UNCLASSIFIED

SIFIED

221

3

15

[redacted]

CLASSIFICATION CANCELLED

Per DOC

By [illegible] 1/14/73

[redacted]

UNCLASSIFIED

LA-221

[redacted]

February 12, 1945

This document contains 18 pages

PROPAGATION OF DETONATION WAVES ALONG THE INTERFACE BETWEEN TWO EXPLOSIVES

WORK DONE BY:

W. A. Bowers

REPORT WRITTEN BY:

W. A. Bowers

ABSTRACT

The ideal problem of a detonation wave propagating along the interface between two semi-infinite explosives has been treated in the case when the point of initiation is infinitely distant. In a coordinate system in which the detonation fronts are stationary, the pressure, density and material velocities behind the front are functions only of the angle with the detonation front. For a given ratio of normal detonation velocities, three cases arise: (1) The angle between the detonation fronts assumes its "normal" value (i.e. its cosine is the detonation velocity ratio), and there is a rarefaction behind each front, (2) For sufficiently great ratio of density of fast explosive to slow, a shock is sent from the fast to the slow explosive, which forces it to travel with greater than its normal velocity, thus reducing the "refractive index", (3) For sufficiently small ratio of densities, the opposite occurs, i.e. the fast explosive is speeded up, thus increasing the "refractive index." Graphs have been plotted showing the limiting values of density ratio at which case (1) goes over into cases (2) and (3) respectively, as a function of normal refraction angle. This has been done assuming (a) an adiabatic index $\gamma = 3$ for reaction products of both explosives, (b) $\gamma = 2$, (c) $\gamma = 2$ for slow and $\gamma = 3$ for fast explosive. Graphs have also been plotted showing the deviation from normal refracting angle as a function of density ratio for some special cases. In a typical case, assuming a normal refractive index of 2, i.e. refracting angle $= 60^\circ$, the deviation is zero from density ratio .08 to .55; above .55 it falls slowly to -5.6° deviation at density ratio 2.0.

[redacted]

PROPAGATION OF DETONATION WAVES ALONG THE INTERFACE BETWEEN TWO EXPLOSIVES

1. We consider here the problem of two semi-infinite explosives bounded by a plane, detonated an infinite distance away. We expect the detonation front in the faster explosive to be a plane perpendicular to the interface, and that in the slower explosive to be a plane making an angle α with the normal to the interface, where in the absence of persistence effects we should have $\cos \alpha$ = ratio of normal detonation velocities in the two media. We shall investigate in particular under what circumstances this is true, and also attempt to calculate α in cases where this is not true.

2. If we transform to a system of co-ordinates in which the detonation fronts are at rest, we have a stationary phenomenon. With the assumptions stated above the problem contains no constant of the dimension of a length, and hence all quantities (pressure, density, material velocity, etc.) can depend only on the angle θ measured from the plane of the fast explosive detonation front, with the origin at the intersection of the detonation fronts and the interface. If we denote the radial and tangential components of material velocity by u_r and u_θ respectively, the equations of motion become:[1)]

$$u_\theta - du_r/d\theta = 0 \qquad (1)$$

$$u_\theta \, (du_\theta/d\theta + u_r) = -\frac{1}{\rho} \frac{dp}{d\theta} \qquad (2)$$

1). Cf. LA-165, Section 12.

[redacted]

[redacted]

and the continuity equation becomes:

$$\frac{du_\theta}{d\theta} + u_r = - \frac{u_\theta}{\rho} \frac{d\rho}{d\theta} \tag{3}$$

Here p and ρ denote pressure and density respectively. Using $dp/d\rho = c^2$, where c is sound velocity, (2) becomes:

$$u_\theta \left(\frac{du_\theta}{d\theta} + u_r \right) = - \frac{1}{\rho} c^2 \frac{d\rho}{d\theta} \tag{2a}$$

Combining this with (3) we have:

$$(u_\theta^2 - c^2)\, d\rho/d\theta = 0 \tag{4}$$

Eq. (4) has two solutions: A) ρ = constant, which leads to $u_\theta = C \cos(\theta + \delta)$, $u_r = C \sin(\theta + \delta)$, which clearly means constant material velocity. B) $u_\theta = \pm c$ where the sign is to be chosen so that c is positive. If we now make the further assumption that the reaction products of the explosive obey an adiabatic law with index γ : $p \sim \rho^\gamma$, so that $c^2 \sim \rho^{\gamma - 1}$, and hence $d\rho/\rho = [2/(\gamma - 1)]\, dc/c = [2/(\gamma - 1)]\, du_\theta/u_\theta$, eq. (2a) becomes:

$$\frac{du_\theta}{d\theta} + u_r = - u_\theta \cdot \frac{2}{\gamma - 1} \cdot \frac{1}{u_\theta} \frac{du_\theta}{d\theta}$$

$$\text{or} \quad \frac{\gamma + 1}{\gamma - 1} \frac{du_\theta}{d\theta} + u_r = 0$$

[redacted]

With eq. (1), this yields:

$$u_r = C \cdot \sqrt{(\gamma + 1)/(\gamma - 1)} \; \sin\left(\theta \sqrt{(\gamma - 1)/(\gamma + 1)} + \delta\right) \qquad (5)$$

$$\pm c = u_\theta = C \cos\left(\theta \sqrt{(\gamma - 1)/(\gamma + 1)} + \delta\right) \qquad (6)$$

This is the familiar Prandtl-Meyer solution for the problem of supersonic flow around a corner.

3. Under certain circumstances a solution of the problem exists of the following nature: behind the detonation front in each explosive there is an expansion (i.e. a Prandtl- Meyer region) which goes over at a certain angle into a region of constant density, material velocity, etc., (cf. Fig. 1; dotted lines indicate expansion region I and IV). The angular width of each expansion region, and the angle at which the interface between the two explosives moves off, can be determined by requiring continuity of material velocity and pressure at the boundary of each expansion region, and continuity of pressure at the interface between the two media. One also obviously requires the vanishing of the tangential material velocity component at the interface, in order to avoid mixing of the two media; the radial component may however be discontinuous (i.e. the interface may be a "slipstream").

We shall now write down the solutions explicitly. Let β be the angle at which the expansion region behind the fast-explosive detonation front ends; let ϕ be the angle at which the interface moves off; let β' be the angle at which the expansion region of the slow explosive ends; the slow-explosive detonation front is at the angle $\pi - \alpha$. Now at the fast-explosive detonation front we have, from the Chapman-Jouguet condition, that the material velocity is $1/(\gamma + 1)$ and the sound velocity is $\gamma/(\gamma + 1)$ in units of the detonation velocity; in our system of co-ordinates (detonation front at rest) this means a material velocity of $\gamma/(\gamma + 1)$; since this is normal to the front, we have that for $\theta = 0$, $u_\theta = \gamma/(\gamma + 1)$, $u_r = 0$, and $c = \gamma/(\gamma + 1)$. We have therefore in region I, $(0 < \theta < \beta)$

$$c = u_\theta = \frac{\gamma}{\gamma + 1} \cos \left(\theta \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right) \tag{7a}$$

$$u_r = \frac{\gamma}{\sqrt{\gamma^2 - 1}} \sin \left(\theta \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right) \tag{7b}$$

In region II, $\beta < \theta < \phi$, we have constant pressure and velocity; the requirement of continuity gives:

$$c_{II} = \frac{\gamma}{\gamma + 1} \cos \left(\beta \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right) \tag{8a}$$

$$u_\theta = A_{II} \cos (\theta - \delta_{II}) \tag{8b}$$

$$u_r = A_{II} \sin (\theta - \delta_{II}) \tag{8c}$$

where

$$A_{II} = \frac{\gamma}{\gamma + 1} \sqrt{1 + \frac{2}{\gamma - 1} \sin^2 \left(\beta \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right)} \tag{8d}$$

$$\delta_{II} = \beta - \tan^{-1} \left[\sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan\left(\beta \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right)\right] \tag{8e}$$

Similarly at the detonation front of the slow explosive, we have the material velocity equal to $\cos\alpha/(\gamma+1)$, since we are assuming here that the slow detonation front is at its "normal" position. Transforming to our system of co-ordinates, we find that at the detonation front we have $u_\theta = -(\gamma\cos\alpha)/(\gamma+1)$, $u_r = \sin\alpha$, $c = (\gamma\cos\alpha)/(\gamma+1)$. This gives the following solution in region IV, ($\beta' < \theta < \pi - \alpha$)

$$c = -u_\theta = \frac{\gamma}{\gamma+1}\sqrt{1-\frac{\sin^2 d}{\gamma^2}}\ \cos\left[(\theta-\psi+\alpha-\pi)\sqrt{\frac{\gamma-1}{\gamma+1}}\right] \tag{9a}$$

$$-u_r = \frac{\gamma}{\sqrt{\gamma^2-1}}\sqrt{1-\frac{\sin^2 d}{\gamma^2}}\ \sin\left[(\theta-\psi+\alpha-\pi)\sqrt{\frac{\gamma-1}{\gamma+1}}\right] \tag{9b}$$

$$\psi = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left[\frac{\sqrt{\gamma^2-1}}{\gamma}\ \tan\alpha\right] \tag{9c}$$

In region III ($\phi < \theta < \beta'$) we have immediately, from the requirement of continuity at $\theta = \beta'$:

$$c_{III} = \frac{\gamma}{\gamma+1}\sqrt{1-\frac{\sin^2\alpha}{\gamma^2}}\ \cos\left[(\beta'-\psi+\alpha-\pi)\sqrt{\frac{\gamma-1}{\gamma+1}}\right] \tag{10a}$$

$$u_\theta = A_{III}\cos(\theta-\delta_{III}) \tag{10b}$$

$$u_r = A_{III}\sin(\theta-\delta_{III}) \tag{10c}$$

$$A_{III} = \frac{\gamma}{\gamma+1}\sqrt{1-\frac{\sin^2\alpha}{\gamma^2}}\ \sqrt{1+\frac{2}{\gamma-1}\ \sin^2\left[(\beta'-\psi+\alpha-\pi)\sqrt{\frac{\gamma-1}{\gamma+1}}\right]} \tag{10d}$$

$$\delta_{III} = \beta' - \pi - \tan^{-1}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}}\ \tan\left[(\beta' - \psi + \alpha - \pi)\sqrt{\frac{\gamma - 1}{\gamma + 1}}\right]\right) \tag{10e}$$

The condition that $u_\theta = 0$ at the interface, $\theta = \phi$, leads to the condition $\phi - \delta_{II} = \phi - \delta_{III} = \pi/2$; hence we must have $\delta_{II} = \delta_{III}$. This gives one equation between β and β'; the other equation must come from the continuity of pressure at $\theta = \phi$. We shall find the relation necessary between c_{II} and c_{III} in order that the pressure be continuous. From $c^2 = \gamma p/\rho$, we have, assuming γ the same in each medium $(c_{II}/c_{III})^2 = \rho_{III}/\rho_{II}$. Further, in each medium, we have $p = p_c(\rho/\rho_c)^\gamma$ where p_c and ρ_c are the Chapman-Jouguet values of pressure and density, respectively. Using $\rho_c = [(\gamma + 1)/\gamma]\rho_o$, where ρ_o is the normal density, and $D^2 = (\gamma + 1)p_c/\rho_o$, where D is the normal detonation velocity, this becomes:

$$p = \frac{1}{\gamma + 1}\left(\frac{\gamma}{\gamma + 1}\right)^\gamma \frac{D^2}{\rho_o{}^{\gamma - 1}}\ \rho^\gamma$$

Continuity of pressure therefore requires:

$$(\rho_{III}/\rho_{II})^\gamma = (\rho_s/\rho_f)^{\gamma - 1}\ (D_f/D_s)^2 \tag{11}$$

$$\text{Hence} \quad c_{II}/c_{III} = (\rho_s/\rho_f)^{(\gamma-1)/2\gamma}\ (1/\cos\alpha)^{1/\gamma} \tag{11a}$$

where ρ_s and ρ_f mean the normal density of the slow and fast explosive respectively.

4. If we now consider the special case $\gamma = 3$, we have

$$c_{III} = c_{II} \cdot \sqrt[3]{(\rho_f/\rho_s)\cos\alpha} \tag{11b}$$

or using 8a and 10a:

$$\sqrt{1-\left(\frac{\sin\alpha}{3}\right)^2}\,\cos\left[(\beta' - \psi + \alpha - \pi)/\sqrt{2}\right] = \sqrt[3]{\rho_f/\rho_s \cos\alpha}\;\cos(\beta/\sqrt{2}) \qquad (12)$$

where $\psi = \sqrt{2}\,\tan^{-1}\left[(\sqrt{8}/3)\tan\alpha\right]$

The condition $\delta_{II} = \delta_{III}$ gives:

$$\beta - \tan^{-1}\left[\sqrt{2}\tan(\beta/\sqrt{2})\right] = \beta' - \pi - \tan^{-1}\left(\sqrt{2}\tan\frac{\beta' - \psi + \alpha - \pi}{\sqrt{2}}\right) \qquad (13)$$

For a given detonation velocity ratio and density ratio of two explosives, therefore, (12) and (13) may be solved for β and β'.

Fig. 1 shows the flow lines in a typical case; the dotted lines indicate the regions of rarefaction behind the detonation fronts. In that case, which corresponds to a detonation velocity ratio 1.6 and density ratio 0.5, one obtains $\beta = 49.55°$, $\phi = 94.8°$, $\beta' = 121.25°$, $\alpha = 51.3°$.

5. For certain ranges of values of ρ_f/ρ_s, it turns out that eqs. (12) and (13) have no solution. In fact, if, keeping α fixed, one varies ρ_f/ρ_s, one finds that as the ratio is increased, β and β' both increase, i.e., the expansion region in the fast explosive increases and that in the slow explosive decreases, until finally $\beta' = \pi - \alpha$ (the slow explosive expansion region disappears); and similarly, if ρ_f/ρ_s is decreased, β and β' decrease until finally $\beta = 0$ (the fast explosive expansion region disappears). That this should be the case is plausible on physical grounds; a fast explosive of great density tends to push into the slow-explosive region, and one of small density is itself prevented from expanding by a dense slow explosive.

 [redacted]

The limiting values of ρ_f/ρ_s for which a solution of the type discussed above can be found, are easily calculated from equations (12) and (13) by setting $\beta = 0$, and eliminating β' (low density ratio limit) or by setting $\beta' = \pi - \alpha$ and eliminating β (high density ratio limit). The resulting curves in the case $\gamma = 3$ in both explosives are shown in Fig. 2, curve No. 1. Curve No. II gives similar limits for the case when $\gamma = 3$ in the fast explosive and $\gamma = 2$ in the slow. The calculation for $\gamma \neq 3$ everywhere is exactly the same as in the original one, except that the form of the condition requiring continuous pressure at $\theta = \phi$ is somewhat more complicated. One has in fact, instead of (11b) the following condition:

$$c_{III}^{4} = (2^8/3^6)\ (\cos^2\alpha)\ (\rho_f/\rho_s)\ c_{II}^{3} \qquad (14)$$

Eq. (13) is changed only by inserting $\sqrt{3}$ in place of $\sqrt{2}$ on the right-hand side.

There have been inserted in Fig. 2 points corresponding to several real explosives. Since not too much is known about the value of γ to be assigned to these explosives - if in fact the assumption of a power law of that type is itself at all valid - one cannot draw many conclusions; it seems however that Baronal used with either pentolite or composition B should probably behave "normally"; the other combinations may be a little outside the allowed regions.

6. In case the density ratio ρ_f/ρ_s is outside the allowed region one may calculate what happens in the following fashion: Since too high values of ρ_f/ρ_s apparently mean that the fast explosive is "pushing into" the slow

[redacted]

 [redacted]

explosive, i.e. the high pressure behind the fast explosive is forcing the pressure behind the slow explosive above its normal value, one may assume that the slow explosive detonation is forced to travel faster than its normal velocity, i.e., α is reduced. In the opposite case, (ρ_f/ρ_s too low) one could argue similarly that the fast explosive is forced to travel faster than normal; but this seems doubtful physically. In this case one probably has to know more about the structure of the detonation front itself, in order to say what happens at the intersection of the fronts in the fast and slow explosives. Since however the lower limits of ρ_f/ρ_s as shown in Fig. 2 are so small, the question is of little practical importance, and we shall not attempt to treat it further here, but will limit ourselves to the first case (ρ_f/ρ_s too high).

The equation of the Hugoniot curve (i.e. the curve of possible pressure - density conditions behind the detonation front) can be written:

$$p = p_o/\left[(\gamma + 1)\ v/v_o - (\gamma - 1)\right] \tag{15}$$

where p_o is the normal, Chapman-Jouguet detonation pressure, v is the specific volume, and v_o the normal specific volume of the explosive. From the Hugoniot conditions, we have further that the detonation velocity D is given by:

$$D^2 = pv_o/(1 - v/v_o) \tag{16}$$

Since the normal detonation velocity D_o is:

$$D_o^2 = (\gamma + 1)p_o v_o \tag{17}$$

[redacted]

[redacted]

We have for the slow explosive:

$$(D_s/D_{so})^2 = \frac{1}{(\gamma+1)(1-v/v_o)\left[(\gamma+1)\ v/v_o-(\gamma-1)\right]} \qquad (18)$$

The new value of α is therefore given by:

$$\cos^2\alpha = (D_s/D_f)^2 = (D_s/D_{so})^2 \cdot (D_{so}/D_{fo})^2$$

$$= \frac{\cos^2\alpha_o}{(\gamma+1)(1-v/v_o)\left[(\gamma+1)\ v/v_o-(\gamma-1)\right]} \qquad (19)$$

where α_o is the "normal" refractive angle. Similarly we have the sound velocity at the slow detonation front $c^2 = \gamma p/\rho$ or:

$$c^2 = \frac{\gamma p_o v}{(\gamma+1)\ v/v_o-(\gamma-1)}$$

$$\therefore (c/D_f)^2 = (c/D_{so})^2\ (D_{so}/D_f)^2 \qquad (20)$$

$$= \frac{\gamma}{\gamma+1} \cdot \frac{v}{v_o} \cdot \frac{\cos^2\alpha_o}{(\gamma+1)\ v/v_o-(\gamma-1)}$$

and the material velocity u at the slow detonation front:

$$u^2 = p(v_o - v) \qquad (21)$$

$$\left(\frac{u}{D_f}\right)^2 = \frac{1}{\gamma+1}\ \frac{(1-v/v_o)\ \cos^2\alpha_o}{(\gamma+1)\ v/v_o-(\gamma-1)}$$

[redacted]

 [redacted]

Transforming to our system of co-ordinates, we have, in units of D_f:

$u_\theta = -(v_o/v)\cos\alpha$, $u_r = \sin\alpha$, $c = \cos\alpha \cdot \sqrt{\gamma(v/v_o)(1 - v/v_o)}$ at the slow detonation front. If we assume we have no expansion region behind the slow detonation front, (which is reasonable since we arrived at this case by going beyond the limit in which such a region disappeared), we have three regions instead of the previous four. Regions I ($0 < \theta < \beta$) and II ($\beta < \theta < \phi$) have precisely the same solutions as before (i.e., equations 7a, b, and 8 a to e). For region III ($\phi < \theta < \pi - \alpha$), we now have:

$$u_\theta = A_{III}\cos(\theta - \delta_{III}) \tag{22a}$$

$$u_r = A_{III}\sin(\theta - \delta_{III}) \tag{22b}$$

$$c_{III}^2 = \gamma(v/v_o)(1 - v/v_o)\cos^2\alpha \tag{22c}$$

where
$$A_{III} = \sqrt{1 - \left(1 - (v/v_o)^2\right)\cos^2\alpha} \tag{22d}$$

$$\delta_{III} = \tan^{-1}\left(\frac{\tan\alpha}{v/v_o}\right) - \alpha \tag{22e}$$

We have again the condition $\delta_{II} = \delta_{III}$. This gives one equation between the three unknowns β, α, v/v_o; Eq. (19) gives another; the third must again come from continuity of pressure at $\theta = \phi$. This gives, by a calculation similar to that at the end of Section 3, the following equation:

$$c_{II}/c_{III} = (\rho_s/\rho_f)^{(\gamma-1)/2\gamma} \cdot \left[(\cos\alpha)^{1/\gamma}\left(\frac{v_o}{v} - \frac{\gamma+1}{\gamma}\right)^{1/2}\left[(\gamma+1)\left(1 - \frac{v}{v_o}\right)\right]^{1/2\gamma}\right]^{-1} \tag{23}$$

which reduces to (11a) when $v/v_o = \gamma/(\gamma + 1)$, i.e. the Chapman-Jouguet value. Inserting the values of c_{II} and c_{III}, we obtain:

[redacted]

$$\frac{\gamma}{\gamma+1}\cos\left(\beta\sqrt{\frac{\gamma-1}{\gamma+1}}\right) = \left(\frac{\rho_s}{\rho_f}\right)^{\frac{\gamma-1}{2\gamma}} \frac{\sqrt{\gamma\cdot\frac{v}{v_o}\left(1-\frac{v}{v_o}\right)}\cos\alpha}{(\cos\alpha)^{1/\gamma}\left[\frac{v}{v_o}\cdot\frac{\gamma+1}{\gamma}\right]^{1/2}\left[(\gamma+1)\left(1-\frac{v}{v_o}\right)\right]^{1/2\gamma}} \tag{23a}$$

and using eq. (19) for cos α and simplifying, we have finally:

$$\cos\left(\beta\sqrt{\frac{\gamma-1}{\gamma+1}}\right) = \left(\frac{\rho_s}{\rho_f}\right)^{\frac{\gamma-1}{2\gamma}} \frac{(\cos\alpha_o)^{1-1/\gamma}}{\left[(\gamma+1)\frac{v}{v_o}-(\gamma-1)\right]^{(1/2)(1-1/\gamma)}} \tag{23b}$$

7. If we consider again the special case $\gamma = 3$, this equation gives:

$$\cos(\beta/\sqrt{2}) = \left[\frac{\rho_s}{\rho_f}\cdot\frac{\cos^2\alpha_o}{(4\,v/v_o - 2)}\right]^{1/3} \tag{23c}$$

Equation (19) becomes:

$$\cos^2\alpha_o = \frac{\cos^2\alpha_o}{4(1-v/v_o)(4\,v/v_o-2)} \tag{24}$$

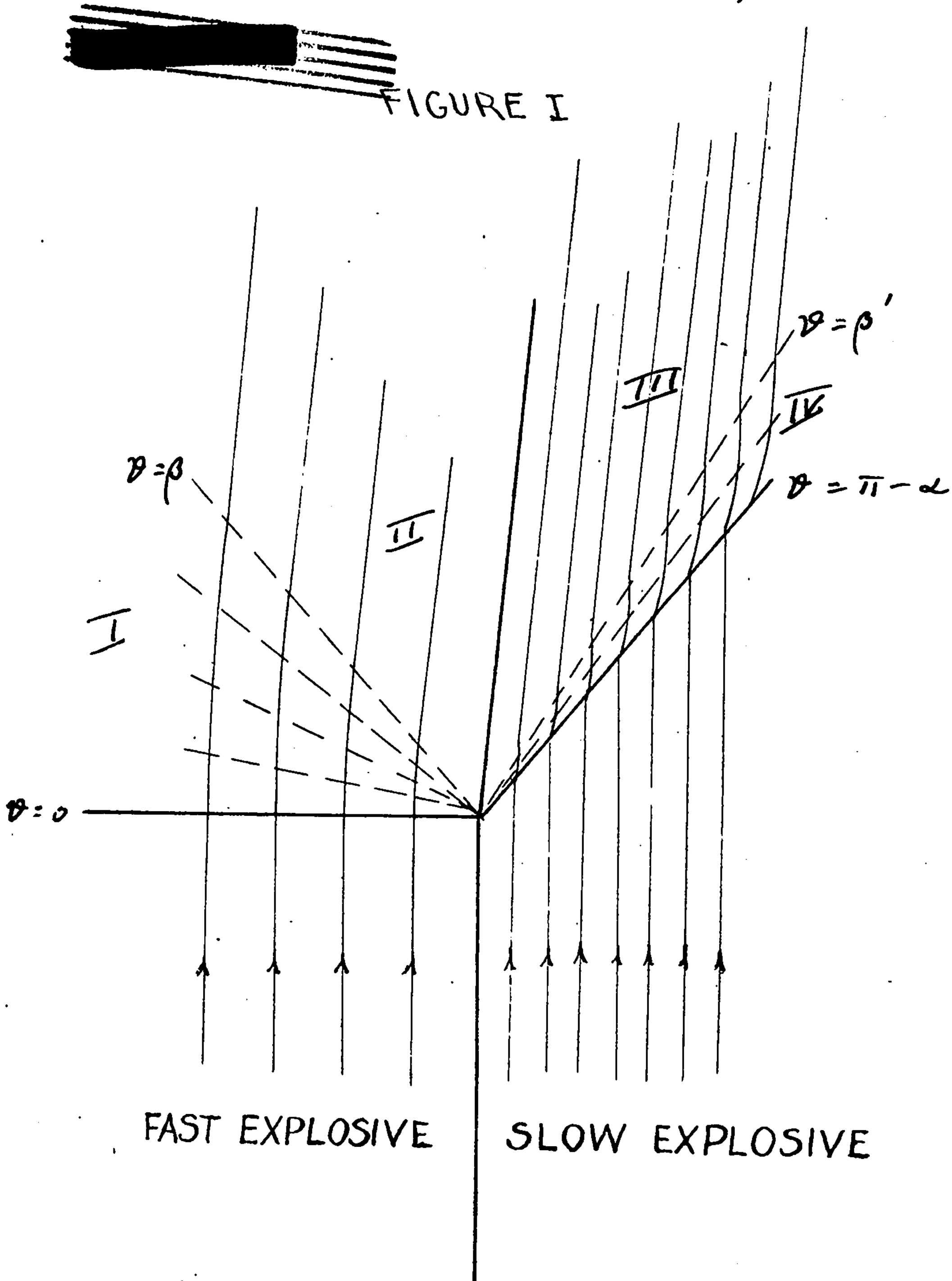
and $\delta_{II} = \delta_{III}$ gives

$$\beta - \tan^{-1}\left[\sqrt{2}\tan(\beta/\sqrt{2})\right] = \tan^{-1}\left(\frac{\tan\alpha}{v/v_o}\right) - \alpha \tag{25}$$

For given values of α_o and ρ_s/ρ_f (where ρ_s/ρ_f is to be chosen above the limiting value of curve No. I, Fig. 2), these equations may be solved for α, β and v/v_o. To calculate the refractive angle α as a function of density, the following scheme is simple: select values of v/v_o less than the Chapman-Jouguet value 3/4, (corresponding to higher than normal pressures in the slow detonation front); from these calculate α from (24); (25) then gives β, and (23c) may be solved for ρ_s/ρ_f. Fig. 3 gives the result of such a calculation for the case $a_o = 60^\circ$, i.e. "normal" index of refraction = 2. According to this curve, the angle of refraction decreases almost linearly from 60° at ρ_f/ρ_s = .55 to about 54.4° at ρ_f/ρ_s = 2.0. These deviations are therefore quite appreciable and should be observed experimentally, if our assumptions about the equation of state of the explosive reaction products are not too far wrong (i.e. equation of state $p \sim \rho^3$).

It should also be remarked that in an experimental setup, these deviations are to be expected only in the neighborhood of the interface; in fact for a distance of the order of the width of the explosives. Beyond such a distance, the detonation waves will have their normal velocities, and will be curved, since it is only in the ideal case of infinite extent of explosive and infinite age of the detonation wave that the fronts are all straight lines.

FIGURE I

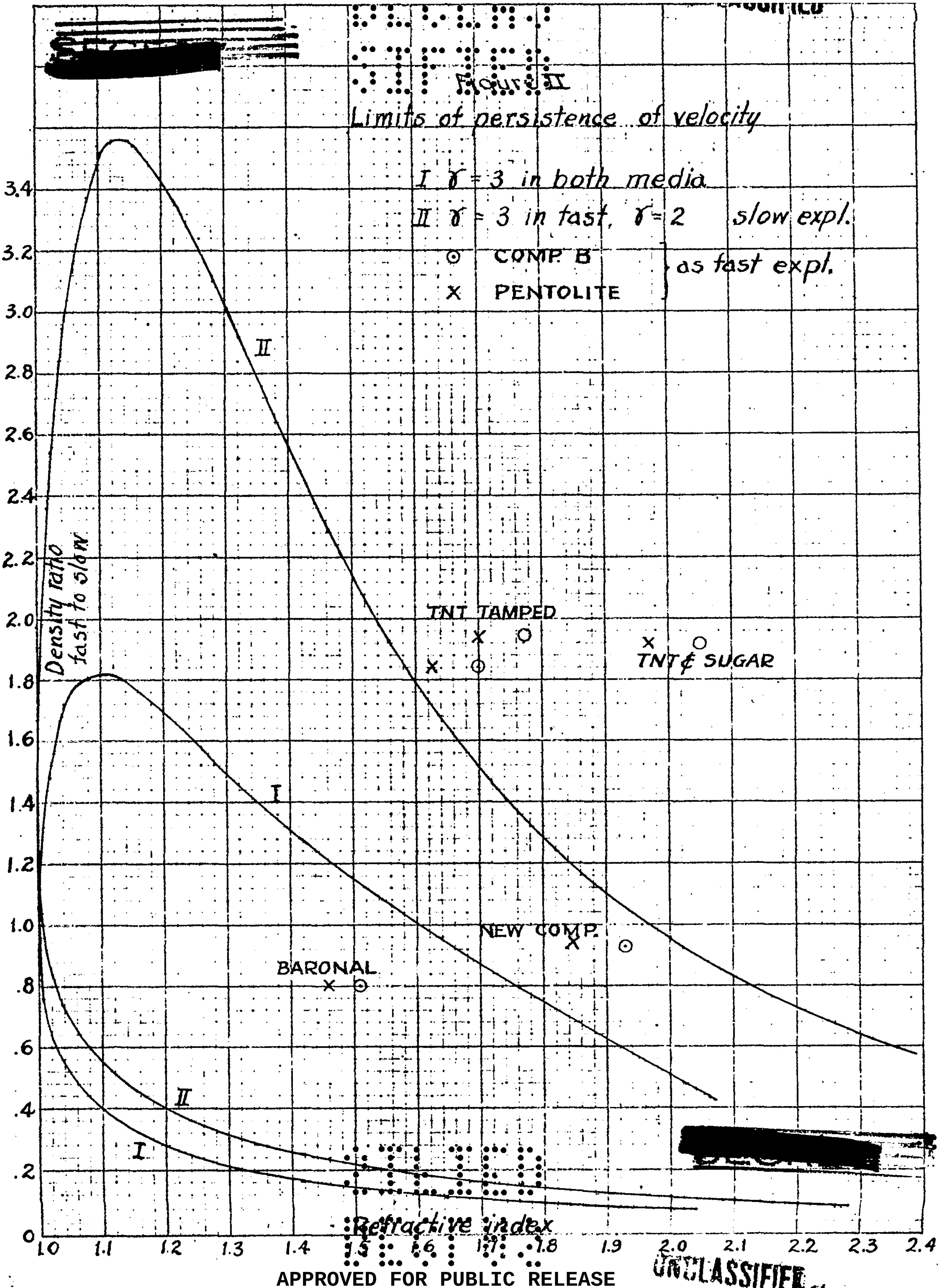


REFRACTION OF DETONATION WAVE

$n = 1.6$

$\rho_f / \rho_s = 0.5$

[redacted]



[redacted]

[redacted]

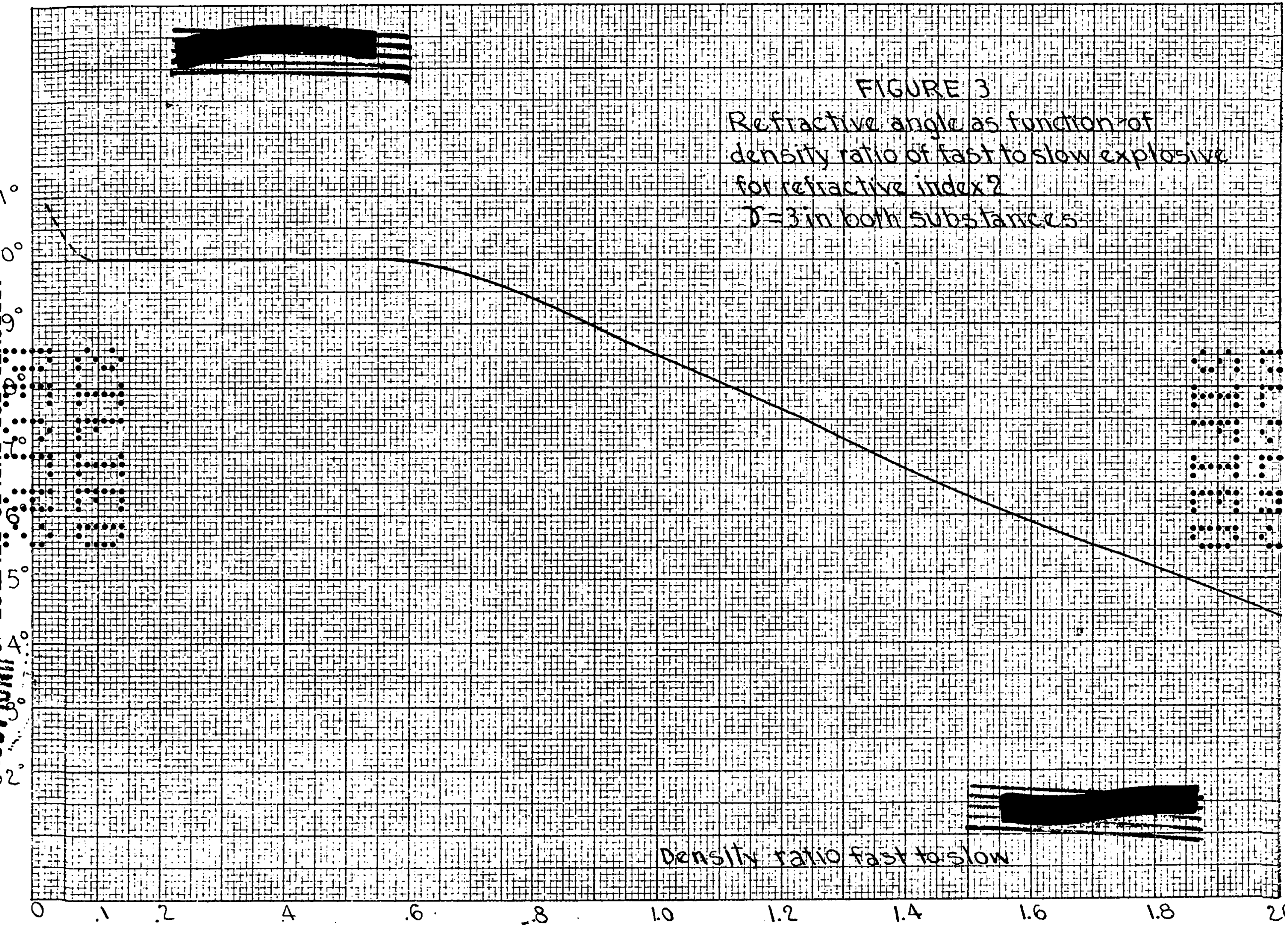


FIGURE 3
Refractive angle as function of density ratio of fast to slow explosive for refractive index 2
$\gamma = 3$ in both substances

[redacted]