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NEUTRON ENERGY AND VELOCITY DISTRIBUTIONS FROM THERMAL D-T REACTIONS

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ABSTRACT

This report collects a number of general formulae which apply to the determination of the neutron energy and velocity distributions from the thermal D-T reaction. Comparison of the predictions of these formulae with the results of a Monte Carlo calculation suggests that the formulae apply to temperatures in excess of 100 keV.

I. INTRODUCTION

Section II presents the general formulae applying to the thermal D-T$\rightarrow n\alpha$ reaction. Section III applies these formulae and compares their predictions to a Monte Carlo calculation of the energy and velocity distributions.

II. GENERAL FORMULAE

For reaction 1

$$1 + 3 \rightarrow 3 + 4 \quad ; \tag{1}$$

the relativistic energy of particle 4 in the center of momentum (mass) frame of 1 and 2 is given by[1]

$$E_{4c} = \left(W^2 - m_3^2c^4 + m_4^2c^4\right) / (2W) \quad , \tag{2}$$

where

$$W \equiv m_1c^2 + m_2c^2 + q \quad . \tag{3}$$

The rest energies of 1, 2, 3, 4 are m_1c^2, m_2c^2, m_3c^2, m_4c^2; q is any additional energy available in the center of momentum (mass) frame of particles 1 and 2; and c is the speed of light. The kinetic energy of 4 is given by

$$KE_4 = E_4 - m_4c^2 \quad . \tag{4}$$

The momentum is given by

$$P_4 = \sqrt{E_4^2 - m_4^2c^4}\,/c \tag{5}$$

$$= \sqrt{KE_4\left(2m_4c^2 + KE_4\right)}\,/c \quad . \tag{5'}$$

The velocity of 4 is given by

$$v_4 = P_4c^2/E_4 \quad . \tag{6}$$

If 1 and 2 have charge numbers Z_1 and Z_2, and if 1 and 2 are in a thermal distribution characterized by a temperature Θ (MeV), q is given by the distribution function which for $\Theta \ll 1$ MeV is approximately[2,3]

$$S(q) \simeq \exp\left[-B(q - q_0)^2\right] \quad , \tag{7}$$

where

$$q_0 = (1/2)^{2/3} E_g^{1/3} \Theta^{2/3} \quad , \tag{8}$$

$$B = 3/\left(2^{4/3} E_g^{1/3} \Theta^{5/3}\right) \quad . \tag{9}$$

E_g is the "Gamow energy" which is defined by

$$E_g = 2M\left(\pi Z_1 Z_2 e^2/\hbar\right)^2 \quad , \tag{10}$$

where M is the reduced mass of 1 and 2. Evaluating this expression,

$$E_g = 0.979\ Y\ (\mathrm{MeV}) \quad , \tag{11}$$

where

$$Y = AZ_1^2 Z_2^2 \tag{12}$$

and

$$A = A_1 A_2 / (A_1 + A_2) \quad . \tag{13}$$

Here A_1 and A_2 are the masses of 1 and 2 in amu.

The expectation value of q is approximately given by

$$\bar{q} = q^0 \tag{14}$$

$$= E_g^{1/3}\, \Theta^{2/3} / 2^{2/3} \tag{15}$$

and the Δq about $\bar{q}$ is

$$\Delta q = \sqrt{\langle q^2 - \bar{q}^2 \rangle} \quad , \tag{16}$$

$$= \sqrt{\frac{1}{2B}} \quad , \tag{17}$$

$$= \frac{1}{\sqrt{3}} \left(2E_G\right)^{1/6} \Theta^{5/6} \quad . \tag{18}$$

The energy of the center of momentum (mass) frame for 1 and 2 in the laboratory has distribution characterized by the temperature Θ. For the decay of this system into particles 3 and 4 where $KE_{40} \gg \Theta$ and for $KE_4 \simeq KE_{40}$, the distribution function is

$$S(KE_4) \simeq \exp\left(- \frac{A_1 + A_2}{A_4} \frac{KE_4 + KE_{40}}{\Theta}\right)$$

$$\times \sinh\left(2 \frac{A_1 + A_2}{A_4} \frac{\sqrt{KE_4 KE_4}}{\Theta}\right) \tag{19}$$

where KE_{40} is the kinetic energy of particle 4 in the center of momentum (mass) frame.

The expectation value of KE_4 is given by

$$\overline{KE_4} = KE_{40} + 3/2\, A_4 / (A_1 + A_2)\, \Theta \tag{20}$$

and the ΔKE_4 about $\overline{KE_4}$ is

$$\Delta KE_4 = \left[2 \frac{A_4}{A_1 + A_2} \Theta\, KE_{40} + 3/2 \left(\frac{A_4}{A_1 + A_2}\right)^2 \Theta^2\right]^{1/2} \quad , \tag{21}$$

or

$$\Delta KE_4 \simeq \sqrt{2 \frac{A_4}{A_1 + A_2} \Theta\, KE_{40}} \tag{21'}$$

for

$$KE_0 \gg \frac{A_4}{A_1 + A_2} \Theta \quad .$$

III. NUMERICAL APPLICATIONS TO THE THERMAL DT→nα REACTION

The following list gives the atomic masses on a carbon 12 scale.[4]

m_e	0.000 548 579
n	1.008 665 22
D	2.014 102 22
T	3.016 049 72
^{4}He	4.002 603 26

The nuclear masses are obtained by correcting for the electron masses and electron binding energies which are 13.595 eV for H and 54.403 eV for He.[5]

n	1.008 665 22
D	2.013 553 66
T	3.015 501 16
α	4.001 506 16

The conversion from amu to MeV is 1 amu = 931.504 MeV. The speed of light used in the calculations is 299.792 50 mm/ns.

Substituting into Eq. (15), the expectation value of q is found to be

$$\bar{q} = 0.6647\ \Theta^{2/3}\ (\mathrm{MeV}) \tag{22}$$

where Θ is in MeV. The associated width is found from Eq. (18) to be

$$\Delta q = 0.6657\ \Theta^{5/6}\ (\text{MeV}) \quad . \tag{23}$$

Propagating Δq through Eq. (2), the resulting spread in the center of momentum frame energy is

$$\Delta KE_{4c} = (1 - E_{4c}/W)\Delta q \tag{24}$$

$$\simeq 4/5\ \Delta q \tag{25}$$

$$\simeq 0.078\ \text{MeV at 100 keV.} \tag{26}$$

On the other hand, ΔKE_4 from Eq. (21) is

$$\Delta KE \simeq \sqrt{(2/5)14\Theta} \tag{27}$$

$$\simeq 0.748\ \text{MeV at 100 keV.} \tag{28}$$

Comparing Eqs. (26) and (28), it is clear that for temperatures below 100 keV the effect of the spread of the energies caused by Δq can be ignored in comparison to the spread of energies caused by ΔKE_4 from Eq. (21).

The velocity of the neutrons is determined by the following scheme. The $\bar{q}$ is calculated from Eq. (22). E_{4c} is calculated from Eq. (2). The kinetic energy, KE_{4_0}, is calculated from Eq. (4). The kinetic energy distribution in the laboratory, KE_4, is obtained from Eq. (19) which may be written as

$$S(KE_4) \simeq \exp\left[-\frac{A_1 + A_2}{A_4}\frac{1}{\Theta}\left(\sqrt{KE_4} - \sqrt{KE_{4_0}}\right)^2\right] \quad . \tag{29}$$

The expected kinetic energy in the laboratory is given by Eq. (20). The neutron velocity at the expected kinetic energy is obtained from Eqs. (5) and (6). The velocity distribution may be obtained from Eq. (29) through the use of Eqs. (4), (5), and (6). Near the peak, Eq. (29) may be approximated by

$$S(KE_4) \simeq \exp\left[-(A_1 + A_2)/(4A_4)(KE_4 - KE_{4_0})^2/(KE_{4_0}\Theta)\right] \ . \tag{30}$$

Table I gives the expected neutron kinetic energy as a function of Θ, the expected ΔKE, the expected velocity, and the Δv for the D-T $\rightarrow$ nα reaction.

TABLE I

ENERGY AND VELOCITY VS TEMPERATURE

Θ (keV)	$\overline{KE_4}$ (MeV)	ΔKE_4 (MeV)	v (mm/ns)	Δv (mm/ns)
0	14.0290	0.000	51.2344	0.000
0.1	14.030	0.024	51.236	0.042
0.2	14.031	0.034	51.238	0.060
0.3	14.031	0.041	51.239	0.073
0.4	14.032	0.047	51.240	0.085
0.5	14.032	0.053	51.241	0.095
0.6	14.033	0.058	51.24	0.10
0.7	14.033	0.063	51.24	0.11
0.8	14.034	0.067	51.24	0.12
0.9	14.034	0.071	51.24	0.13
1.0	14.035	0.075	51.24	0.13
2.0	14.04	0.11	51.25	0.19
3.0	14.04	0.13	51.26	0.23
4.0	14.04	0.15	51.26	0.27
5.0	14.05	0.17	51.26	0.30
6.0	14.05	0.18	51.27	0.33
7.0	14.05	0.20	51.27	0.35
8.0	14.05	0.21	51.28	0.38
9.0	14.05	0.22	51.28	0.40
10.0	14.06	0.24	51.28	0.42
20.0	14.07	0.34	51.31	0.60
30.0	14.09	0.41	51.34	0.73
40.0	14.10	0.48	51.37	0.85
50.0	14.12	0.53	51.39	0.95
60.0	14.13	0.58	51.41	1.0
70.0	14.14	0.63	51.43	1.1
80.0	14.15	0.67	51.45	1.2
90.0	14.16	0.72	51.47	1.3
100.0	14.17	0.75	51.49	1.3

A Monte Carlo code was written which sampled the D and T from a Maxwellian distribution, evaluated the cross section to obtain the reaction probability and produced the neutron isotropically in the center of momentum frame of the D and T, and translated the neutron back to the laboratory frame in a relativistically correct manner. Table II compares the calculated Monte Carlo results with the numbers from Table I.

At 1 and 10 keV, the kinetic energy distributions were compared to the distribution predicted by Eq. (29) where KE_{4_0} was the mean calculated by

TABLE II

ENERGY AND VELOCITY COMPARED TO THE RESULTS OF A MONTE CARLO INTEGRATION

	Table I		MC Code		Table I		MC Code	
Θ (keV)	KE (MeV)	ΔKE (MeV)	KE (MeV)	ΔKE (MeV)	v (mm/ns)	Δv (mm/ns)	v (mm/ns)	Δv (mm/ns)
1	14.035	0.075	14.036	0.077	51.24	0.13	51.25	0.14
10	14.06	0.24	14.06	0.24	51.28	0.42	51.30	0.43
100	14.17	0.75	14.14	0.77	51.5	1.3	51.4	1.4

the Monte Carlo code. To within the statistics for the calculation, the two distributions agreed.

Table II suggests that the formulae presented here are valid for temperatures in excess of 100 keV.

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