$ to obey

$$\cos^2\theta < \frac{6}{25}\ \frac{x^2 + \beta^2}{a^2\ \ x^2}\quad (6\ c^2 - x^2) \tag{40}$$

The right hand side of eq.(40) becomes unity when $x^2 = c^2$ (hence $x^2 + \beta^2 = \frac{5}{6}\ a^2$) so that when $x^2 < c^2$ the limits on $\cos\theta$ are ± 1.

The fraction of 4π solid angle through which the recoiling nucleon has enough energy to escape is then

$$F(x,\ a,\ \beta,\ \theta) = \frac{1}{2}\left[1 - \cos\theta_{\ell}\right] \tag{41}$$

This may be written:

$$F(x,\ a,\ c,\ \theta) = \frac{1}{2}\left[\ 1 - \frac{7\ x^2 + 5\ a^2 - 12\ c^2}{5\sqrt{(a^2 + x^2)^2 - 4\ a^2\ x^2\ \cos^2\theta}}\right] \tag{42}$$

The expectation values for threshold cross sections therefore have the form

$$\bar{\bar{\sigma}} = \left\{\int_0^c N\ (x)\ dx \int_{-1}^{1} d\cos\theta + \int_c^{\sqrt{6}\,c} N\ (x)\ dx \int_{-\Lambda}^{\Lambda} d\cos\theta\right\} \times$$

$$\frac{\sigma_o}{\alpha^2 + a^2 + x^2 - 2\ a\ x\cos\theta}\ \frac{1}{2}\left[1 - \frac{7\ x^2 + 5\ a^2 - 12\ c^2}{5\sqrt{(a^2 + x^2)^2 - 4\ a^2x^2\cos^2\theta}}\right] \tag{43}$$

where Λ stands for

$$\Lambda = \frac{\sqrt{6\ (x^2 + \beta^2)\ (6\ c^2 - x^2)}}{5\ a\ x}\ .$$

Let

$$q = \alpha^2 + a^2 + x^2$$

$$s = \frac{2}{5}\sqrt{6\,(x^2 + \beta^2)(5\,a^2 - 6\beta^2 - x^2)}$$

$$u = \sqrt{\alpha^4 + 2\alpha^2\,(a^2 + x^2)}$$

$$w = \frac{1}{5}\,(7\,x^2 + 12\beta^2 - 5\,a^2)$$

Then the averaged cross section contains the forms, after integrating eq.(43) over θ:

$$\bar{\bar{\sigma}} = \sigma_o \int_0^c \frac{N(x)}{4\,a\,x}\left[\ln\frac{q + 2\,a\,x}{q - 2\,a\,x} - 2\,\frac{w}{u}\tan^{-1}\frac{2\,a\,u\,x}{q(a^2 - x^2)}\right]dx$$
$$+ \;\sigma_o \int_c^{c\sqrt{6}} \frac{N\,(x)}{4\,a\,x}\left[\ln\frac{q + s}{q - s} - 2\,\frac{w}{u}\tan^{-1}\frac{u\,s}{q\,w}\right]dx \qquad (44)$$

In application, eq.(44) pertains to the four values of α^2 indicated by eq.(22) and the two forms of N (x), eqs.(12) and (13). The integral over $N_2(x)$ is readily carried out, of course, and it results in a negligible contribution to the n - 2 n cross section (3 mb at 13.6 Mev, and that without allowing for competition with the n - D reaction). Similarly, the result in this region for the n - D reaction is very small so we shall merely add it to that for r > R even though in principle the corrections for orthogonality, etc., are not the same for the two regions.

In the region $r > R$, eq.(44) was integrated by a digital computer. The result of integration is not yet the expected value of the cross section, however, because we have still to consider effects of spatial correlation (for the n - D process), orthogonality to compound states and competition.

We have noted previously that the neutron and proton in Li^6 are in the same substate of a p-orbital and that the radius of the deuteron (intensity) is large. This indicates that an appropriate estimate of the geometrical probability is given by allowing the formation of a deuteron if both nucleons (being outside $r = R$) are on the same side of the α-core at the instant of collision. This gives a factor $\frac{1}{2}$.

With respect to orthogonality to the states forming the compound nucleus we have seen that about 3/4 of all collisions appear to be in P-states. Thus as a rough estimate we should discount $\frac{5}{9} \times \frac{3}{4} = \frac{5}{12}$, or about 40% of the integral. However, we are specifically interested in the fraction of collisions which occur outside $r = R$ and which are in P-waves with respect to the Li^6. We shall estimate this fraction by intersecting the radial wave-function, eq.(8), with two cylinders, concentric with $\phi(r)$ and of radii $\lambda\!\!\!^{-}$ and $2\lambda\!\!\!^{-}$. The intensity of $\phi(r)$ lying between these cylinders will be considered to be effectively in P-wave collisions. Inside $\lambda\!\!\!^{-}$ they will be s-waves and outside $2\lambda\!\!\!^{-}$, higher L-values than <u>one</u>.

Since we are concerned with the intensity at $r > R$ the fraction inside $\lambda\!\!\!^{-}$, which is denoted by ϕ_o, can be estimated as the ratio of surfaces

$$\phi_o = \frac{2\pi \lambda\!\!\!^{-2}}{4\pi R^2} = \frac{7}{12\ a^2} \tag{45}$$

where we use the reduced energy for the n - Li^6 system and eq.(21).

Outside the cylinder of radius $2\lambda\!\!\!^{-}$ we have the fraction ϕ_1 given by

$$\phi_1 = \frac{1}{I\ (R)} \int_{2\lambda\!\!\!^{-}\csc\theta}^{\infty} \frac{1}{2} \int_0^{\pi} \phi(r)^2\ r^2\ dr \sin\theta\ d\theta \tag{46}$$

$$\phi_1 = \frac{1}{4}\ \frac{6.994}{(\kappa R)^3\ I(R)} \int_0^{\pi} \left(1 + \frac{\sin\theta}{\kappa\lambda\!\!\!^{-}} \right) e^{-4\kappa\lambda\!\!\!^{-}\csc\theta} \sin\theta\ d\theta$$

The integrand in eq. (46) has a strong maximum at $\theta = \frac{\pi}{2}$ (in the range of energy of interest) and is closely approximated by a Gaussian of equal amplitude and second derivative. Integrating the Gaussian we get,

$$\phi_1 = 2.495 \left(1 + \frac{1}{\kappa\lambda\!\!\!^{-}} \right) e^{-4\kappa\lambda\!\!\!^{-}}\ \frac{1}{2} \sqrt{\frac{2\pi(1 + \kappa\lambda\!\!\!^{-})}{4\ (\kappa\lambda\!\!\!^{-})^2 + 5\kappa\lambda\!\!\!^{-} + 2}} \tag{47}$$

$$\kappa\lambda\!\!\!^{-} = \frac{7}{6}\ \frac{\kappa R}{a} = 1.803 \sqrt{\phi_o}$$

Now $1 - \phi_o - \phi_1$ is the fraction of the region which is effective in p-wave collisions. The n - 2 n and n - D cross sections have therefore to be multiplied by a factor which is given by (assuming that the average cross section is representative of that in the p-wave region)

$$\phi_{eff} = \frac{1}{9} (4 + 5\phi_o + 5\phi_1) \tag{48}$$

Finally, one must correct for the competition. Since this is influenced by a number of factors which are more conveniently considered in a summary, the subject will be finalized in a later section.

In this section we note one factor in deciding the relative probability of emitting a single nucleon or a deuteron or triton is the density of states in the phase space of the emitted system. For two particles this may be written

$$\phi_{ph} = p^2 \frac{dp}{dE} \Delta\Omega = Mp\Delta\Omega \tag{49}$$

where $\Delta\Omega$ is the fraction of solid angle which is available and M and p are the reduced mass and momentum of the system. The solid angle factor is already contained in $\bar{\bar{\sigma}}$, eq. (44).

<u>Effect of the Pick-up Reaction</u>

The considerations made, so far, about direct interactions concern the possibility that one of the nucleons in a p-orbital recoil from a collision with sufficient energy to escape either singly, or if its partner be nearby, in a stable deuteron. Under much more restricted conditions all three nucleons escape as a triton.

We have still to consider the cross section for the orbital proton to form a deuteron with the bombarding neutron without going through the compound state. This is the pick-up process well known to be of significance at high energy in heavier nuclei. It turns out to be of importance also in our energy range of neutrons on Li^6, because of the large size and low momenta of the exponential tail of the proton wave.

The theory of this cross section has been presented by Chew and Goldberger,[3] who used the Born approximation but avoided the error to which this leads in the neutron-proton interaction matrix element by using the wave equation for the deuteron.

Let M_1 be the reduced mass of the neutron on Li^6, and M_2 that of the deuteron-He^5 system. In Born approximation, the differential cross section for going from wave number $\vec{k}$ of the incident system to $\vec{K}$ of the latter is

$$d\sigma' = \frac{3}{4} \frac{2\pi M_1}{\hbar^2 k} |H|^2 \rho_E d\Omega$$
$$\rho_E = \frac{p^2}{8\pi^3\hbar^3} \frac{dp}{dE} = \frac{M_2 K}{8\pi^3\hbar^2} \tag{50}$$

The factor $\frac{3}{4}$ gives the a priori probability of a triplet state. In the following we shall deal with averages over angles so we replace $d\Omega$ by 4π and get

$$\sigma' = \frac{45}{49\pi} \frac{M^2}{\hbar^4} \frac{K}{k} |H|^2 \tag{51}$$

Let $\varphi_D(\vec{r})$ be the wave function of the deuteron in the coordinates $\vec{r} = \vec{r}_n - \vec{r}_p$; then H may be written (cf. ref. 3)

$$H = \int e^{i(\vec{K}-\vec{k})\cdot\vec{r}_p} \psi_f \psi_i^* \, d\vec{r}_p d\zeta \int e^{i(\vec{k} - \frac{1}{2}\vec{K})\cdot\vec{r}} V(r)\varphi_D(\vec{r}) d\vec{r} \tag{52}$$

where ψ_i and ψ_f are initial and final states of the nuclei and ζ stands for all their coordinates except that of the proton. Assuming the integral over $d\zeta$ to be unity, the first integral in eq.(53) has been found above and is just $(2\pi)^{3/2} \chi\left(|\vec{K} - \vec{k}|\right)$, eq.(11).

The second integral in eq.(52) was transformed and shown to be essentially

$$\int \cdots d\vec{r} = 4\pi \frac{\hbar^2}{M} \sqrt{\frac{\alpha}{2\pi(1 - \alpha\rho)}} \tag{53}$$

where ρ is the effective range of the triplet forces and α is the reciprocal radius of the deuteron wave function

$$\alpha = \sqrt{\frac{MB_D}{\hbar^2}} = 2.314 \times 10^{12} \text{ cm}^{-1}$$

$$\rho = 1.6 \times 10^{-13} \text{ cm.} \tag{54}$$

(B_D is the binding energy of the deuteron.)

The form of $|H|^2$ is then

$$|H|^2 = \frac{64\pi^4\alpha}{1 - \rho\alpha} \frac{\hbar^4}{M^2} \frac{N_1(|\vec{K}-\vec{k}|)}{|\vec{K} - \vec{k}|^2} \frac{3 \cos^2 \varphi_1}{4\pi} \tag{55}$$

with $N_1(k')$ given by eq.(12).

Following ref. (3), let B_{if} be the energy difference between initial and final nuclear levels. In our case, B_{if} is 4.653 Mev when the He^5 is left in the quartet state, which is 2/3 of the time, and 3 or 4 Mev greater when it is left in the doublet state (probability = 1/3). The magnitude of K is determined by

$$\frac{\hbar^2 K^2}{2M_2} - B_D = \frac{\hbar^2 k^2}{2M_1} - B_{if} \tag{56}$$

Averaging over the orientations of the p-orbital in Li^6 the factor, $3 \cos^2 \varphi_1$, in eq.(55) becomes unity. Next we average over φ_2, the angle between $\vec{K}$ and $\vec{k}$ in

$$|\vec{K} - \vec{k}|^2 = K^2 + k^2 - 2Kk \cos \varphi_2 \tag{57}$$

by computing

$$\overline{N}(K,k) \equiv \frac{8\pi^3}{2R^3} \int \frac{N_1(|\vec{K} - \vec{k}|)}{|\vec{K} - \vec{k}|^2} \, d \cos \varphi_2 \tag{58}$$

for each k and corresponding values of K, viz., $K_{3/2}$ for the $He_{3/2}^5$ state and $K_{1/2}$ for the $He_{1/2}^5$; the resulting expression for the cross section is

$$\sigma' = \frac{90\,\alpha\,R^3}{49\pi(1 - \rho\alpha)} \left[\frac{2}{3}\,\frac{K_{3/2}}{k}\,\overline{N}(K_{3/2},k) + \frac{1}{3}\,\frac{K_{1/2}}{k}\,\overline{N}(K_{1/2},k) \right] \qquad (59)$$

As mentioned above, the effectiveness of the $P_{1/2}$ level of He^5 presents an uncertainty in the prediction. This level appears in scattering experiments to be very broad and centered about 4 Mev above the $P_{3/2}$ which is much sharper. In order to make the computations definite we assume that the $P_{1/2}$ level is also sharp and 4 Mev above the $P_{3/2}$ level. The integrations for $\overline{N}(K,k)$ and resulting cross sections are given in Table VI. The pick-up cross section σ_{pu} is computed by multiplying σ', eq. (59), with the orthogonality factors, ϕ_{eff} of eq. (48), and ϕ_α, the orthogonality to n - α channels given in eq. (61):

$$\sigma_{pu} = \phi_{eff}\,\phi_\alpha\,\sigma'. \qquad (60)$$

TABLE VI

Results for Pick-Up Cross Section (mb)

E_{lab} (Mev)	3.97	5.67	6.80	9.07	11.34	13.61
$\overline{N}(K_{3/2})$	15.33	12.54	10.52	7.76	6.09	5.07
$\overline{N}(K_{1/2})$			5.85	7.76	6.24	5.35
σ'	414	339	363	314	249	209
ϕ_α	.959	.956	.951	.944	.939	.935
ϕ_{eff}	.692	.640	.627	.629	.651	.683
σ_{pu}	275	207	216	186	152	133

DISCUSSION

From the points of view of relative magnitudes of cross sections and amount of information available, the most important reaction on Li^6 is $Li^6 + n \rightarrow \alpha + d + n$. The most complete experimental results have been presented by L. Rosen and L. Stewart[4] in LA-2643.

In the preceding pages three different mechanisms for producing this reaction have been studied and we now investigate their probable importance. The compound nuclear method leads to $\alpha + d + n$ in two ways: either the Li^{7*} emits a neutron going to a ^3D state of Li^6 which then emits the deuteron, or Li^{7*} emits the deuteron directly leaving He^5 in either its ground state or the excited, $P_{1/2}$, state. In making the computations on partial widths it was assumed that the $P_{1/2}$ level lay 4 Mev above the $P_{3/2}$ level of He^5. The positions of the ^3D levels of Li^6 are those assigned in Table I. Being lowest the $^3D_3$ contributes most except when selection rules prevent its being reached, as they do when $P_{1/2}$ intermediate states are excited. On the other hand, the statistical weights of these states are small.

In Table VII we give the results of the production of the n,dn reaction by the various methods and compare them with the experimental values. The latter are subject to estimated errors of $\pm$ 50 mb at high energy and $\pm$ 100 at the low end. The comparison is shown graphically in Fig. 1. Ignoring direct interaction for the moment, the sum of values from compound nuclear theory and from pickup fits quite well with experiment except at the lowest energies where theory is a little

TABLE VII

Results for $Li^6 n,dn$ Reaction

E_{lab} (Mev)	3.97	5.67	6.80	9.07	11.34	13.61
n,dn X section (mb)						
Observed	550	540	500	350	350	290
Compound	150	205	225	216	188	166
Pickup*	286	217	228	197	162	143
Max. Direct Int.	43	69	80	96	108	119
Comp.+ Pickup	436	422	453	413	350	309
Comp. with n,α	134	190	214	209	184	163
Add. $^3D_3$ contr.	130	107	77	41	29	25
Total	539	504	507	436	365	321

*These differ from σ_{pu} of Table VI because competition with n-α reaction is not yet included.

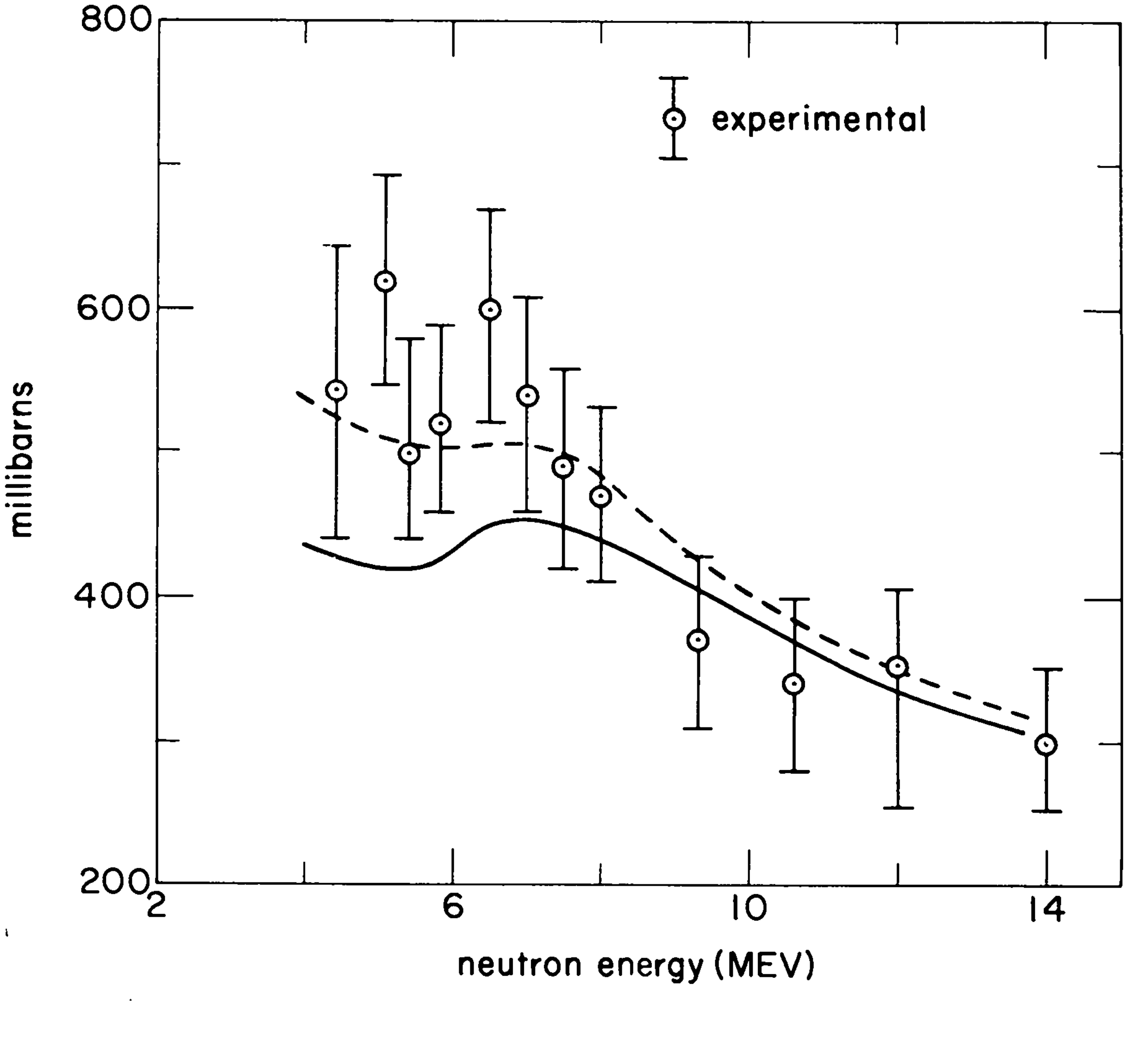

Fig. 1. Cross Section for n,dn Reaction on Li^6. The solid line is the result of theory, including the compound nuclei and pickup. If an additional mechanism excites the 2.2 Mev state, the observed cross section for the latter leads to the dashed curve.

low. The dip in the theoretical curve near 5 Mev comes from the loss of the effect of the $P_{1/2}$ state of He^5. Hence, if we should assume the effective position of that state to be lower than 4 Mev, the dip disappears and the fit improves.

It will be noted in Table VII that the energy dependence of these two parts is very different, the pickup cross section being relatively stronger at low energy. Therefore, if the pickup process can compete with formation of the compound state, i.e. ϕ_{eff}, eq. (48), is larger than we have assumed, the pickup contribution becomes larger and the compound part smaller.

In making these calculations it has been assumed that the n - α reaction does not take place through the compound states because the symmetry in space does not match. It is likely, however, that this symmetry (the quantum number C) is not strongly conserved, especially in the process of forming the triton for which the release of kinetic energy is high. This point is of minor importance to the n-d cross section, however, because the n - α effect is so small that whether it competes does not change final values very much. This correction is added later.

Also shown in Table VII are values for direct interaction under the assumptions made above, viz., if the collision took place in a state which is orthogonal to the compound state and the recoiling nucleon had sufficient energy to liberate a deuteron, and if the two nucleons were on the same side of the α-core at the time of collision, then the n-d reaction resulted. The assumptions are certainly optimistic because of the large geometric factor allowed and because the only requirement on the dynamics is that energy be conserved. Even so, the calculations shown in Table VII are rather small, at least at the lower energies. In fact the energy dependence is the opposite of what is required to improve the fit. This is interpreted to mean that the direct interaction (through recoil) should be ignored in the production of deuterons.

On the other hand, it does not seem justified to arbitrarily improve the fit to total cross section by increasing the pickup at the expense of the compound part. One reason for this is that the angular distributions of the deuterons coming from the two parts are quite different and there are measurements of differential cross section with which to compare. The theory shows that the compound nuclear angular distribution is almost spherical, whereas that of the pickup process, i.e., the integrand of eq. (58), is strongly peaked in the forward angles.

The experimental results of Rosen and Stewart are presented by them as differential cross sections for the emission of the particles into one of ten zones of equal solid angle. These results for deuterons have been fitted by least squares to quadratic functions of the cosine of the laboratory angle ($\cos \psi_2$) of emission relative to the direction of the incident neutron beam. The same process was followed in treatment of the theoretical results after transforming them to the laboratory frame. The calculations were made for a simple spherical distribution (in the C.M. system) and for the mixture with the pickup process represented in Table VII.

In Fig. 2 is given the comparison of results for the coefficients of $\cos \psi_2$. The upper curve comes from the theory presented above and the lower curve is what would result from a spherical distribution of deuterons if the total cross section were the same as the theory. It is seen that the experimental points fall mostly in the region between these curves, suggesting that some forward peaking exists but perhaps not as much as the theory predicts.

Departure from spherical symmetry is indicated also in the coefficient of $\cos^2\psi_2$. This is shown in Fig. 3. The experimental values are negative below 8 Mev, reflecting an enhancement in the intensity perpendicular to the beam axis. This can only be an effect of the compound nucleus, such as certain compound states dominating particular energy regions, and can not be expected to appear as markedly in a theory that has been averaged over all level positions. For example, the sign of the $\cos^2\psi_2$ term in the He^5,d reaction is different for a $^4P_{5/2}$ intermediate state than for the $^4P_{3/2}$ (the latter is negative). Also these coefficients are comparable to the constant term in magnitude.

Additional evidence of imperfections in the model appears in comparing the results on the cross section for formation of the 2.2 Mev state of Li^6. In our model, this is the $^3D_3$ state and is formed only through formation of a compound state. However, in Fig. 4 we see that the theory falls short of experiment and especially at low energy. This may be interpreted either as arising from the difference between a real nucleus and the model or as reflecting an additional mechanism for forma-

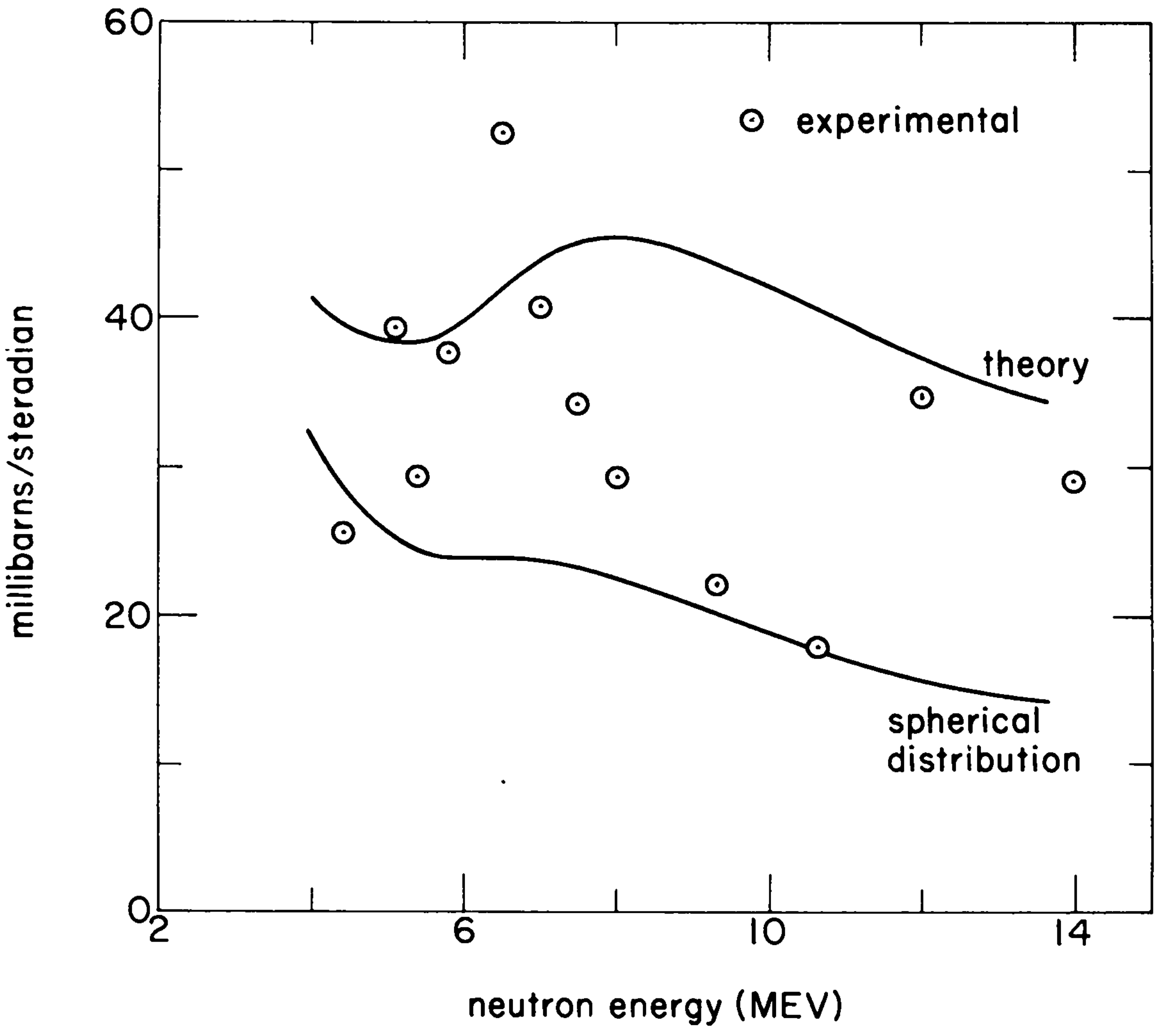

Fig. 2. Coefficient of Cosine ψ_2 in n,dn
Differential Cross Section.

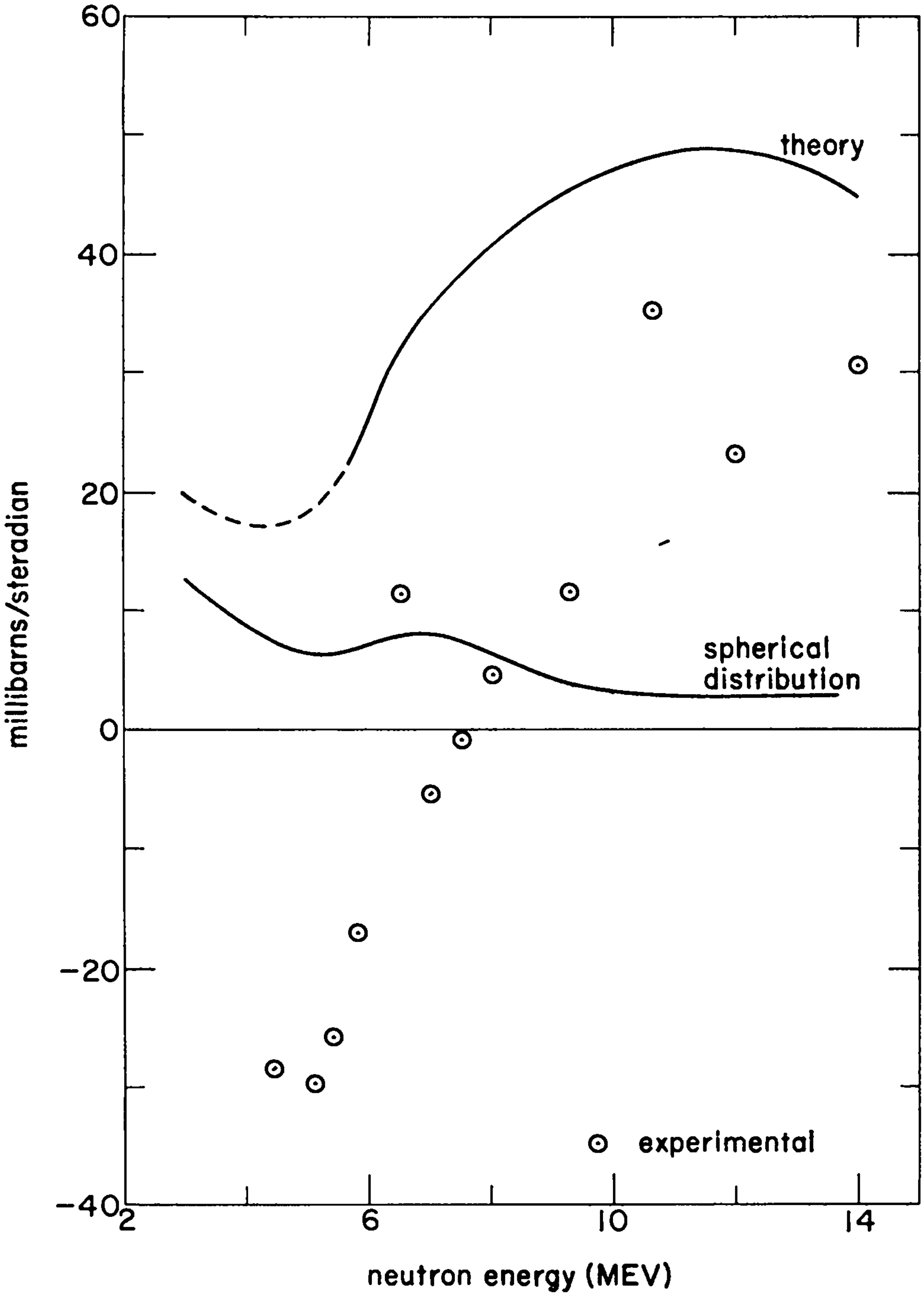

Fig. 3. Coefficient of $\cos^2\psi_2$ in n,dn Differential Cross Section.

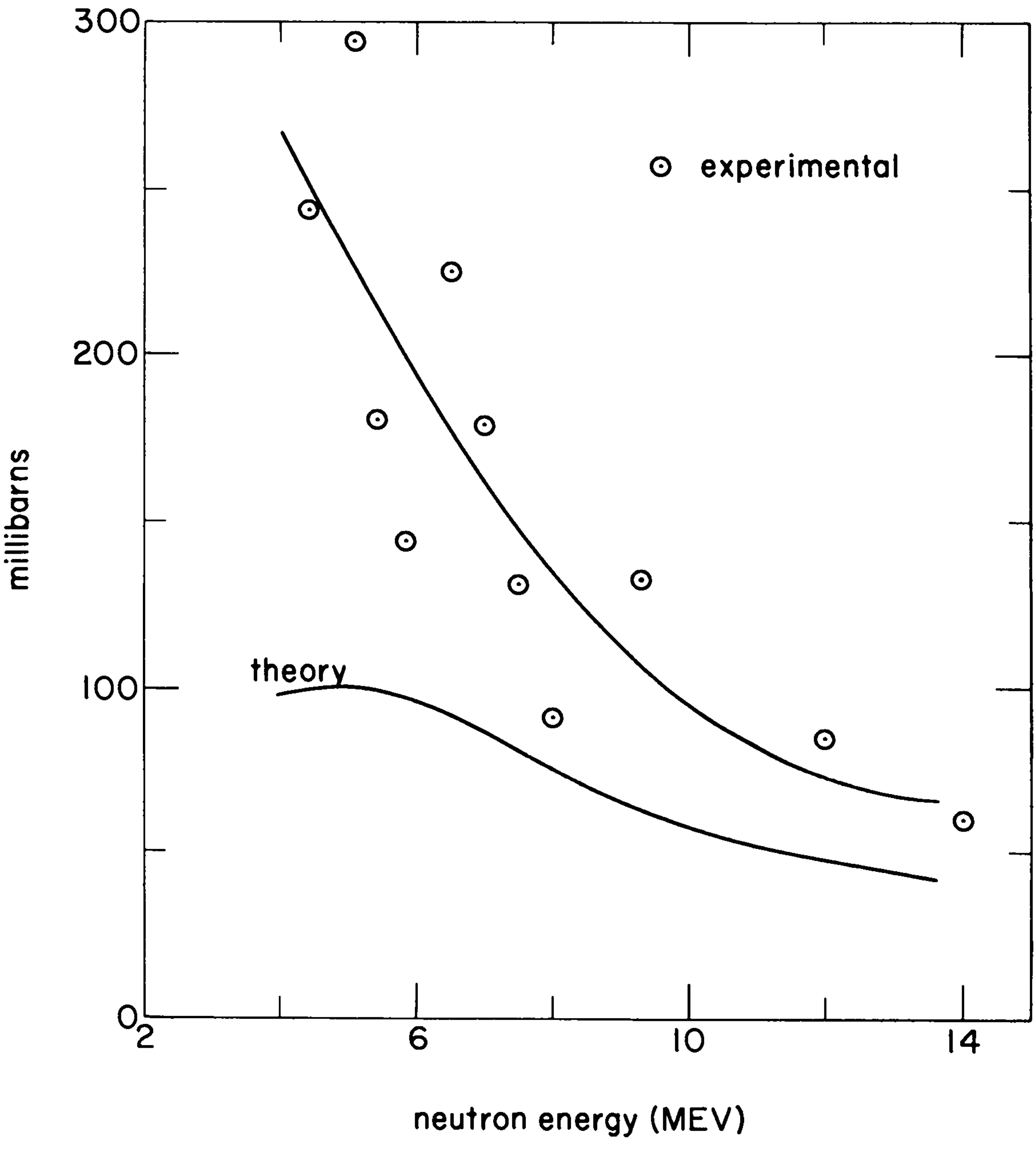

Fig. 4. Cross Section for Excitation of the 2.2 Mev ($^3D_3$) State of Li^6.

tion of the $^3D_3$ state, or both. In light of the apparent importance of the pickup process it seems likely that there is some direct excitation of Li^6 in states orthogonal to the compound states. This has the added feature that it represents an additional source of deuterons which has the correct dependence on energy to account for the difference between theory and experiment in total cross section, Fig. 1.

As mentioned above these results are obtained without allowing competition from the n,α reaction. The theory of the latter has several unknown factors in it, the main ones being the importance of symmetry and the influence of dynamic effects, other than energy, in its production through direct interaction. In Table VIII we show the maximum cross section the theory will allow for the compound nuclear process and for direct interaction. It will be noted that the sum of the two maximum

TABLE VIII

Results for n,α Reaction (mb)

E_{lab} (Mev)	3.97	5.67	6.80	9.07	11.34	13.61
$\sigma_{n\alpha,obs}$	109	70	54	38	31	27
Max. Comp. Nuc.	68.1	44.5	33.3	19.6	12.7	9.6
Max. Dir. Int.	65.3	48.3	44.6	39.3	35.6	33.3
$\sigma_{comp} + .53\,\sigma_{dir}$	103	70	57	40	32	27

contributions exceeds the experimental results at all energies and also that dependence upon energy is very different for the two processes. There must therefore be some optimum mixture of the two theoretical cross sections. If we determine the optimizing factors by least squares we find that the compound nuclear contribution should have a weight 1.16 and the direct process a weight .42. In order to remain consistent with the theoretical approach we shall assume that the compound process has weight unity and give the recoil method an average weight .53. This appears as $\sigma_{comp} + .53\ \sigma_{dir}$ in Table VIII, and the agreement with experiment is obviously satisfactory. The experimental points used to estimate $\sigma_{n\alpha,obs}$ were taken from BNL 324[5].

This result means that even though the symmetry quantum number, C, is 3 for the triton-alpha state and is 0 for the five compound states, the latter appear to be fully effective in producing a triton when the spin is right. It also means that triton emission competes with other processes and, in particular, with deuteron emission from the compound nucleus. The lower cross sections are shown in Table VII under n,dn X-section, Comp. with n,α. On the other hand, we have found evidence for an additional mechanism for the formation of the 2.2 Mev level and hence for the emission of deuterons. The estimated additional cross sections appear under n,dn X section, Add. $^3D_3$ contr. Adding these two to the pickup cross section, from Table VI, for each energy we get the result shown as Total. It is clear that this agrees very well with experiment. It is shown as the dashed curve in Fig. 1. If one does admit the addition, of unspecified origin, to formation of the $^3D_3$ state he widens the latitude in the interpretation of the angular distribution, of course.

Another source of information for comparison is the report of J. F. Barry[6], on the "Cross Section of the Reaction $Li^6(n,p)\ He^6$". So far as our theory is concerned, this reaction goes only through doublet, compound states. It is therefore small but greatly affected by the competition with the n,α reaction. In Fig. 5 are shown the experimental results and the two curves predicted by compound nuclear theory. The upper, solid curve is that predicted by the theory without emission of tritons, the lower, dashed curve is with competition. Even the solid curve falls below the observations, and in light of the results on the n,α cross section one would rather expect the lower curve to represent the contribution of the compound states. It follows that states orthogonal to these appear to contribute about an equal amount to the cross section. The obvious mechanism is one in which the incident neutron knocks the proton out but is itself caught in the ground state of He^6. This possibility has not been included here.

The remaining cross section of interest is for the n,p2n reaction. The contributions to this final product are several, viz., if Li^{7*} emits a neutron and leaves Li^6 in a P-state, the latter, being even, can not in turn emit a deuteron but will emit a neutron or proton (this is the only method by which the reaction occurs through ^4P states); if Li^{7*} goes to the ^1D state of Li^6, or to the proton leaving He^{6*}, or the singlet deuteron and He^5, or to Li^5 and the "dineutron", the result is

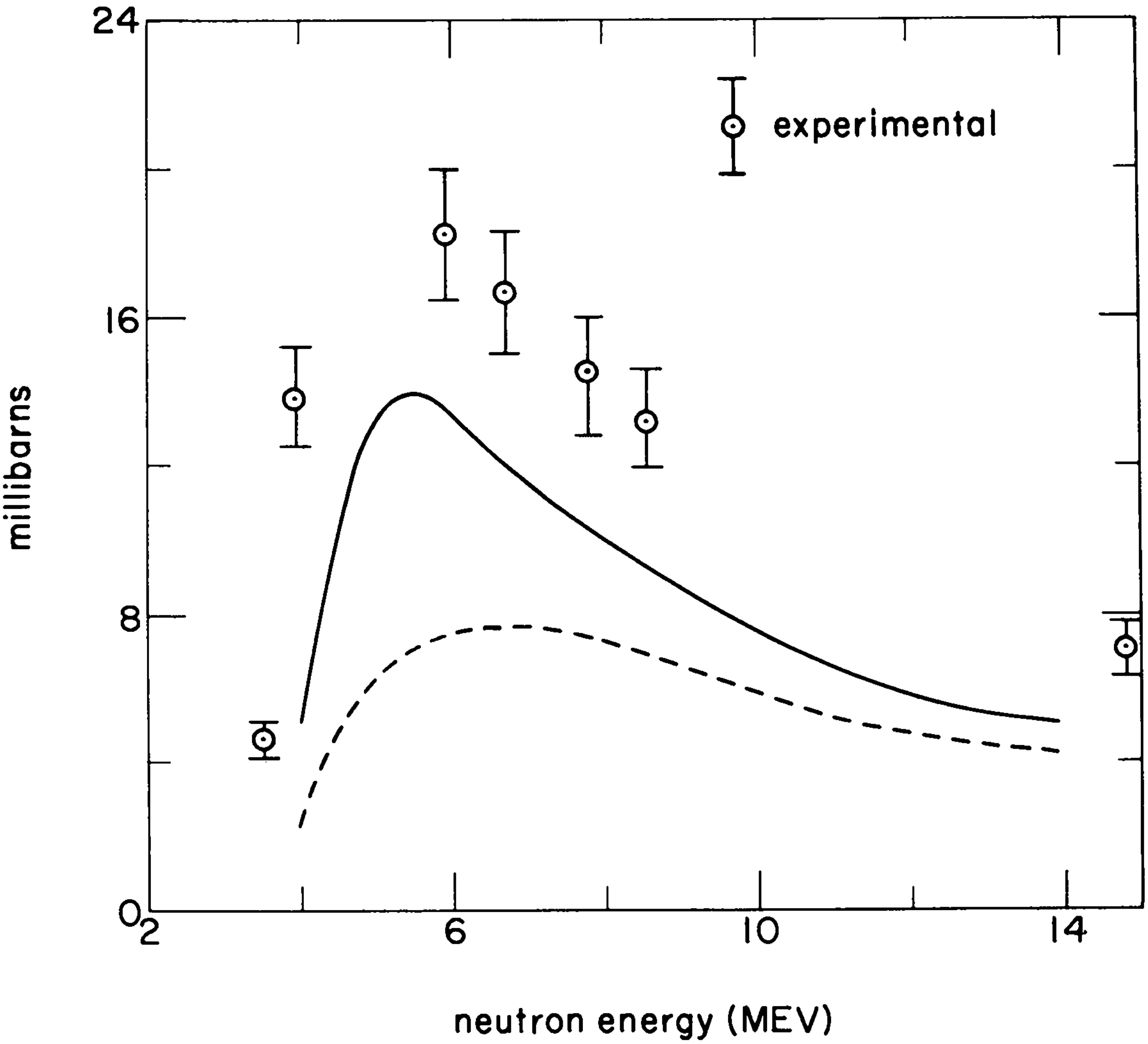

Fig. 5. Cross Section for n-p, He^6
Reaction on Li^6.

an n-2n reaction. The theoretical cross sections summed over these possibilities and allowing for full competition from the n - α reaction are shown in Table IX.

In addition to the compound nuclear mechanism, the n,2n process takes place directly through collisions in states orthogonal to these. We have found that this is apparently unimportant to the n-d reaction and of less than full importance to the n - α cross section. For the n,2n cross section, however, there are no estimates of proximity needed and no dynamic requirements other than that the recoiling nucleons have sufficient energy. Hence, except for reduction through competition, the direct interaction should be fully effective in the n,2n cross section.

Competition with the n - α process will take place only in that fraction of collisions which are favorable to formation of a triton. Practically all of the collisions which permit the ejection of a neutron are in the region $r > R$ and they are in states which are orthogonal to the compound states. The collisions which permit the formation of a triton are orthogonal because of the symmetry requirement. Collisions which are orthogonal to the compound states at $r > R$ present a cross section $\phi_{eff}\, \sigma(r > R)$, and of these the cross section for ejecting the nucleon is $\sigma_{n,2n}$. Now all collisions leading to tritons must occur among these same states, so that the probability that a collision which is right for ejection of a nucleon is also right for making the triton is

TABLE IX

Results for n,p2n Reaction (mb)

E_{lab}(Mev)	3.97	5.67	6.80	9.07	11.34	13.61
$\sigma_{n,2n,\ comp}$		.5	6.5	29.0	50.1	55.6
$\sigma_{n,2n,\ max,dir}$			7.3	62.0	116.0	156.4
ϕ_{eff}	.692	.640	.627	.629	.651	.683
$\phi_{eff}\ \sigma_{max,dir}$			4.6	39.0	75.5	106.8
$.53\ \sigma_{n\alpha,dir}$	34.6	25.6	23.6	20.8	18.9	17.6
$\sigma(r > R)_{dir}$	1221	902	769	589	477	397
$\phi_{eff}\ \sigma(r > R)_{dir}$	845	577	482	370	311	271
$.53\ \sigma_{n\alpha}/\phi_{eff}\ \sigma(r > R)$	.041	.044	.049	.056	.061	.065
ϕ_{α}	.959	.956	.951	.944	.939	.935
$\phi_{\alpha}\ \phi_{eff}\ \sigma_{max,dir}$			4.4	36.8	70.9	99.9
$\sigma_{n,2n}$		.5	10.9	65.8	121.0	155.5

$$\frac{.53\ \sigma_{n,\alpha}\ (r > R)_{dir}}{\phi_{eff}\ \sigma(r > R)_{dir}}$$

The statistical factor for emission of the triton, instead of a nucleon, eq. (49), is so overwhelmingly favorable to the triton that we assume a correction factor for competition with the n,2n process to be

$$\phi_\alpha = 1 - \frac{.53\ \sigma_{n,\alpha}\ (r > R)_{dir}}{\phi_{eff}\ \sigma(r > R)_{dir}} \tag{61}$$

There remains the possibility that competition with deuteron emission can lower the predicted n,2n cross section. For, although the direct interaction (other than pickup) is of no noticeable importance to the total n-d cross section, an amount of, say, 20 or 30 millibarns would not be noticed in that comparison, Fig. 1. Hence, the results given in Table IX for the n,2n cross section are probably upper limits. On the other hand they are to be compared with the measurements[7] made at 14 Mev indicating a cross section of 122^{+50}_{-40} mb.

CONCLUSION

Simple independent particle models of Li^6 and Li^7 lead to the result that about half the observed reaction cross sections with neutrons on Li^6 can be accounted for as having gone through compound states. The calculations were made by averaging over possible positions of the latter. The additional mechanisms of reaction are assumed to be effective only in states orthogonal to possible compound states and the orthogonality factor estimated on geometrical grounds.

Most prominent of the additional modes is the pickup reaction giving an n-d reaction. This was estimated by using the Fourier transform of a neutron wave in the square well representing its binding to Li^6, instead of that of a proton wave, but the two should be very similar. The pickup reaction, however, produces somewhat more of a forward peak in the deuteron distribution than is observed. On the other hand, the observations are more peaked than one would expect from the compound states alone (practically spherical in center of mass).

There is no convincing evidence that deuterons are formed by nucleons recoiling from what is called direct interaction, i.e. neutron-nucleon collisions calculated as though the two were free particles. However, the effect would be small compared with the total n-d cross section and could be of importance only to the n,2n cross section by presenting competition.

Evidence for direct interaction comes in the n-α reaction which, first, requires all that the compound nuclear states can supply and in addition 53% of the maximum estimate for direct interaction. This indicates that, of the quantum numbers (C, S, T), the latter two are conserved (and give a g-factor $\frac{1}{2}$) but that the space symmetry is not conserved. It also means that allowance must be made for competition with the n-α reaction when the doublet compound states are formed or when direct interaction is important.

Additional reaction mechanisms are suggested by comparing the theory with experimental results on the excitation of the 2.2 Mev level of Li^6 and on the production of He^6. Both of these are observed to be higher than the compound nuclear theory alone, by about a factor two, or less in some regions. Assuming that there exist such additional mechanisms, the excitation of the 2.2 Mev level represents an added source of deuterons. If we add this to that from the compound nucleus, plus pickup reaction, (corrected for n-α competition) we get very good agreement for the n-d cross section.

Finally, the n,2n cross section is calculated from compound plus direct interaction theories. Owing to the zero binding energy of the singlet deuteron there is no pickup effect predicted. The result at 14 Mev is within range of the experimental result, but it must be remembered that the direct interaction contribution might be smaller than calculated because no allowance is made for deuteron formation by this process.

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APPENDIX I

p^2 CONFIGURATIONS

We are concerned here with the dependence of wave functions upon spin, isotopic spin, and angle. Since only p-orbitals for individual nucleons are being considered, the wave function of particle 1 is determined by the quantum numbers m_ℓ, S_z, and T_z.

$$\psi^{(1)} = \psi^{(1)}(m_\ell, S_z, T_z) \tag{A1}$$

The dependence upon m_ℓ is given by the normalized Legendre functions $Y m_\ell$

$$Y_1 = \sqrt{\frac{3}{8\pi}}\, e^{i\phi} \sin\theta, \qquad Y_0 = \sqrt{\frac{3}{4\pi}} \cos\theta, \qquad Y_{-1} = -\sqrt{\frac{3}{8\pi}}\, e^{-i\phi} \sin\theta \tag{A2}$$

The two values of S_z will be indicated by the arrows $\uparrow$ and $\downarrow$ representing normalized spinors. Similarly, the two values of T_z will be denoted n and p, for neutron and proton; these are also normalized spinors.

We use these symbols to abbreviate eq. (A1). Thus if particle 1 is a neutron with positive spin component

$$\psi^{(1)} = n_{m_\ell}\uparrow$$

or a proton in Y_0 and negative spin

$$\psi^{(1)} = p_0\downarrow$$

and so on. The scalar product then represents a sum over spin, isotopic spin, and integral over θ and ϕ.

The antisymmetrized two-particle (p^2) wave function has the form

$$\Psi^{(1,2)} = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi^{(1)}(m_\ell,S_z,T_z) & \psi^{(2)}(m_\ell,S_z,T_z) \\ \psi^{(1)}(m_\ell',S_z',T_z') & \psi^{(2)}(m_\ell',S_z',T_z') \end{vmatrix} \tag{A3}$$

Suppose $\psi^{(1)}$ is given by the abbreviated form $n_1\uparrow$ and $\psi^{(2)}$ by $p_0\downarrow$; then the antisymmetry of $\Psi^{(1,2)}$ and its normalization can be retained by considering $n_1\uparrow$ and $p_0\downarrow$ to be anticommuting unitary operators whose order determines the sign of $\Psi^{(1,2)}$. Thus we represent Ψ, in this example, simply by

$$\begin{aligned} \Psi^{(1,2)} &= n_1\uparrow \ p_0\downarrow \\ &= -p_0\downarrow \ n_1\uparrow \end{aligned} \tag{A4}$$

In the first form particle 1 has the quantum numbers indicated by $n_1\uparrow$, and in the second form it has those of $p_0\downarrow$.

In this notation the ${}^3D_3$ state of Li^6 has the form

$$(M=3) \qquad n_1\uparrow p_1\uparrow \tag{A5}$$

The function for M=2 can then be generated by an infinitesimal rotation which carries $\uparrow$ into $\downarrow$ and n_1 into $\sqrt{2}n_0$, etc., and results in the sum

$$(M=2) \qquad n_1\uparrow p_1\downarrow + n_1\downarrow p_1\uparrow + \sqrt{2}\,(n_1\uparrow p_0\uparrow + n_0\uparrow p_1\uparrow) \tag{A6}$$

The sum of squares of the coefficients is 6, so this is normalized by dividing with $\sqrt{6}$. The choice of signs in eq. (A2) is such that an infinitesimal rotation carries p_0 into $\sqrt{2}$ p-1, and n_0 into $\sqrt{2}$ n-1, of course.

We see from eq. (A6) that there are three states orthogonal to ${}^3D_3$ which have a maximum M=2. These are, of course, ${}^3D_2$, ${}^1D_2$, and ${}^3P_2$. In this way the composition of all possible p^2 configurations as sums of determinants similar to eq. (A4) can be developed. The remaining quantum number C is then determined from the symmetry with regard to exchange of subscripts m_ℓ and m_ℓ'.

It is convenient to present the coefficients in these superpositions in tabular form. Four tables are required (Tables X to XIII), one each for M = 3,2,1,0. The waves for negative M may be obtained from those of positive M by interchanging ↑ with ↓ and subscripts 1 with -1. The coefficients remain the same except for states having $J = 2$, and in these all signs change.

The quantum numbers of the p^2 states, J, L, and S, appear in the usual spectroscopic notation, whereas C and T appear in separate rows. The value of T is zero if the wave function vanishes when p is replaced by n, as in eq. (A5). If it does not vanish, $T = 1$.

TABLE X

$M = 3$

C	1
T	0
	$^3D_3$
$n_1{\uparrow}\, p_1{\uparrow}$	1
N^{-2}	1

TABLE XI

$M = 2$

C	1	1	1	-1
T	1	0	0	1
	$^1D_2$	$^3D_2$	$^3D_3$	$^3P_2$
$n_1\uparrow\ p_1\downarrow$	1	q	1	
$n_1\downarrow\ p_1\uparrow$	-1	q	1	
$n_0\uparrow\ p_1\uparrow$		-1	q	1
$n_1\uparrow\ p_0\uparrow$		-1	q	-1
N^{-2}	2	6	6	2

$q = +\sqrt{2}$

Here we have used q to represent $+\sqrt{2}$. Also, we include the sum of squares of coefficients for each state, N^{-2}. Hence N is the normalization factor for the sum. These notations appear in all the following tables.

TABLE XII

$M = 1$

C	1	1	1	1	-1	-1	-1	1
T	0	0	0	0	1	1	0	1
	$^3S_1$	$^3D_3$	$^3D_2$	$^3D_1$	$^3P_2$	$^3P_1$	$^1P_1$	$^1D_2$
$n_0\uparrow\ p_0\uparrow$	-1	2	-2q	2q				
$n_1\uparrow\ p_{-1}\uparrow$	1	1	-q	q	-q	-q		
$n_{-1}\uparrow\ p_1\uparrow$	1	1	-q	q	q	q		
$n_1\uparrow\ p_0\downarrow$		q	1	-3	-1	1	1	1
$n_1\downarrow\ p_0\uparrow$		q	1	-3	-1	1	-1	-1
$n_0\uparrow\ p_1\downarrow$		q	1	-3	1	-1	-1	1
$n_0\downarrow\ p_1\uparrow$		q	1	-3	1	-1	1	-1
$n_1\downarrow\ p_1\downarrow$		1	2q	6q				
N^{-2}	3	15	24	120	8	8	4	4

TABLE XIII

$M = 0$

	$^3P_1$	$^3D_2$	$^3P_2$	$^3P_0$	$^3D_3$	$^3D_1$	$^3S_1$	$^1S_0$	$^1D_2$	$^1P_1$
C	-1	1	-1	-1	1	1	1	1	1	-1
T	1	0	1	1	0	0	0	1	1	0
$n_0\downarrow\ p_1\downarrow$	-1	1	1	-q	q	3				
$n_1\downarrow\ p_0\downarrow$	1	1	-1	q	q	3				
$n_{-1}\uparrow\ p_0\uparrow$	1	-1	1	-q	q	3				
$n_0\uparrow\ p_{-1}\uparrow$	-1	-1	-1	q	q	3				
$n_{-1}\downarrow\ p_1\uparrow$			q	1	1	-q	1	-1	-1	1
$n_{-1}\uparrow\ p_1\downarrow$			q	1	1	-q	1	1	1	-1
$n_1\downarrow\ p_{-1}\uparrow$			-q	-1	1	-q	1	-1	-1	-1
$n_1\uparrow\ p_{-1}\downarrow$			-q	-1	1	-q	1	1	1	1
$n_0\uparrow\ p_0\downarrow$					2	-2q	-1	-1	2	
$n_0\downarrow\ p_0\uparrow$					2	-2q	-1	1	-2	
N^{-2}	4	4	12	12	20	60	6	6	12	4

APPENDIX II

p^3 CONFIGURATIONS

The procedure outlined in Appendix I is followed in finding the p^3 configurations. The rule for determining the function for $-M$ is now changed to the following: interchanging $\uparrow$ with $\downarrow$ and $m_\ell = 1$ with -1 gives the negative of the wave function when the coefficients for positive M are used, except for $J = \frac{5}{2}$ and $J = \frac{1}{2}$; in these functions the sign remains the same.

Also, for convenience, the value of 2T is tabulated instead of T. For three particles a given S, L, and J may occur more than once, and it is necessary to classify the states according to eigenvalues T and C. The results are presented in Tables XIV through XVIII.

TABLE XIV

$M = \frac{7}{2}$

C	3	0
2T	1	1
	${}^2F_{7/2}$	${}^4D_{7/2}$
$n_1\uparrow\ p_1\uparrow\ n_1\downarrow$	1	
$n_1\uparrow\ p_1\uparrow\ n_0\uparrow$		1
N^{-2}	1	1

TABLE XV

$M = \frac{5}{2}$

C	0	0	0	0	0	3	3
2T	1	1	1	3	1	1	1
	$^4P_{\frac{5}{2}}$	$^4D_{\frac{7}{2}}$	$^4D_{\frac{5}{2}}$	$^2D_{\frac{5}{2}}$	$^2D_{\frac{5}{2}}$	$^2F_{\frac{7}{2}}$	$^2F_{\frac{5}{2}}$
$n_1\uparrow p_1\uparrow n_{-1}\uparrow$	-1	q	-3				
$n_1\uparrow p_0\uparrow n_0\uparrow$	1	q	-3				
$n_1\uparrow p_1\downarrow n_0\uparrow$		1	2q	1	1		
$n_1\downarrow p_1\uparrow n_0\uparrow$		1	2q	-1		-q	1
$n_1\uparrow p_1\uparrow n_0\downarrow$		1	2q		-1	q	-1
$n_1\uparrow p_0\uparrow n_1\downarrow$				-1	1	q	-1
$n_1\uparrow p_1\downarrow n_1\downarrow$						1	3q
N^{-2}	2	7	42	3	3	7	21

TABLE XVI

$M = \frac{3}{2}$

C	-3	0	0	0	0	0	0	0	0	0	0	0	3	3	3
2T	3	1	1	1	1	1	3	3	1	1	1	3	1	1	1
	${}^4S_{\frac{3}{2}}$	${}^4D_{\frac{7}{2}}$	${}^4D_{\frac{5}{2}}$	${}^4D_{\frac{3}{2}}$	${}^4P_{\frac{3}{2}}$	${}^4P_{\frac{5}{2}}$	${}^2D_{\frac{5}{2}}$	${}^2D_{\frac{3}{2}}$	${}^2D_{\frac{5}{2}}$	${}^2D_{\frac{3}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2F_{\frac{7}{2}}$	${}^2F_{\frac{5}{2}}$
$n_1\uparrow p_0\uparrow n_{-1}\uparrow$	-1	2	-6q	2q											
$n_0\uparrow p_1\uparrow n_{-1}\uparrow$	1	1	-3q	q	3	-q									
$n_1\uparrow p_{-1}\uparrow n_0\uparrow$	1	1	-3q	q	-3	q									
$n_1\uparrow p_1\downarrow n_{-1}\uparrow$		q	1	-2	-q	-1	q	-1	q	-1	-1	1			
$n_1\uparrow p_0\downarrow n_0\uparrow$		q	1	-2	q	1	q	-1	q	-1	1	-1			
$n_1\uparrow p_0\uparrow n_0\downarrow$		q	1	-2	q	1	-q	1				1	-1	2	-2q
$n_1\downarrow p_1\uparrow n_{-1}\uparrow$		q	1	-2	-q	-1	-q	1				-1	-2	-1	q
$n_1\downarrow p_0\uparrow n_0\uparrow$		q	1	-2	q	1			-q	1	-1		1	-2	2q
$n_1\uparrow p_1\uparrow n_{-1}\downarrow$		q	1	-2	-q	-1			-q	1	1		2	1	-q
$n_1\downarrow p_1\downarrow n_0\uparrow$		1	4q	2q					1	2q				-q	-5
$n_1\uparrow p_1\downarrow n_0\downarrow$		1	4q	2q			1	2q						q	5
$n_1\downarrow p_1\uparrow n_0\downarrow$		1	4q	2q			-1	-2q	-1	-2q					
$n_0\uparrow p_1\uparrow n_0\downarrow$							q	-1	-q	1	-1	-1	-1	2	-2q
$n_1\uparrow p_{-1}\uparrow n_1\downarrow$							-q	1	q	-1	-1	-1	2	1	-q
$n_1\uparrow p_0\downarrow n_1\downarrow$							-1	-2q	1	2q				q	5
N^{-2}	3	21	210	60	30	10	15	30	15	30	6	6	15	21	105

TABLE XVII

$M = \frac{1}{2}$ Doublets

C	3	3	3	3	0	0	0	0	0	0	-3	0	0
2T	1	1	1	1	3	3	3	3	1	1	1	1	1
	${}^2F_{\frac{7}{2}}$	${}^2F_{\frac{5}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2P_{\frac{1}{2}}$	${}^2D_{\frac{5}{2}}$	${}^2D_{\frac{3}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2P_{\frac{1}{2}}$	${}^2D_{\frac{5}{2}}$	${}^2D_{\frac{3}{2}}$	${}^2S_{\frac{1}{2}}$	${}^2P_{\frac{3}{2}}$	${}^2P_{\frac{1}{2}}$
$n_0\uparrow p_0\uparrow n_0\downarrow$	2q	-6	-3q	3									
$n_1\uparrow p_{-1}\downarrow n_1\downarrow$	1	2q	2	2q	-q	-3	-1	-q	q	3		-1	-q
$n_0\uparrow p_1\downarrow n_0\downarrow$	2	4q	-1	-q	q	3	-1	-q	-q	-3		-1	-q
$n_1\uparrow p_0\uparrow n_{-1}\downarrow$	q	-3	q	-1	-1	q	q	-1	-1	q	-1	q	-1
$n_1\downarrow p_0\uparrow n_{-1}\uparrow$	q	+3	-q	1	-1	q	-q	1	-1	q	-1	-q	1
$n_1\downarrow p_{-1}\uparrow n_0\uparrow$	-q	3	-q	1	1	-q	q	-1	-2	2q	1		
$n_0\uparrow p_1\uparrow n_{-1}\downarrow$	q	-3	q	-1	1	-q	-q	1	-2	2q	1		
$n_1\uparrow p_{-1}\uparrow n_0\downarrow$	q	-3	q	-1	-2	2q			1	-q	1	-q	1
$n_0\downarrow p_1\uparrow n_{-1}\uparrow$	-q	3	-q	1	-2	2q			1	-q	1	q	-1
$n_1\uparrow p_0\downarrow n_0\downarrow$	2	4q	-1	-q					q	3		1	q
$n_1\downarrow p_1\downarrow n_{-1}\uparrow$	-1	-2q	-2	-2q					q	3		-1	-q
$n_1\uparrow p_1\downarrow n_{-1}\downarrow$	1	2q	2	2q	q	3	1	q					
$n_1\downarrow p_0\downarrow n_0\uparrow$	-2	-4q	1	q	q	3	-1	-q					
$n_1\downarrow p_0\uparrow n_0\downarrow$					-q	-3	1	q	-q	-3		-1	-q
$n_1\downarrow p_1\uparrow n_{-1}\downarrow$					-q	-3	-1	-q	-q	-3		1	q
$n_0\uparrow p_1\downarrow n_{-1}\uparrow$					1	-q	q	-1	1	-q	-2	-q	1
$n_1\uparrow p_{-1}\downarrow n_0\uparrow$					1	-q	-q	1	1	-q	-2	q	-1
$n_1\uparrow p_0\downarrow n_{-1}\uparrow$					2	-2q			2	-2q	2		
N^{-2}	35	210	45	45	30	90	18	18	30	90	18	18	18

TABLE XVIII

$M = \frac{1}{2}$ Quartets

C	0	0	0	0	-3	0	0	0
2T	1	1	1	1	3	1	1	1
	$^4D_{\frac{7}{2}}$	$^4D_{\frac{5}{2}}$	$^4D_{\frac{3}{2}}$	$^4D_{\frac{1}{2}}$	$^4S_{\frac{3}{2}}$	$^4P_{\frac{5}{2}}$	$^4P_{\frac{3}{2}}$	$^4P_{\frac{1}{2}}$
$n_1\uparrow\ p_0\uparrow\ n_{-1}\downarrow$	2	-2q	2q	-2q	-1			
$n_1\downarrow\ p_0\uparrow\ n_{-1}\uparrow$	2	-2q	2q	-2q	-1			
$n_1\downarrow\ p_{-1}\uparrow\ n_0\uparrow$	1	-q	q	-q	1	q	-1	-q
$n_0\uparrow\ p_1\uparrow\ n_{-1}\downarrow$	1	-q	q	-q	1	-q	1	q
$n_1\uparrow\ p_{-1}\uparrow\ n_0\downarrow$	1	-q	q	-q	1	q	-1	-q
$n_0\downarrow\ p_1\uparrow\ n_{-1}\uparrow$	1	-q	q	-q	1	-q	1	q
$n_1\uparrow\ p_0\downarrow\ n_0\downarrow$	q	5	-3			1	2q	1
$n_1\downarrow\ p_1\downarrow\ n_{-1}\uparrow$	q	5	-3			-1	-2q	-1
$n_1\uparrow\ p_1\downarrow\ n_{-1}\downarrow$	q	5	-3			-1	-2q	-1
$n_1\downarrow\ p_0\downarrow\ n_0\uparrow$	q	5	-3			1	2q	1
$n_1\downarrow\ p_0\uparrow\ n_0\downarrow$	q	5	-3			1	2q	1
$n_1\downarrow\ p_1\uparrow\ n_{-1}\downarrow$	q	5	-3			-1	-2q	-1
$n_0\uparrow\ p_1\downarrow\ n_{-1}\uparrow$	1	-q	q	-q	1	-q	1	q
$n_1\uparrow\ p_{-1}\downarrow\ n_0\uparrow$	1	-q	q	-q	1	q	-1	-q
$n_1\uparrow\ p_0\downarrow\ n_{-1}\uparrow$	2	-2q	2q	-2q	-1			
$n_1\uparrow\ p_{-1}\uparrow\ n_{-1}\uparrow$	q	-9	-3	6		1	-3q	3
$n_0\uparrow\ p_0\uparrow\ n_{-1}\uparrow$	q	-9	-3	6		-1	3q	-3
$n_1\downarrow\ p_1\downarrow\ n_0\downarrow$	1	6q	6q	6q				
N^{-2}	35	420	180	180	9	20	90	36

APPENDIX III

CALCULATION OF g-FACTORS

Consider the collision between a neutron with spin $+\frac{1}{2}$ and a Li^6 nucleus with M = 1. The angular and spin dependence of the neutron is then $n_o\uparrow$, and from Table XII the composition of the $^3S_1$ state is

$$\left({}^3S_1\right)_1 = \frac{1}{\sqrt{3}}\left[n_1\uparrow p_{-1}\uparrow - n_o\uparrow p_o\uparrow + n_{-1}\uparrow p_1\uparrow\right]$$

Taking the antisymmetric product of this with $n_o\uparrow$, the second term vanishes and for the incident three body state we have

$$\left({}^3S_1\right)_1 n_o\uparrow = \frac{1}{\sqrt{3}}\left(n_1\uparrow p_{-1}\uparrow n_o\uparrow + n_{-1}\uparrow p_1\uparrow n_o\uparrow\right)$$

$$= \frac{1}{\sqrt{3}}\left(n_1\uparrow p_{-1}\uparrow n_o\uparrow - n_o\uparrow p_1\uparrow n_{-1}\uparrow\right)$$

This has $n = \frac{3}{2}$; looking at Table XVI we find

$$n_1\uparrow p_{-1}\uparrow n_o\uparrow = \frac{1}{\sqrt{3}}\,{}^4S_{3/2} + \frac{1}{\sqrt{21}}\,{}^4D_{7/2} - \frac{3\sqrt{2}}{\sqrt{210}}\,{}^4D_{5/2} + \frac{\sqrt{2}}{\sqrt{60}}\,{}^4D_{3/2}$$

$$- \frac{3}{\sqrt{30}}\,{}^4P_{3/2} + \frac{\sqrt{2}}{\sqrt{10}}\,{}^4P_{5/2}$$

and $n_0\uparrow p_1\uparrow n_{-1}\uparrow$ is the same with reversed signs on the ^4P terms. Hence

$$\left({}^3S_1\right)_1 n_o\uparrow = \frac{2}{\sqrt{3}}\left(\frac{1}{\sqrt{5}}\,{}^4P_{5/2} - \sqrt{\frac{3}{10}}\;{}^4P_{3/2}\right)_{3/2}$$

The partial g_a^i factors for $^4P_{5/2}$ and $^4P_{3/2}$ are therefore 4/15 and 2/5, respectively. Analogous calculations for the other two incident states produce the results in Table XIX.

As an illustration of calculation of g_f^i, consider the $M = 3/2$ state of $^4P_{5/2}$. According to Table XVI this is

$$\left({}^4P_{5/2}\right)_{3/2} = \frac{1}{\sqrt{10}}\Big[-\sqrt{2}\,n_o\uparrow p_1\uparrow n_{-1}\uparrow + \sqrt{2}\,n_1\uparrow p_{-1}\uparrow n_o\uparrow$$

$$- n_1\uparrow p_1\downarrow n_{-1}\uparrow + n_1\uparrow p_o\downarrow n_o\uparrow + n_1\uparrow p_o\uparrow n_o\downarrow$$

$$- n_1\downarrow p_1\uparrow n_{-1}\uparrow + n_1\downarrow p_o\uparrow n_o\uparrow - n_1\uparrow p_1\uparrow n_{-1}\downarrow\Big]$$

Taking the scalar product of this with $n_{-1}\uparrow$, say, we get the amplitude factor for emitting the neutron with $m_\ell = -1$ and $S_z = \frac{1}{2}$, viz.

$$A_{-1}\uparrow = \frac{1}{\sqrt{10}}\Big[-\sqrt{2}\;n_o\uparrow p_1\uparrow - n_1\uparrow p_1\downarrow - n_1\downarrow p_1\uparrow\Big]$$

Consulting Table XI we find this wave to be expressed in p^2-terms as

TABLE XIX

Values of $g_a^{\,i}$

	$\left({}^3S_1\right)_1 n_o\uparrow$	$\left({}^3S_1\right)_1 n_o\downarrow$	$\left({}^3S_1\right)_o n_o\uparrow$	Ave $g_a^{\,i}$
C = 0				
${}^4P_{5/2}$	4/15	2/15	4/15	2/9
${}^4P_{3/2}$	2/5	2/135	4/135	4/27
${}^4P_{1/2}$		2/27	4/27	2/27
${}^2P_{3/2}$		4/27	2/27	2/27
${}^2P_{1/2}$		2/27	1/27	1/27
			Sum	5/9
C = 3				
${}^2P_{3/2}$		10/27	5/27	5/27
${}^2P_{1/2}$		5/27	5/24	5/54
			Sum	5/18

$$A_{-1}\uparrow = \frac{1}{\sqrt{10}} \left\{ -\sqrt{2} \left[-\frac{1}{\sqrt{6}}\ {}^3D_2 + \frac{\sqrt{2}}{\sqrt{6}}\ {}^3D_3 + \frac{1}{\sqrt{2}}\ {}^3P_2 \right] \right.$$

$$- \left[\frac{1}{\sqrt{2}}\ {}^1D_2 + \frac{\sqrt{2}}{\sqrt{6}}\ {}^3D_2 + \frac{1}{\sqrt{6}}\ {}^3D_3 \right]$$

$$\left. - \left[-\frac{1}{\sqrt{2}}\ {}^1D_2 + \frac{\sqrt{2}}{\sqrt{6}}\ {}^3D_2 + \frac{1}{\sqrt{6}}\ {}^3D_3 \right] \right.$$

$$A_{-1}\uparrow = \frac{1}{\sqrt{10}} \left\{ -\frac{1}{\sqrt{3}}\ {}^3D_2 - \frac{4}{\sqrt{6}}\ {}^3D_3 - {}^3P_2 \right\}$$

The probability factors for emission into each of these states are then

$n_{-1}\uparrow \cdot {}^3D_2$	1/30
$n_{-1}\uparrow \cdot {}^3D_3$	4/15
$n_{-1}\uparrow \cdot {}^3P_2$	1/10

The values of $g_f^{\,i}$ given in Table III are the sums of such factors over the six possible neutron states. For example one can reach the ${}^3D_3$ state from ${}^4P_{5/2}$ by the following paths, with the probability factors:

$$n_{-1}\uparrow,\ n_{-1}\downarrow,\ n_o\uparrow,\ n_o\downarrow,\ n_1\uparrow,\ n_1\downarrow$$

$${}^4P_{5/2} \quad \frac{4}{15} + \frac{1}{10} + \frac{16}{75} + \frac{1}{30} + \frac{2}{25} + \frac{1}{150} = \frac{7}{10}\,.$$

APPENDIX IV

CONSTANTS

Calculations of wave numbers and neutron wave $\phi(r)$, eq. (8), are based upon the following numerical values

Mass of neutron	$M = 1.008986$ A.M.U.
	$= 1.6739 \times 10^{-24}$ gm
Mass of Li^5	$= 5.013948$ A.M.U.
Planck's constant	$\hbar = 1.0545 \times 10^{-27}$ erg sec
10^6 electron volts	1.60209×10^{-6} erg

For reduction of masses in the cross sections for direct interaction the integral values of the masses were used.