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A SIMPLIFIED METHOD FOR CALCULATING CRITICAL SIZES

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ABSTRACT

Serber's new approximate method for calculating critical sizes is analyzed. It can be reduced to the condition that a certain quantity referring to the tamper alone is equal to a corresponding quantity referring to the core alone.

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A SIMPLIFIED METHOD FOR CALCULATING CRITICAL SIZES

Serber has recently developed a very simple method for calculating critical masses and multiplication rates in the case of one neutron velocity. (See report of Group T-2 in LAMS-192, and a forthcoming LA Report). This method is based on assuming a simple exponential distribution of neutrons in the tamper which is normalized by the condition that all neutrons produced in the core are absorbed in the tamper. Similarly the neutron distribution in the core is assumed to be (sin kr)/kr with k determined from the usual integral-theory formula. The critical radius is then determined by the condition that the number of neutrons arriving at the center in an integral treatment is equal to the value unity required by the assumed distribution.

The formula thus obtained has been tested by Kurath against the known results for critical radii obtained by Glauber by the spherical harmonic method. The agreement was found to be very good for a great variety of ratios of tamper to core mean free path and of f values in core and tamper.

This result seems to be of considerable importance not only because it will greatly simplify and make possible calculations for composite tampers and for more than one neutron

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velocity but also because it permits a very simple physical interpretation. It is easily seen that in Serber's method it is possible to divide the problem into one for the core and one for the tamper. Serber's fundamental equation can be written:

$$(1 + f)\,\frac{k^2}{f}\,(\sigma a)^2\,\frac{-\mathrm{Im.P.}\;\; E_1(\sigma a(1 + ik))}{\sin k\sigma a - k\sigma a \cos k\sigma a}\; e^{\sigma a}$$

$$= (1-g)\,\frac{h^2}{g}\,(\sigma' a)^2\,\frac{E_1(\sigma' a\,(1 + h))}{1 - h\sigma' a}\; e^{\sigma' a(1 + h)} \equiv 3T \qquad (1)$$

where the unprimed quantities, referring to the core, are found in the first number of the equation whereas the primed quantities, referring to the tamper, are all in the second. The notation used in Eq. (1) is essentially the same as in the report by Reines LA-173, h being the analog in the tamper of the usual k and $-g$ the analog of f, while a is the core radius. σ is the transport cross section in the core and σ' that in the tamper. In subsequent formulae n denotes the neutron density and $\sigma_{r'}$ the capture cross section in the tamper. E_1 is the exponential integral,

$$E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t}\, dt$$

The physical significance of the expression on the second number of the equation is very simple. It is 4n

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times the flux (nv) of neutrons per unit solid angle coming in from the tamper to the interface along a radius, divided by the <u>net</u> flux of neutrons entering the tamper per unit area of the interface. This ratio can easily be calculated since the distribution of neutrons in the tamper is supposed to be given, viz.

$$n = \frac{B}{r} e^{\sigma' h(r-a)} \qquad (2)$$

The net flux per unit surface area is given by the number of neutrons absorbed in the tamper, i.e.

$$F = \frac{4n\sigma_r \int_a^\infty n(r)r^2dr}{4na^2} \quad \frac{\sigma' g}{a^2} \int_a^\infty n(r)r^2dr \qquad (3)$$

The flux of neutrons coming to the interface along a radius per unit solid angle is

$$n_{in}(a) = \left(\frac{1}{4n}\right) \int_a^\infty n(r)e^{-\sigma'(r-a)}dr \qquad (4)$$

Inserting Eq. (2) into Eq. (3) and (4) Serber's result, namely the second member of Eq. (1), is obtained.

The second number of Eq. (1) thus represents a measure of the quality of the tamper which of course depends on the radius of the interface. Two tampers having the same value of the "reflection coefficient" Γ will be equivalent. This makes it possible to find a simple infinite tamper

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equivalent to a given composite and possible finite tamper.

The reason why the tamper reflection coefficient comes in in this form is of course that the number of neutrons arriving at the center of the gadget from the tamper is given by $Te^{-\sigma a}$. From the point of view of the core, T therefore represents $e^{\sigma a}$ times the dificiency of neutrons arriving at the center from the core. From this definition it is easy to derive the left hand side of Eq. (1).

It is easy to see that the tamper reflection coefficient T goes over into the correct diffusion-theory limit. In diffusion theory $\sigma' a$ becomes very large and h becomes very small. In that case the right hand side of Eq. (1) simplifies to

$$T \rightarrow \frac{1}{(1/\sigma' a) + h} \tag{5}$$

In diffusion theory the neutron flux at any one point is approximately isotropic so that 4n times the flux per unit solid angle is equal to v times the neutron density at that point. Therefore from diffusion theory

$$\frac{n(a)\, v}{-\lambda' v (dn/dr)_a} = \frac{\sigma'}{-(d \log n/dr)_a} = \frac{\sigma'}{(1/a) + \sigma' h} \tag{6}$$

In the last transformation we have used the form of the neutron distribution in the tamper, Eq. (2).

While Serber's expression reduces to the diffusion theory expression in the limit, it is far from it in practical cases. The following table gives the ratio of the value of T from Eq. (1) to the diffusion theory value for various tamper properties. This demonstrates the well-known fact that the value of a tamper is not as great in integral theory as it is in diffusion theory. The opposite fact is of course observed for active material.

TABLE OF $T/T_{diff.}$

σ'a	$T/T_{diff.}$		
	h = .20	h = .50	σ'ah = 1
.5	.404	.304	---
1	.513	.379	0
1.2			.212
1.5	.573		.327
2	.611	.444	.447
3	.661	.473	.576
5	.709	.497	.719
10	.754	.530	.835
large	.813	.564	$1-2/\sigma$'a

The left hand side of Eq. (1) can also be shown to go over into the correct limit in differential theory.

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