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AN ANALYSIS OF Li^6 NEUTRON CROSS SECTIONS

by

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PHYSICS

ABSTRACT

An isolated level Breit-Wigner analysis of the neutron elastic scattering and (n, t) cross section over the region 0 to 0.5 Mev is presented. In order to depict these cross sections more accurately, the effect of six levels from -0.5193 to 3.52 Mev neutron energy was considered; thus cross sections to 3.0 Mev were studied. The parameters of the theory were obtained directly from experiment, from fitting experimental cross-section curves, and from the predictions from other theories.

ACKNOWLEDGMENTS

I have benefited greatly from conversations with many scientists, and it is therefore a pleasure to acknowledge discussions with R. G. Thomas, R. F. Taschek, the late H. C. Martin, F. L. Ribe, S. Bame, and others of the Los Alamos Scientific Laboratory; with Harvey Willard of Oak Ridge; and D. J. Hughes and J. A. Harvey of Brookhaven National Laboratory. I have enjoyed the computational assistance of J. E. Powers, M. Johnson, C. E. Moss, and D. Glassgow.

CONTENTS

I. Introduction

For the convenience of those who are interested only in the results of this study, we present our curves immediately following this introduction. Order of discussion is, therefore, Results, Theory, Probable Errors, and Suggestions for Further Experimental Study.

This study was primarily undertaken to obtain the $Li^6(n,t)He^4$ cross section below the 0.256 Mev resonance. Since the 0.256 Mev resonance is a $^4P_{5/2}$ (Refs. 1-3) and the high thermal $\frac{1}{v}$ (n,α) cross section indicates that the closest negative energy level is likely to be S wave for neutrons, the single or isolated level Breit-Wigner formula suggests itself as appropriate for this problem. Calculation of the total cross section, using such a theory for two levels only, gives a good fit to the thermal and epithermal cross section and to the 0.256 Mev resonance, except at 0.3 Mev and higher. Since the 0.256 Mev level was itself very broad and since, further, the total cross section (Refs. 2, 4) and the (n,α) cross section (Ref. 5) are known at higher energies, it was decided that the effect of higher levels should and could be included at least to some extent.

We therefore have attempted to fit the total and (n, t) cross section from 0 to 3 Mev using six levels, the parameters of which are derived (1) directly from experiment, (2) to lead to best agreement with known cross sections, and (3) from the choices offered by theory. The curves so derived fit experiment in all but one, or possibly two, cases better than experimental error over the region 0 to 3 Mev.

We discuss the discrepancies in Section IV. Our accuracy throughout, then, is only as good as the thermal cross section and as the 0.256 Mev resonance total cross section; it is limited by the known failure of the linear amplitude interference for levels of the same character (Ref. 6) under this theory, and to the accuracy with which the higher level influence can

1) R. G. Thomas, Los Alamos Scientific Laboratory, private communication, P-169, (1954).

2) Johnson, Willard, and Bair, Phys. Rev. 96, 985 (1954).

3) Darlington, Haughness, Mann, and Roberts, Phys. Rev. 90, 1049 (1952); Solano and Roberts, Phys. Rev. 89, 892 (1953).

4) D. J. Hughes and J. A. Harvey, Brookhaven National Laboratory Report BNL-325 (1955).

5) Fred L. Ribe, private communication (1953), and Phys. Rev. 87, 205A (1952).

6) The approximation of adding amplitudes of distinct levels is, however, a good one. Refer to E. P. Wigner, Phys. Rev. 70, 609 (1946).

be calculated by the isolated level approximation. Quantitative estimates for our accuracy are given in Section IV.

II. Results

1. Radiative capture negligible ($\sigma_c < 0.1$ barn at thermal - Ref. 4).
2. Inelastic scattering identically zero to 2.189 Mev (first excited level in Li^6 - Ref. 7).
3. Threshold of $Li^6(n;d,n)He^4$ is 1.722 Mev (Ref. 8).
4. Q $Li^6(n,p)He^6$ = -2.77 Mev (Ref. 7). Threshold at E_n = 3.23 Mev.
5. Q $Li^6(n,d)He^5$ = -2.43 Mev (Ref. 7). Threshold at E_n = 2.84 Mev.
6. Graph 1: σ_T, $\sigma(n,t)$ vs E_n; $0.01 \leq E_n \leq 100$ ev.
7. Graph 2: σ_T, $\sigma(n,t)$ vs E_n; $100 \text{ ev} \leq E_n \leq 1$ Mev.

Probable errors are estimated in Section IV.

III. Theory

The theory used is that of a single level Breit-Wigner formula for an unpolarized beam as given, for example, by Blatt and Weisskopf (Ref. 9), omitting spin flip, and as modified to add amplitudes of neutron scattering in terms of the same character, and to add the effect of level shifts. We use the Wigner and Eisenbud (Ref. 10) boundary conditions rather than the Weisskopf, since the former lead to smaller level shifts, especially for low energy neutrons. In fact, the neutron contribution to the level shift of an $\ell = 1$ resonance goes to zero with decreasing neutron energy for Wigner-Eisenbud boundary conditions and to a constant for Weisskopf boundary conditions.

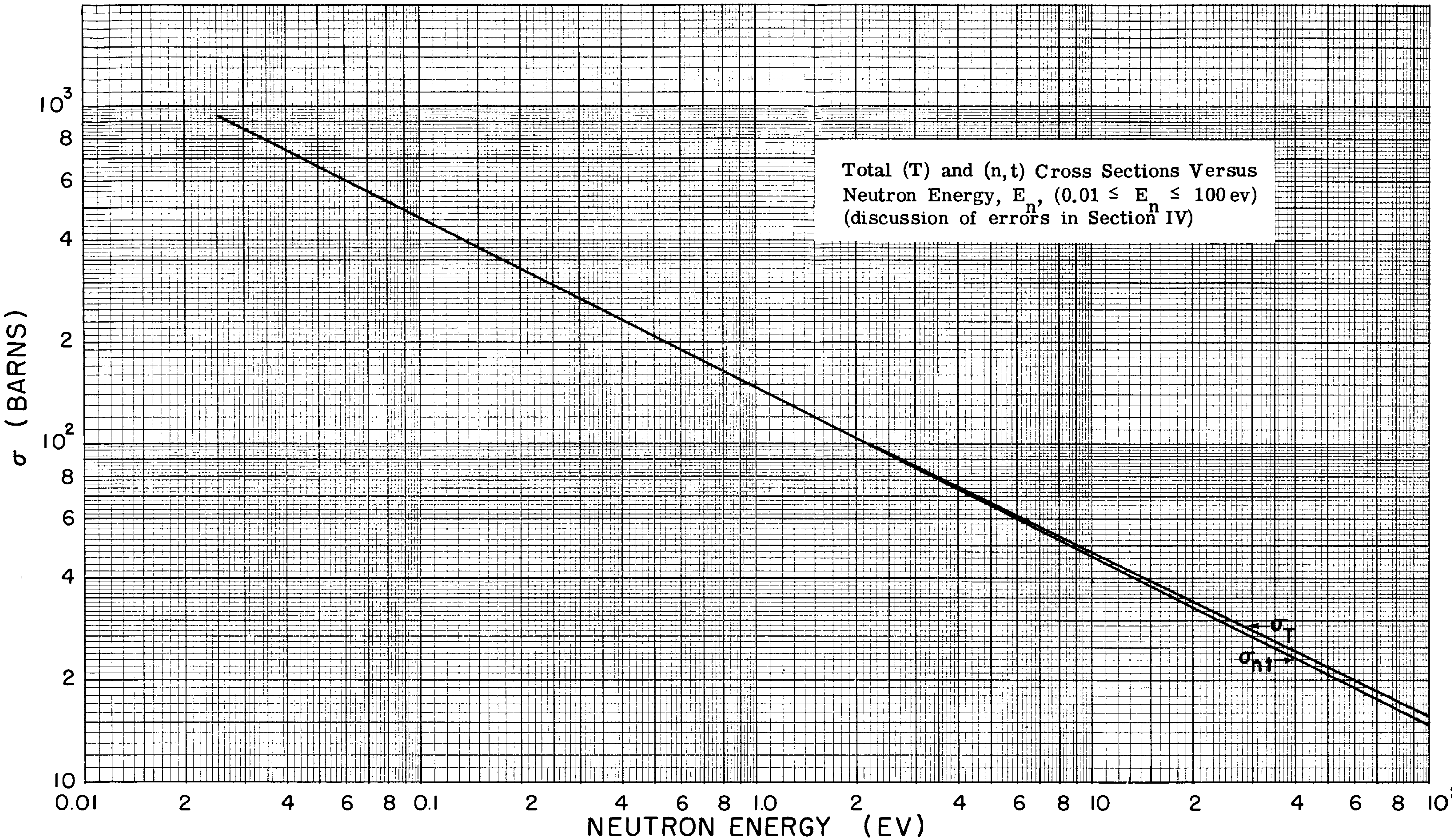
Before we can write down our formulas, we must assign spins and parities to the levels, since the existence of interference depends on whether the levels are of like character. We must first define what we mean by a level position. We define the position of a resonance, i, to be that neutron energy, E'_i, at which the corresponding energy part of the denominator in our resonance formulas is zero (i. e., where the Breit-Wigner denominator is equal to one-half the total width squared). We further define our energy dependent widths at this energy, E'_i. This definition has the advantage of being the same for all processes and being very close to the experimentally observed maxima. The latter are, by the way, not quite identical for

7) F. Ajzenberg and T. Lauritsen, Revs. Mod. Phys. 27, 77 (1955).

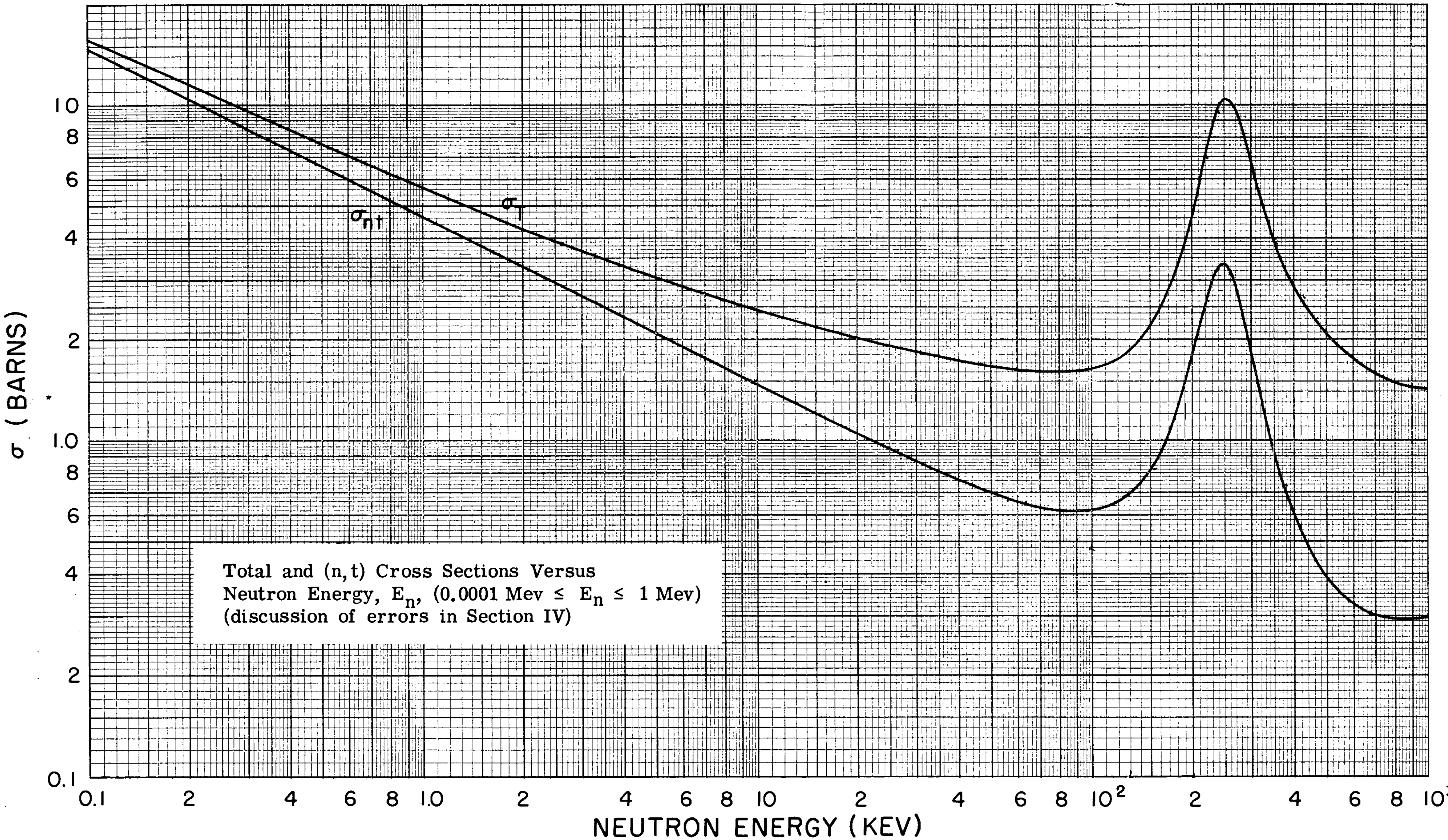
8) Calculated from mass values given by J. E. Drummond, Phys. Rev. 97, 1004 (1955).

9) Blatt and Weisskopf, "Theoretical Nuclear Physics," John Wiley & Sons, Inc., New York, 1952; especially Chap. VIII, formulas 10.28 and 10.32.

10) Wigner and Eisenbud, Phys. Rev. 72, 29 (1947).


Total (T) and (n,t) Cross Sections Versus
Neutron Energy, E_n, $(0.01 \leq E_n \leq 100$ ev)
(discussion of errors in Section IV)
σ (BARNS)
NEUTRON ENERGY (EV)
σ_T
σ_{nt}
10
2
4
6
8
10^2
10^3
0.01
0.1
1.0

Graph 1



σ (BARNS)
10
8
6
4
2
1.0
8
6
4
2
0.1
0.1
2
4
6
8
1.0
2
4
6
8
10
2
4
6
8
10²
2
4
6
8
10³
NEUTRON ENERGY (KEV)
σnt
σT
Total and (n,t) Cross Sections Versus
Neutron Energy, En, (0.0001 Mev ≤ En ≤ 1 Mev)
(discussion of errors in Section IV)

Graph 2

different processes as can be seen from our results and also from experiment.

The compound nucleus resonance, E_i, is a non-physical quantity and its position is determined from the rather arbitrary Wigner-Eisenbud boundary conditions (Ref. 10).

The reactions we consider are elastic scattering and $Li^6(n,t)He^4$. The exclusion of other reactions up to 1.7 Mev has been exhibited in Section II. We will later include a mock-up for this competition above 1.7 Mev. We take the spins to be $Li^6 = 1$ (Ref. 11); the neutron of course, $\frac{1}{2}$; triton is $\frac{1}{2}$ (Ref. 11); and He^4 is, of course, 0.

Level No. 1 we put at a neutron energy, $E'_1 = -0.5193$ (Refs. 12-14, 17); compound nucleus spin and parity, $J_1 = \frac{3^+}{2}$ (Refs. 1, 12); incoming neutron orbital angular momentum, $\ell_{n1} = 0$ (Ref. 15); outgoing alpha-triton orbital angular momentum, $\ell_{\alpha 1} = 2$ (Ref. 16). The apparent total width of this level is about 0.62 Mev (Ref. 17); the alpha width, $\Gamma'_{\alpha 1} = 0.615$ Mev·(Ref. 18); and the neutron width at $E_n = 0.2563$ Mev, $\Gamma''_{n1} = 0.0345$ Mev (Ref. 18).

Level No. 2 is put at a neutron energy $E'_2 = 0.2563$ Mev in order to obtain the best fit to the total neutron cross section of Johnson et al. (Ref. 2), $J_2 = \frac{5^-}{2}$ (Refs. 1-3), $\ell_{n2} = 1$ (Refs. 1-3), whence $\ell_{\alpha 2} = 3$ (Ref. 16); $\Gamma'_{\alpha 2} = 0.0995$ and $\Gamma'_{n2} = 0.04839$ are also obtained from the best fit to the total neutron cross section of Johnson et al.

Level No. 3 is expected to be in the region $E_n = 0.96$ to 1.4 Mev, and probably at 1.1 Mev with total width of the order of 0.5 Mev (Refs. 17, 19). In order to make the best fit to the total and (n,t) cross section (Refs. 2, 4, 5), we have chosen $E'_3 = 1.306$ Mev, $\Gamma'_{n3} = 0.722$, $\Gamma'_{\alpha 3} = 0.41$ Mev. We place the spin of this level to be $\frac{1^+}{2}$.

11) Way et al., National Bureau of Standards, Circ. 499, "Nuclear Data" (1950).

12) Ajzenberg and Lauritsen, Revs. Mod. Phys. 27, 77 (1955), and Erdos et al., Nuovo cimento 12, 639 (1954), give a level at 6.8 Mev excitation from $Li^7(\gamma,t)He^4$ of $\frac{1^+}{2}$ or $\frac{3^+}{2}$ spin and parity.

13) Ajzenberg and Lauritsen, ibid., give the binding energy of the last neutron in Li^7 to be 7.245 Mev.

14) Li, Whaling, Fowler, Lauritsen, Phys. Rev. 83, 512 (1951) give for the mass of the neutron: 1.008982 amu, and for the mass of Li^6: 6.017021 amu whence $E_{relative} = 0.85639\ E_n$ (laboratory).

15) Based in part on the assumption that part of the thermal cross section is due to the next negative level.

16) Derived from the parity of the level, and the positive intrinsic parities of the neutron, proton, and alpha particle.

17) Stoll, Helv. Phys. Acta 27, 401 (1954). Erdos, Stoll, Wachter, and Wataghin, Nuovo cimento 12, 639 (1954).

18) The choice of widths will be explained later in this section when the entire fitting will be discussed.

19) D. L. Allen, Nature 174, 267 (1954).

This choice was made from the levels suggested by the theories of D. R. Inglis (Ref. 20), and from the fact that the total cross section, which is also proportional to (2J + 1), is very low in this region (Refs. 2, 4). The parity was chosen positive in order to give the simple orbital angular momenta: $\ell_{n3} = 0 = \ell_{\alpha 3}$ (Ref. 21).

Level No. 4 is expected to be in the range 1.58 to 2.17 Mev, and probably at 2 Mev, with a total width of the order of 0.6 Mev (Refs. 17, 19). For a best fit we have chosen $E'_4 = 1.994$ Mev, $\Gamma'_{n4} = 0.384$ Mev, and $\Gamma'_{\alpha 4} = 0.116$ Mev. For the identical reasons given in the discussion of level No. 3 we have chosen $J_4 = \frac{1^+}{2}$, $\ell_{n4} = 0 = \ell_{\alpha 4}$ (Ref. 21).

Level No. 5 is expected to be in the range of neutron energies of 2.40 to 2.87 Mev and probably at 2.75 Mev, with a total width of the order of 0.5 Mev (Refs. 12, 17). Inglis' theories no longer have any $J = \frac{1}{2}$ levels in this region. We choose then $J = \frac{3^+}{2}$ with $\ell_{n5} = 0$ (Ref. 21); $\ell_{\alpha 5} = 2$ since the penetrabilities and formulae have already been computed for level No. 1, and since this choice is not out of line with Inglis' level schemes, and is also suggested by the magnitude of the cross sections (Refs. 2, 4, 5). Again for a best fit we have chosen $E'_5 = 2.556$ Mev, $\Gamma'_{n5} = 0.48$ Mev, and $\Gamma'_{\alpha 5} = 0.1$ Mev.

Level No. 6 is expected to lie in the region 3.100 to 4.058 Mev and to be of the order of 1 Mev in width (Ref. 17). We have chosen $E'_6 = 3.52$ Mev, $\Gamma'_{n6} = 0.65$ Mev, and $\Gamma'_{\alpha 6} = 0.8$ Mev, not only to fit the data as best we can but also to make up somewhat for cutting our analysis off beyond level No. 6. Thus level No. 6 is intended to summarize (as far as that can be done) the effect of all higher levels to the region less than 3 Mev. From the rise in the total cross section (Refs. 2, 4) and the theory (Ref. 20), we choose $J_6 = \frac{7^-}{2}$. We choose negative parity and $\ell_{\alpha 6} = 3$ for ease in calculation because we have already calculated $\ell_\alpha = 3$ penetrabilities for level No. 2. It therefore follows that $\ell_{n6} = 3$ (Ref. 21).

Having assigned spins and angular momenta we are able to write our cross section formulae with levels 1 and 5, and 3 and 4 interfering (linearly in amplitude) with each other and with potential scattering. These interferences are the only interlevel interferences accounted for in this approximation. A discussion of the errors obtained by not considering interference in reaction cross sections, for example, is given in Section IV.

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20) D. R. Inglis, Revs. Mod. Phys. 25, 390 (1953), especially pp. 411-414. These theories are in good agreement with the later compilation of Ajzenberg and Lauritsen, see note under Ref. 12, at all lower energies.

21) Weddell and Roberts, Phys. Rev. 95, 117 (1954), report $\ell = 0$ and $\ell = 1$ neutron waves predominate in $Li^6(n,\alpha)H^3$ for neutrons of 1.1, 1.5 and 2.0 Mev. Higher ℓ is not completely excluded.

The (n,t) or (n,α) cross section is given by:

$$\sigma_{n,\alpha}(\text{barns}) = \frac{0.8067}{E_n}\left(\frac{\Gamma_{n1}\,\Gamma_{\alpha 1}}{(E_n - E_1 - \delta E_1)^2 + 0.3409(\Gamma_{n1} + \Gamma_{\alpha 1})^2}\right)$$

$$+ \frac{1.210}{E_n}\left(\frac{\Gamma_{n2}\;\Gamma_{\alpha 2}}{(E_n - E_2 - \delta E_2)^2 + 0.3409(\Gamma_{n2} + \Gamma_{\alpha 2})^2}\right.$$

$$+ \frac{0.40335}{E_n}\left(\frac{\Gamma_{n3}\,\Gamma_{\alpha 3}}{(E_n - E_3')^2 + 0.3409(\Gamma_{n3} + \Gamma_{\alpha 3})^2}\right)$$

$$+ \frac{0.40335}{E_n}\left(\frac{\Gamma_{n4}\;\Gamma_{\alpha 4}}{(E_n - E_4')^2 + 0.3409(\Gamma_{n4} + \Gamma_{\alpha 4})^2}\right)$$

$$+ \frac{0.8067}{E_n}\left(\frac{\Gamma_{n5}\;\Gamma_{\alpha 5}}{(E_n - E_5 - \delta E_5)^2 + 0.3409(\Gamma_{n5} + \Gamma_{\alpha 5} + \Gamma_{05})^2}\right)$$

$$+ \frac{1.61335}{E_n}\left(\frac{\Gamma_{n6}\,\Gamma_{\alpha 6}}{(E_n - E_6 - \delta E_6)^2 + 0.3409(\Gamma_{n6} + \Gamma_{\alpha 6} + \Gamma_{06})^2}\right)$$

The neutron scattering cross section is given by:

$$\sigma_{sc}(\text{barns}) = \frac{0.8874}{E_n}\left\{\left(\frac{1.168\;\Gamma_{n2}(E_n - E_2 - \delta E_2)}{(E_n - E_2 - \delta E_2)^2 + 0.3409(\Gamma_{n2} + \Gamma_{\alpha 2})^2}\right.\right.$$

$$\left. + \frac{2}{1 + x^2}\,(\cos x + x \sin x)\,(\sin x - x \cos x)\right)^2$$

$$+ \left(\frac{-0.682\;\Gamma_{n2}\,(\Gamma_{n2} + \Gamma_{\alpha 2})}{(E_n - E_2 - \delta E_2)^2 + 0.3409(\Gamma_{n2} + \Gamma_{\alpha 2})^2} + \frac{2}{1 + x^2}\,(\sin x - x \cos x)^2\right)^2 +$$

$$+ \frac{2}{3}\left(\frac{1.168\ \Gamma_{n1}(E_n - E_1 - \delta E_1)}{(E_n - E_1 - \delta E_1)^2 + 0.3409(\Gamma_{n1} + \Gamma_{\alpha 1})^2} \right.$$

$$\left. + \frac{1.168\ \Gamma_{n5}(E_n - E_5 - \delta E_5)}{(E_n - E_5 - \delta E_5)^2 + 0.3409(\Gamma_{n5} + \Gamma_{\alpha 5} + \Gamma_{05})^2} + 2 \sin x \cos x \right)^2$$

$$+ \frac{2}{3}\left(\frac{-0.682\ \Gamma_{n1}(\Gamma_{n1} + \Gamma_{\alpha 1})}{(E_n - E_5 - \delta E_1)^2 + 0.3409(\Gamma_{n1} + \Gamma_{\alpha 1})^2} \right.$$

$$\left. + \frac{-0.682\ \Gamma_{n5}(\Gamma_{n5} + \Gamma_{\alpha 5} + \Gamma_{05})}{(E_n - E_5 - \delta E_5)^2 + 0.3409(\Gamma_{n5} + \Gamma_{\alpha 5} + \Gamma_{05})^2} + 2 \sin^2 x \right)^2$$

$$+ \frac{1}{3}\left(\frac{1.168\ \Gamma_{n3}(E_n - E'_3)}{(E_n - E'_3)^2 + 0.3409(\Gamma_{n3} + \Gamma_{\alpha 3})^2} \right.$$

$$\left. + \frac{1.168\ \Gamma_{n4}(E_{n4} - E'_4)}{(E_n - E'_4)^2 + 0.3409(\Gamma_{n4} + \Gamma_{\alpha 4})^2} + 2 \sin x \cos x \right)^2$$

$$+ \frac{1}{3}\left(\frac{-0.682\ \Gamma_{n3}(\Gamma_{n3} + \Gamma_{\alpha 3})}{(E_n - E'_3)^2 + 0.3409(\Gamma_{n3} + \Gamma_{\alpha 3})^2} \right.$$

$$\left. + \frac{-0.682\ \Gamma_{n4}(\Gamma_{n4} + \Gamma_{\alpha 4})}{(E_n - E'_4)^2 + 0.3409(\Gamma_{n4} + \Gamma_{\alpha 4})^2} + 2 \sin^2 x \right)^2$$

$$+ \frac{4}{3}\left(\frac{\Gamma_{n6}^2}{(E_n - E_6 - \delta E_6)^2 + 0.3409(\Gamma_{n6} + \Gamma_{\alpha 6} + \Gamma_{06})^2} \right)$$

$$\left. + \frac{8}{1 + x^2}(\sin x - x \cos x)^2 \right\}$$

The total neutron cross section is then:

$$\sigma_T(\text{barns}) = \sigma_{n\alpha} + \sigma_{sc} + \frac{\Gamma_{05}}{\Gamma_{\alpha 5}} (\sigma_{n\alpha 5} + \sigma_{n\alpha 6})$$

where we have put the energy variation of δE_3 and δE_4, the level shifts of resonance 3 and 4, to be negligible. The neutron contribution is in fact zero. The alpha contribution is only a few per cent variation of the shift from -0.6 to 2.3 Mev. These facts lead to further simplifications in the alpha widths, $\Gamma_{\alpha 3}$ and $\Gamma_{\alpha 4}$, exhibited below.

No potential scattering term is included for resonance No. 6, nor was any $\ell = 2$ potential scattering term included, since the magnitude of such terms corresponds to less than 0.01 barn out of 2 barns (< 7 per cent effect in interference) at 4 Mev, and this study is only up to 3 Mev for fitting the data.

The meanings of the yet undefined symbols used are given as follows: E_n is the laboratory neutron energy in Mev.

Level 1:

$$E_1 = E'_1 - \delta E_1(E'_1)$$

$$\Gamma_{n1} = \Gamma''_{n1} \cdot \frac{x}{x_2}$$

$$\Gamma_{\alpha 1} = 4.4528877 \ \Gamma'_{\alpha 1} \cdot \frac{x_\alpha}{A_2^2}$$

$$\delta E_1 = 4.4528877 \ \Gamma'_{\alpha 1} \cdot \left(1 - 0.106628 \ x_\alpha^2 - \frac{0.674525}{x_\alpha}\right)$$

Level 2:

$$E_2 = E'_2 - \delta E_2(E'_2)$$

$$\Gamma_{n2} = \Gamma'_{n2} \cdot \frac{1 + x_2^2}{x_2^3} \cdot \frac{x^3}{1 + x^2}$$

$$\Gamma_{\alpha 2} = 28.964003 \ \Gamma'_{\alpha 2} \frac{x_\alpha}{A_3^2}$$

$$\delta E_2 = -\frac{\Gamma_{n2}}{2x} - 28.964003\ \Gamma'_{\alpha 2}\left(1.5 - 0.0479991\ x_\alpha^2 - \frac{2.02210}{x_\alpha}\right)$$

Level 3:

$$E_3 = E'_3$$

$$\Gamma_{n3} = \Gamma'_{n3} \cdot \frac{x}{x_3}$$

$$\Gamma_{\alpha 3} = 0.60117629\ \Gamma'_{\alpha 3} \cdot x_\alpha$$

$$\delta E_3 = 0$$

Level 4:

$$E_4 = E'_4$$

$$\Gamma_{n4} = \Gamma'_{n4} \cdot \frac{x}{x_4}$$

$$\Gamma_{\alpha 4} = 0.56804700\ \Gamma'_{\alpha 4} \cdot x_\alpha$$

$$\delta E_4 = 0$$

Level 5:

$$E_5 = E'_5 - \delta E_5(E'_5)$$

$$\Gamma_{n5} = \Gamma'_{n5} \frac{x}{x_5}$$

$$\Gamma_{\alpha 5} = 1.7284708 \cdot \Gamma'_{\alpha 5} \cdot \frac{x_\alpha}{A_2^2}$$

$$\Gamma_{05} = 7\ \Gamma_{\alpha 5}\ \exp\left[-\left(\frac{3.8}{E_n}\right)^2\right]$$

$$\delta E_5 = -1.7284703\, \Gamma'_{\alpha 5} \left(1 - 0.106628\, x_\alpha^2 - \frac{0.674525}{x_\alpha}\right)$$

Level 6:

$$E_6 = E'_6 - \delta E_6(E'_6)$$

$$\Gamma_{n6} = \Gamma'_{n6} \cdot \frac{225 + 45\, x_6^2}{x_6^7} \cdot \frac{x^7}{225 + 45\, x^2}$$

where the approximation has been made that $(225 + 45\, x^2) >> (6x^4 + x^6)$ up to 3.5 Mev.

$$\Gamma_{\alpha 6} = 5.9059088\, \Gamma'_{\alpha 6}\left(\frac{x_\alpha}{A_3^2}\right)$$

$$\Gamma_{06} = 7\Gamma_{\alpha 6} \exp\left[-\left(\frac{3.8}{E_n}\right)^2\right]$$

$$\delta E_6 = -\frac{\Gamma_{n6}}{2x} - 5.9059088\, \Gamma'_{\alpha 6}\left(1.5 - 0.0479991\, x_\alpha^2 - \frac{2.0221}{x_\alpha}\right)$$

In the above formulae,

$$A_2^2 \equiv 10 \exp\left(0.17829 + \frac{1.171762}{x_\alpha} - 0.0926154\, x_\alpha^2\right)$$

and

$$A_3^2 \equiv 10 \exp\left(- 0.532344 + \frac{3.51272}{x_\alpha} - 0.0416912\, x_\alpha^2\right)$$

and these are the Coulomb functions (Refs. 9, 22):

$$A_\ell \equiv \left|F_\ell^2 + G_\ell^2\right|^{\frac{1}{2}}$$

and we have fitted these functions with

$$A \equiv 10 \exp\left(a + \frac{b}{x_\alpha} + c\, x_\alpha^2\right)$$

the form of which is suggested by the true Geiger-Nuttall approximation of alpha decay (Ref. 23). The accuracy of this approximation is far better than 7 per cent in the magnitude of A_ℓ over the entire range of interest.

$$x_\alpha \equiv 0.637224\sqrt{E_n + 5.582156} \qquad \text{(Ref. 24)}$$

which is the wave number times the alpha-triton radius, R_α, which we have set equal to 2.4×10^{-13} cm on the following grounds:

1. The corrected data of Blair and Holland (Ref. 25), being high at the high energy side, suggest that the alpha width is rapidly increasing through the resonance. A rapid variation in alpha width can be achieved by a large barrier obtained from a small radius, R_α.

2. On the other hand, the interaction radius cannot be made arbitrarily small. Under the concept of the Coulomb barrier rising where the short range nuclear attractive force is overcome by the long range Coulomb repulsion (Ref. 26), we can associate the effective inner Coulomb barrier radius with the radius at which nuclear forces begin to be strong. Because of the greater size of the triton, use of a neutron as a probe for the nuclear force range of He^4 is expected to give a lower bound to the alpha-triton radius, R_α, of about 2.4×10^{-13}cm (Refs. 23,27).

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22) I. Bloch et al., private communication, "Coulomb functions for reactions of Protons and Alpha Particles with the Lighter Nuclei," Yale University. Breit, Abramowitz, et al., "Tables of Coulomb Wave Functions," National Bureau of Standards, A. M. S. 17 (1952).

23) See, for example, J. J. Devaney, LA-1506, Los Alamos Scientific Laboratory (1953), or Ph. D thesis, Mass. Inst. Tech. (1950).

24) Q $Li^6(n,t)$ = 4.780 ± 0.006 Mev, Ajzenberg and Lauritsen, Revs. Mod. Phys. 24, 321 (1952).

25) Blair and Holland, private communication to F. C. Hoyt, August, 1950, as corrected by the revised fission cross section of U^{235} given by Barschall and Henkel, LA-1714, Los Alamos Scientific Laboratory (1954), and by Ferguson, AERE, England, private communication (Geneva) from B. Diven, LASL (1955). Note that $\sigma Li^6(n,\alpha)H^3$ as given in BNL-325 and AECU 2040 is uncorrected (D. Hughes, private communication).

26) See, for example, J. J. Devaney, Phys. Rev. 91, 587 (1953).

27) Bashkin, Petree, Mooring, and Peterson, Phys. Rev. 77, 748A (1950) and S. Bashkin, private communication.

One might possibly argue that the alpha-triton radius is as low as 2.4×10^{-13} cm by holding that the triton charge is not shared by all nucleons, but is concentrated on the proton, and we have a "heavy" proton-alpha system at least insofar as the barrier is concerned. In In view of paragraph 1, we adopt such a lower limit and choose $R_\alpha = 2.4 \times 10^{-13}$ cm.

Returning again to the explanation of symbols, we define:

$$x \equiv 0.18816886 \sqrt{E_n} \cdot R_n \cdot 10^{13}$$

$$x_i \equiv 0.18816886 \sqrt{E_i'} \cdot R_n \cdot 10^{13}$$

where x's are the neutron wave number times the n-Li^6 interaction radius, R_n. R_n is determined from the total cross section fit away from resonances to be 3.382×10^{-13} cm. (See the discussion of fitting experiment below.)

E_i are the Wigner-Eisenbud boundary condition resonances of the compound system, as stated earlier in this section.

δE_i are the level shifts.

Γ_{ni} are the neutron widths.

$\Gamma_{\alpha i}$ are the alpha-triton widths.

Γ_{0i} is the width of all other processes than scattering and (n, t) (see Section II for thresholds). The lowest threshold of these processes is given by that of $Li^6(n;d,n)He^4$ and is 1.722 Mev (Ref. 8). Assuming that the (n, t) cross section, in the absence of these processes, would behave somewhat similar to the total cross section above 1.722 Mev (Refs. 2, 4), and comparing this assumption with the actual (n, t) cross section as given by Ribe (Ref. 5), we mock up the competition by the following very crude formula:

$$\Gamma_{0i} = 7\, \Gamma_{\alpha i} \exp\left[-\left(\frac{3.8}{E_n}\right)^2\right]$$

This formula has the essential property that Γ_0 is negligible compared with Γ_α below 1.7 Mev.

Before adjusting the theory to experiment one must eliminate extraneous broadening of the levels. Insofar as level No. 2 is concerned, the quoted experimental resolution in the total cross section is so low that we feel that the experimental cross section is very closely the true cross section. The (n, t) part of resonance No. 2 measurements is quoted at a resolution of 25 kev (Ref. 25). The Maxwellian-Doppler width, $D \equiv 2\sqrt{(mE_n kT)/A}$, where m is the neutron mass, A is the target mass, and kT is $\frac{2}{3}$ the thermal energy (Ref. 27a), is about 70 ev at 0.25 Mev and less than 280 ev at 4 Mev. Since all the widths that we are concerned with are large fractions of a Mev, we are led to neglect Doppler broadening.

Now that we have determined the character and number of levels and have exhibited our formulae, we can conclude our discussion of how the experimental work was fitted. We have already shown how we obtained R_α. We shall discuss the fitting of the curves starting at low energy and progressing to higher energy.

The thermal (n, t) cross section is given by Hughes and Harvey (Ref. 28) to be 945 barns ± 1 per cent, and the total cross section at 3 ev is 81.7 barns ± 3 per cent and is consistent with $\frac{1}{v}$ (Ref. 29) at higher energies. The 3 ev cross section is below a $\frac{1}{v}$ extrapolation of the thermal (n, t) cross section by more than the quoted errors. In this formalism such behavior could be accounted for by a very thin level (~3 ev) just below thermal. This level has not been observed nor expected, nor is a level of such a small width expected. (See the discussion of probable error of the formalism in Section IV). To fit both data as closely as possible we want to make σ_{sc} small; hence we choose $\Gamma'_{\alpha 1}$ quite large and Γ''_{n1} small, placing the total width and position at the values derived from Stoll (Ref. 17); further, we adopt the compromise thermal value of 935 barns (a 1 per cent change, therefore within quoted error).

In the case of resonance No. 2, we first attempted to fit the total cross section of Hughes and Harvey, BNL-325 (Ref. 4) and the corrected (n, t) cross section of Blair and Holland (Ref. 25). We found it impossible to fit both simultaneously, and we took the total cross section as being more accurate than the (n, t) (Ref. 29a), and chose to accurately fit the total

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27a) P. Morrison in E. Segrè, Ed., Experimental Nuclear Physics, Vol. II, John Wiley and Sons, Inc., New York, 1953.

28) Hughes and Harvey, BNL-325 (1955); errors are from a private communication of D. J. Hughes, 1955.

29) Private communication, J. A. Harvey (1955).

29a) The Blair and Holland measurement (Ref. 25) of the ratio of counting rates of $Li^6(n,t)He^4$ to U^{235}(fission) has an estimated statistical accuracy of 2 per cent; we estimate the accuracy in $\sigma_f(U^{235})$ to be less than 10 per cent. Their energy resolution is put at 25 kev. The most serious criticism of using the Blair and Holland preliminary (1950) values is that their measurements exhibit an internal inconsistency (corrected) of from 12 to 28 per cent. This inconsistency is that the (n, t) cross section of Li as obtained by multiplying the Li^6(n, t) cross section by the isotopic abundance exceeds that measured by them directly, by 12 to 28 per cent. In emphasizing to Dr. Frank Hoyt that their work is very preliminary, they have further stated that they have not always been able to reproduce early counting rates. In short, Blair and Holland have made every effort to caution users of their preliminary data, and we therefore will no longer use their data as criteria for our fit. We have only considered this work, because (1) it is the only (n, t) work that we know of at this energy, and (2) others have been using the uncorrected Blair-Holland work.

cross section alone. In doing this, we found that the total cross-section data of BNL-325 had a shape that couldn't be as accurately fit as the data of Johnson et al. (Ref. 2). Our best fit, therefore, accurately follows the Johnson curve, except where it lies between the Johnson and BNL-325 curves. However, both experiments and this theory lie well within the experimental error of each other over the resonance, so that these differences are not really significant.

It is interesting to note, however, that the peak of the cross section curve of Johnson et al. lies higher than that of BNL-325. Moreover, use of the Johnson data brings the (n, t) peak of the theory closer to the (n, t) peak position of the Blair and Holland results. The maxima of the total cross section and the (n, t) cross section lie at about 0.255 Mev and 0.250 Mev, respectively, in this analysis.

The neutron radius, R_n, was adjusted to give the experimental (Johnson) cross section between the $\frac{1}{v}$ type rise and the 0.256 Mev resonance. The value was found to be $R_n = 3.382 \times 10^{-13}$ cm. This large value is consistent with the concept of Li^6 as an alpha particle plus a deuteron, and is somewhat consistent with the suggestion of Dabrowski and Sawicki (Ref. 30) that the interaction radius for $Li^6(n,t)He^4$ is about 4×10^{-13} cm. Also trial calculations using the two level formulas favored large R_n and small R_α out of a limited choice of radii.

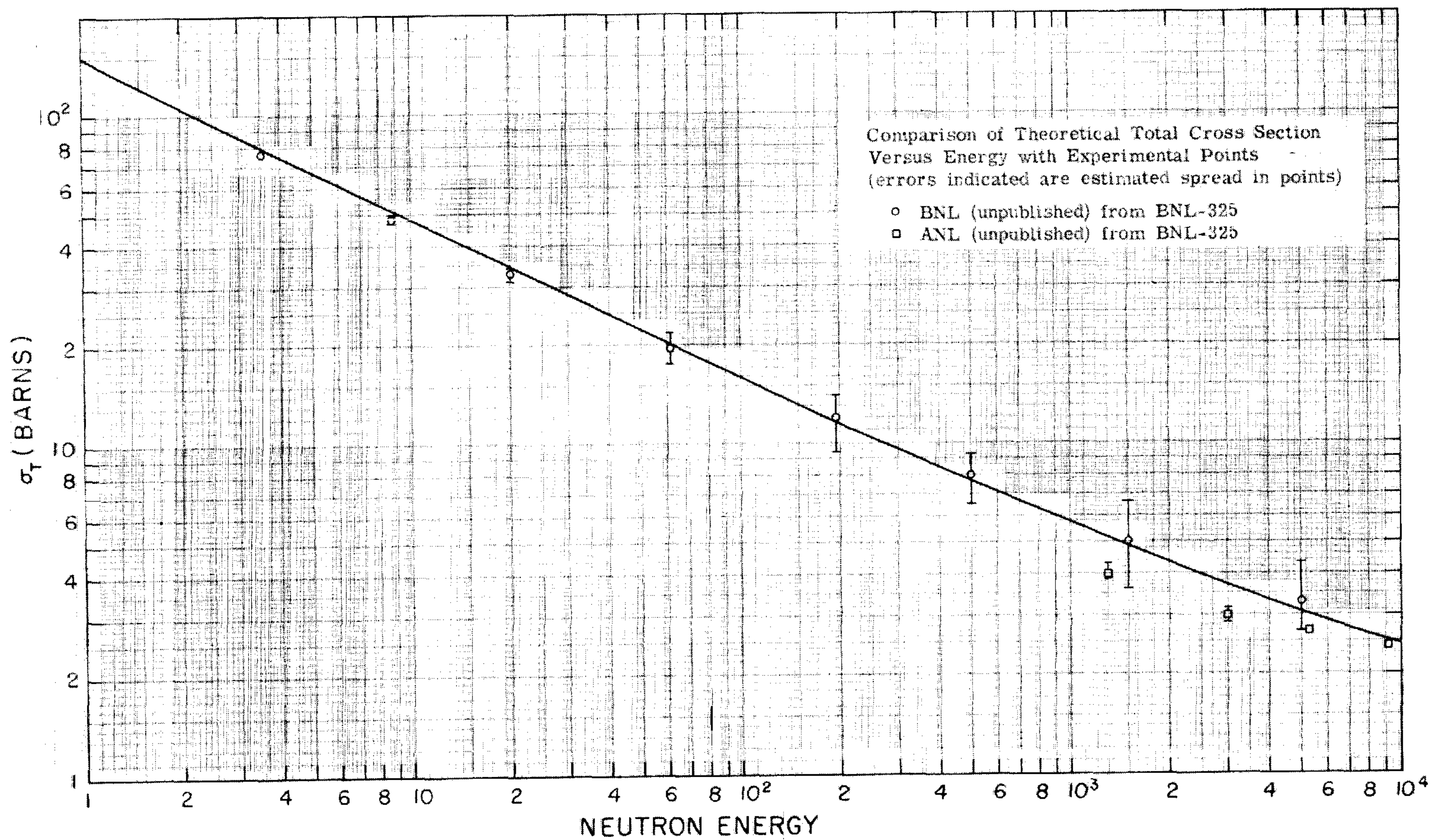
The contribution of resonances 3 to 6 was obtained by varying position and widths to get the best fit to the total cross section (Refs. 2, 4) from 0.6 to 3.2 Mev and to the (n, t) cross section (Ref. 5) from 1 to 3.2 Mev, while staying within the bounds set by the experimental work on $Li^7(\gamma,t)He^4$ of Stoll et al. (Ref. 17).

The accuracies of the theoretical fit are exhibited in Section IV. It should be noted that the theory must fit not only a few points, but all points where the two agree, and so in a sense is a form of statistical average, and is therefore better than an individual experimental point. Where the two are in definite disagreement, however, the question is still open, and the next section will try to make clear the failings of this analysis so that the reader can judge for himself which curves are the more accurate.

IV. Discussion of Probable Errors

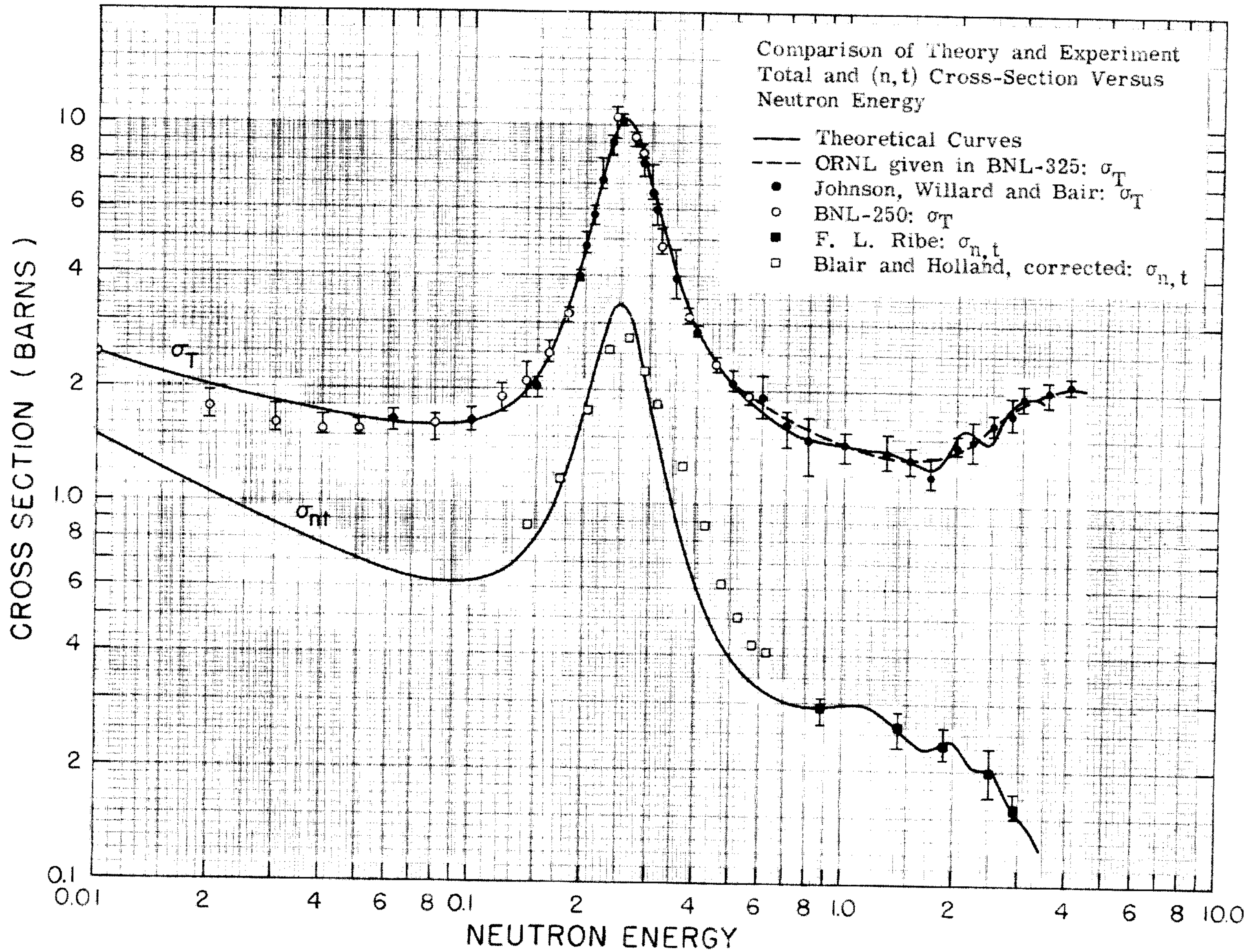
Graphs 3 and 4 show the theoretical curves together with representative experimental curves and points. In order to reduce the number of experimental points, we have averaged the points of a region to one point and have exhibited their spreads and statistical errors by the usual probable error limits on each point.

30) Dabrowski and Sawickie, Phys. Rev. 97, 1002 (1955), using the work of Lauritsen, Huus, and Nilsson, Phys. Rev. 92, 1501 (1953).



Comparison of Theoretical Total Cross Section Versus Energy with Experimental Points
(errors indicated are estimated spread in points)
○ BNL (unpublished) from BNL-325
□ ANL (unpublished) from BNL-325
σ_T (BARNS)
NEUTRON ENERGY

Graph 3


Comparison of Theory and Experiment
Total and (n, t) Cross-Section Versus
Neutron Energy
Theoretical Curves
ORNL given in BNL-325: σ_T
Johnson, Willard and Bair: σ_T
BNL-250: σ_T
F. L. Ribe: $\sigma_{n,t}$
Blair and Holland, corrected: $\sigma_{n,t}$
σ_T
σ_{nt}
CROSS SECTION (BARNS)
NEUTRON ENERGY
10
8
6
4
2
1.0
0.1
0.01
0.1
1.0
10.0

Graph 4

The first discrepancy pointed out by this study is that between a $\frac{1}{v}$ extrapolation of the thermal (n, t) cross section to 3 ev (86.3 barns) and the measured total cross section at 3 ev (81.7 barns). (Our fit compromised between these values.) It is by no means certain that a complete analysis will show that there truly is a discrepancy here. As a matter of fact, the two level approximation (Ref. 31) does show destructive interference between two reaction levels of the same character, and with our choice of parameters, interference between levels 1 and 5 can reduce the (n, t) cross section at 3 ev by as much as 13.5 barns, an effect, which if selective at 3 ev and not at 0.025 ev, could easily bring the theoretical curves into agreement with experiment at both 0.025 and 3 ev.

The rub is in the word "selective." With our choice of parameters, for example, the choice of maximum interference produces identical changes at both 0.025 ev and at 3 ev, the $\frac{1}{v}$ law is not affected, and the discrepancy apparently still exists, interference or no. However, there is nothing sacred about our parameters or even about our choice of level position and character. Their only claim to fame is that they fit the present experiment, but others could perhaps be made to fit as well, and experiment is none too detailed on the matter. In addition, the interference theory is not as simple as we have sketched. In short, if the present experimental values are confirmed, we have a qualitative excuse for the failure of the $\frac{1}{v}$ law, but until the positions, characters, and widths of the Mev levels are known, it is fruitless to attempt a quantitative study using the general theory. We therefore quite arbitrarily assign an error of 6 per cent to the $\frac{1}{v}$ law section of our curves on the basis of the existing discrepancy with experiment.

The choice of the Johnson, Willard and Bair total cross section over the BNL-325 total cross section at resonance No. 2 is, as we have said, not significant, since all three curves (two experiment and one theory) lie within experimental error of each other. It is interesting to note, however, that it was impossible for this author to fit the BNL-325 data at all closely, whereas the Johnson data were relatively easy to fit. Moreover, the Johnson work is fitted using a higher value for the resonance, E'_2, which leads to a much better agreement in the (n, t) curve than the BNL-325 does.

We used the total cross sections exclusively to obtain the parameters of level No. 2 on the grounds that Blair and Holland evidently never did have a chance to complete their work (Ref. 29a), and on the grounds that the theory would not fit both, and that the total cross section was accurately measured. The discrepancy so obtained between the theory and experiment in the (n, t) cross section over resonance No. 2 may possibly be explained by poor

31) E. P. Wigner, Phys. Rev. 70, 609 (1946).

experimental energy resolution, especially on the high side of the resonance, or even by a weak $J = \frac{1}{2}$ resonance just above resonance No. 2. No indication of the latter in the literature is known to this author, however, and the former appears unlikely.

Our knowledge of the character, position, and magnitude of the levels 3 to 6 is indeed weak, although accurately fitting both the (n, t) and the total cross section within the bounds of positions and sizes given by the $Li^7(\gamma, t)He^4$ data is a stringent restriction. We are led to believe, therefore, that a calculation of the contribution of the poorly known levels to the (n, t) cross section in the region of the well known level No. 2 will give some idea, indeed a conservative idea, of the probable error. Equating our error to the fractional contribution of levels 3 to 6 will be very conservative, if we have done a good job in the Mev region, and of course becomes more and more conservative as we increase our energy above level 2, until at 1 Mev it becomes 81 per cent whereas our true error is actually experimental (i. e., of the order of 7 per cent), and could be made less with better experiments. Another factor tending to make our error estimation conservative is the extra broadness of the levels, which have been so made to compensate in some degree for cutting off the analysis above 3.5 Mev. Interference, on the other hand, with its subtraction occurring in amplitude and not in cross section can invalidate these arguments, especially if we have chosen the wrong spins and parities of the levels.

Together with the 6 per cent error in the region 0.01 to 1000 ev, quoted above, we may equate the per cent contribution of levels 3 to 6 to a conservative error in the region 0.03 to 0.5 Mev, viz.,

Energy, E_n, Mev	Contribution of levels 3 to 6 to $\sigma(n, t)$ per cent
0.09	35
0.2	10
0.250	4.8
0.3	9
0.5	43
1.0	81

Since we actually fit the cross sections to within their experimental errors above 0.5 or 0.8 Mev to 3.4 Mev, we are therefore as accurate as experiment in this region, and the error criterion of contribution of levels 3 to 6 is absurd here.

V. Suggestions for Further Experimental Study

1. The failure of the $\frac{1}{v}$ law from thermal to 3 ev might be affirmed by more careful measurement, but in this author's opinion such remeasurement should only be attempted if relatively easy. The reason for this limited recommendation is that such remeasurement will emphasize the errors without providing data on which more detailed calculations can be based.

2. Since the validity of a $\frac{1}{v}$ (or to be more accurate, an "isolated level") approximation is in question, it would be helpful to determine both the scattering and the (n, t) cross section at the same low energy, say below 1 kev and preferably close to thermal, in order to fix the relative magnitudes of the neutron and alpha widths of the negative energy level (No. 1).

3. The difference between this author's prediction and the Blair and Holland work on the (n, t) cross section over the 0.256 Mev resonance, as exhibited in Graph 4, should be resolved.

4. Any work which gives the spins, angular momentum, parity, widths, decay probability, and/or cross sections of the levels, especially the levels above 0.256 Mev, will contribute to a better understanding of this problem, and will certainly lead to a better analysis. If $Li^7(\gamma, t)He^4$ experiments can be improved in resolution to do better in this matter, this author urges that such work be continued.