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FORMULA FOR THE CRITICAL RADIUS FOR ONE VELOCITY

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ABSTRACT

By combining the results of various numerical computations, a formula is obtained for the critical core radius when the mean free path differs in core and tamper. It is limited to one neutron velocity, and the tamper is infinite.

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FORMULA FOR THE CRITICAL RADIUS FOR ONE VELOCITY

An "empirical" formula relating the radius of the core to the properties of the core and tamper has been obtained for the case in which the tamper is infinite and only one velocity is considered:

$$k(a - z) = 3.168 - \frac{3.95}{1.453 + \lambda + ha} \qquad (1)$$

where a is the radius of the core in units of the mean free path in the core;

$\lambda = \lambda_t/\lambda_c$, ratio of tamper mean free path to core mean free path.

$k = \sqrt{3f\ (1 + .8f)}$, $f = \left[(\nu - 1)\sigma_f - \sigma_r\right]/\sigma_h$ = net number of neutrons liberated per collision in the core.

$h = \sqrt{3g(1 - .8g)}$, g = ratio of absorption cross section to transport cross section in the tamper. (g was formerly called $-f'$)

$z = .149 - .114\lambda + .08g$

In this form k can be calculated directly if a is given. f can then be obtained from the formula, $f = -.625 + \sqrt{.390625 + k^2/24}$.

A formula from which the radius can be obtained directly results from solving (1).

$g \neq 0$; $a = -G + \sqrt{G^2 + H}$, where $G = (B - A)/2$, $H = AB - C$ and

$A = 3.168/k + z$, $C = 3.95/hk$, $B = (1.453 + \lambda)/h$

$$g = 0; \qquad a = \frac{3.168(1.453 + \lambda) - 3.95}{k(1.453 + \lambda)} + .149 - .114\lambda$$

The data on which this formula is based were obtained from the integral theory of Frankel and Nelson in the cases of equal mean free path

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in core and tamper and of an untamped sphere. For other cases the polynomial method introduced by Marshak as calculated by Group T-IV and by Glauber was used.

The curves of radius vs. the various parameters can be used in the following way: let us assume that it is desired to determine a value of the radius a for a set of values of f, λ, and g with λ and g intermediate between those on the given curves. Choose 2 families, one for a λ on each side of the given λ. Interpolate on g by sight, (or more accurately interpolate linearly in $\sqrt{g}$), in each family, obtaining 2 values of the radius. Use linear interpolation in λ on these 2 numbers to get the radius corresponding to the given f, λ, and g.

The accuracy of the radius formula is 2.5 per cent or better. Despite the fact that over a wide range of the variables

$$0.8 > f > .13; \quad .5 > g > 0; \quad 2 > \lambda > .2;$$

most of the radii calculated from the formula are 1.5 per cent or closer to the polynomial- and integral-theory data, it is not possible to give any more favorable limit for the accuracy of the formula because of the uncertainties associated with the polynomial method.

Curves have been included for g = .7 but they are outside the known range of validity of the formula. The dotted curve for g = 1 is not derived from the formula, but is the essentially exact result of the variational method and the extrapolated end-point method for untamped spheres.

Systems in which neutrons multiply as $e^{\alpha t}$ can be calculated by assuming an additional absorption cross section equal to α/v both in the core and in the tamper, where v is the neutron velocity.

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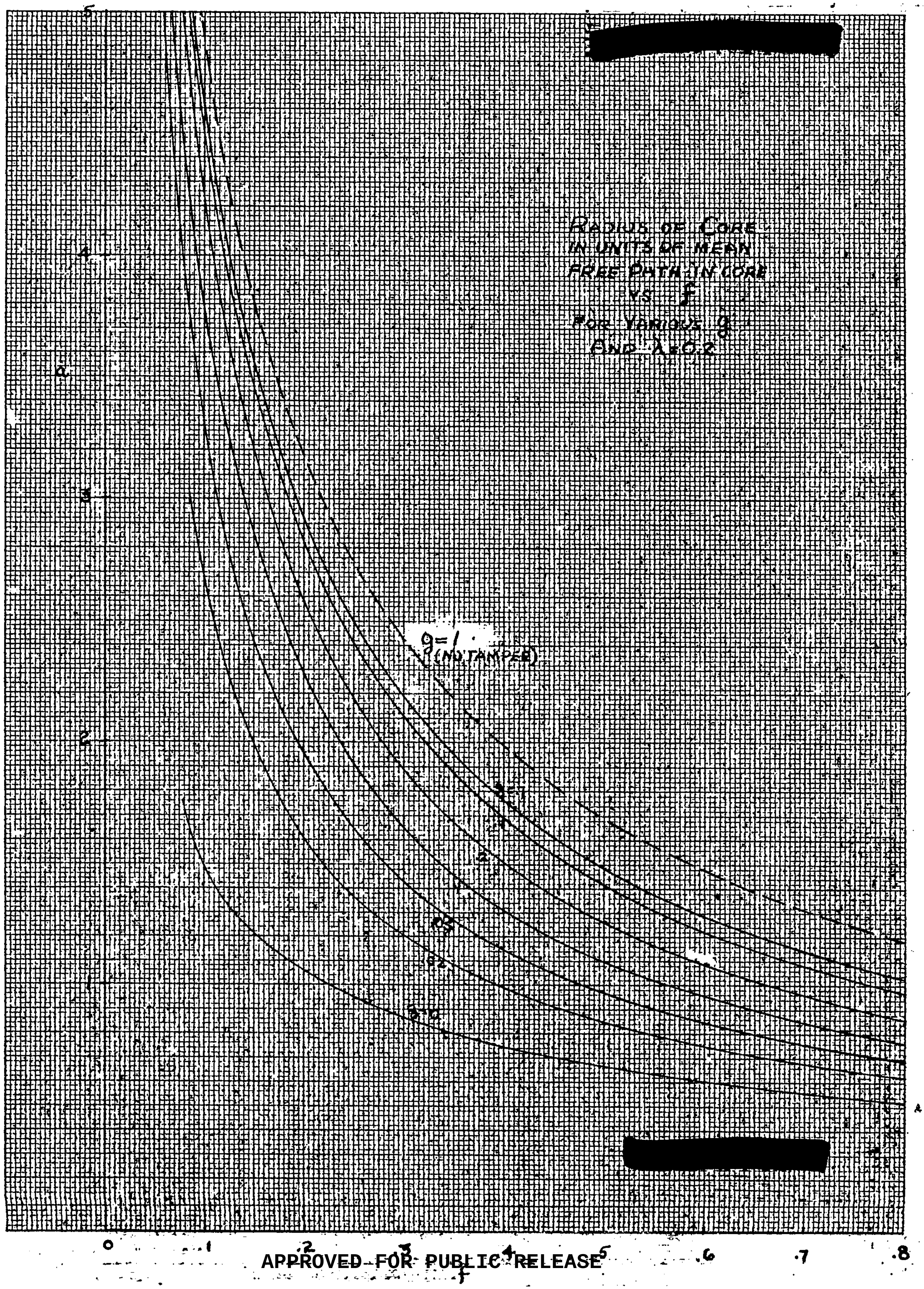
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RADIUS OF CORE
IN UNITS OF MEAN
FREE PATH IN CORE
VS. f
FOR VARIOUS g
AND λ=0.2
a
g=1
(NO TAMPER)
0
.1
.2
.3
.4
.5
.6
.7
.8
f
1
2
3
4
5
[redacted]

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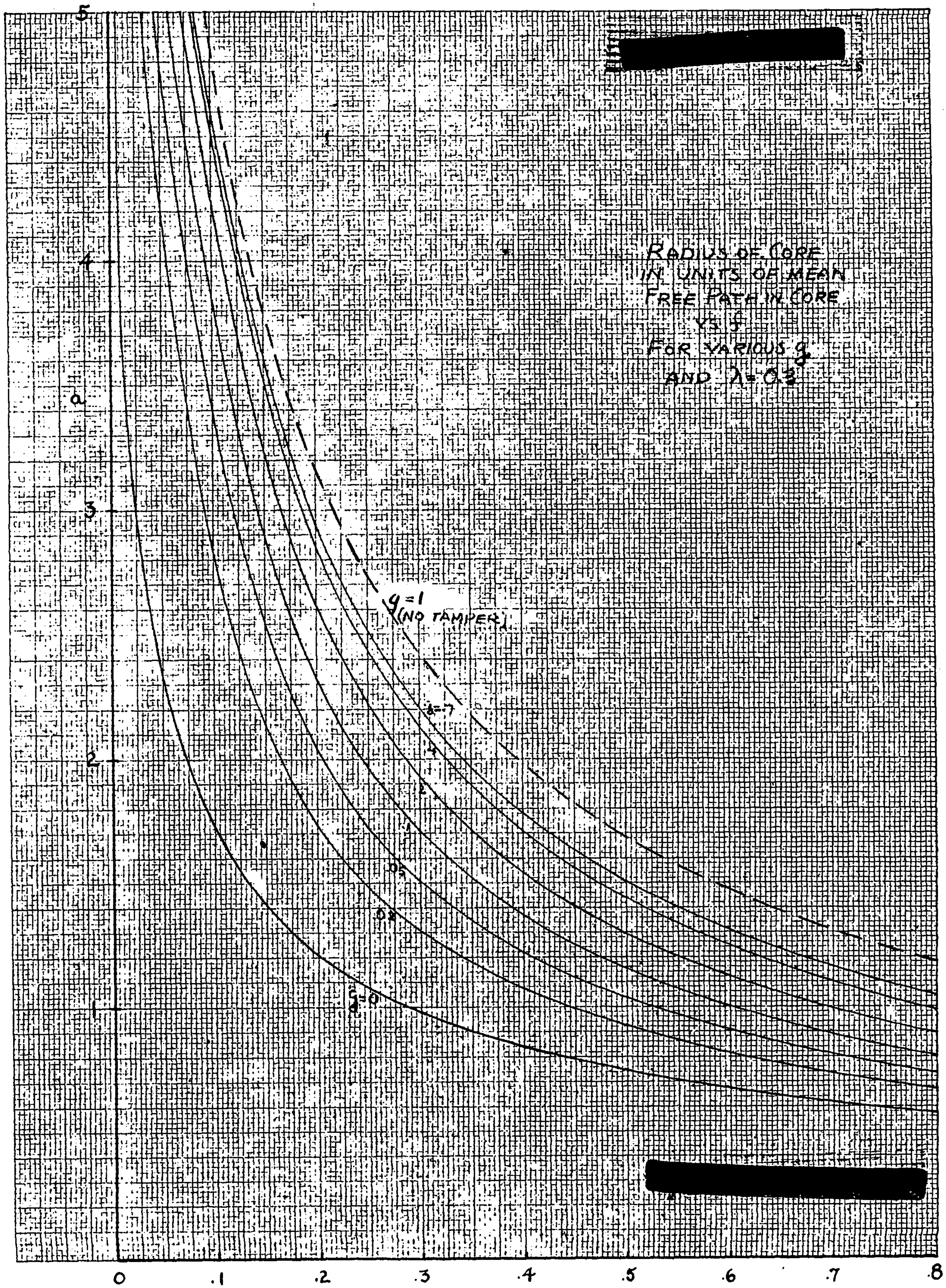

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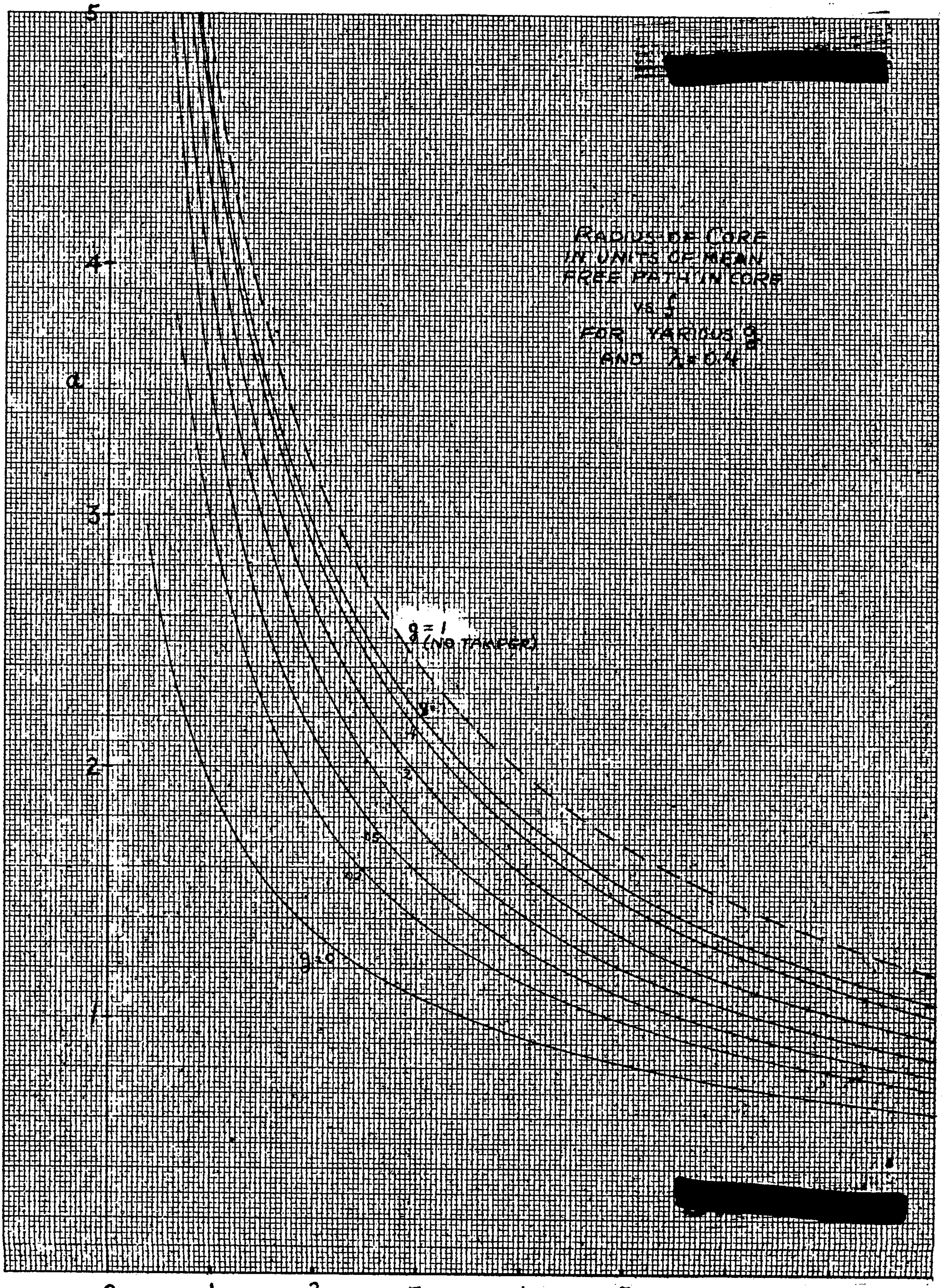

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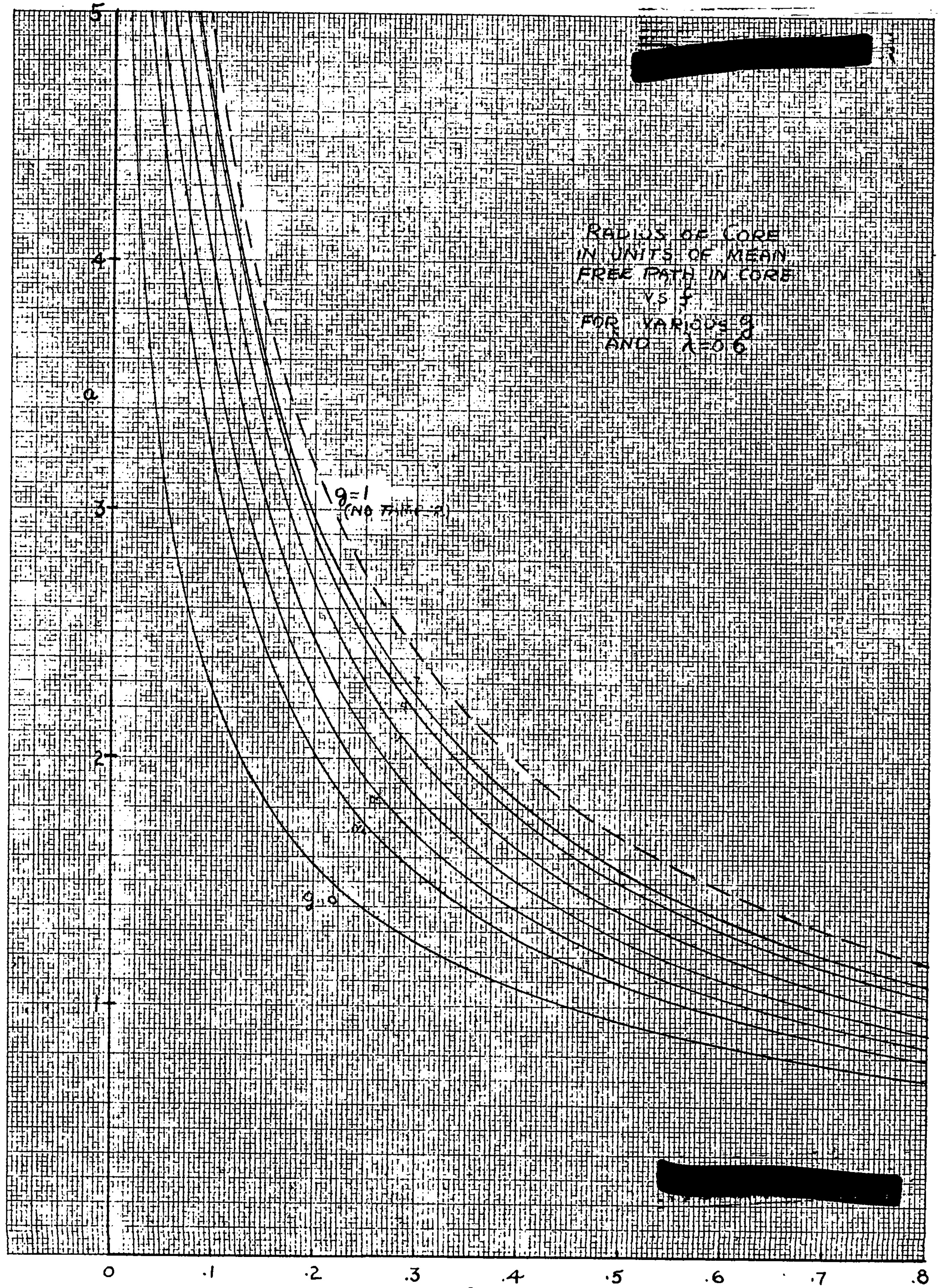

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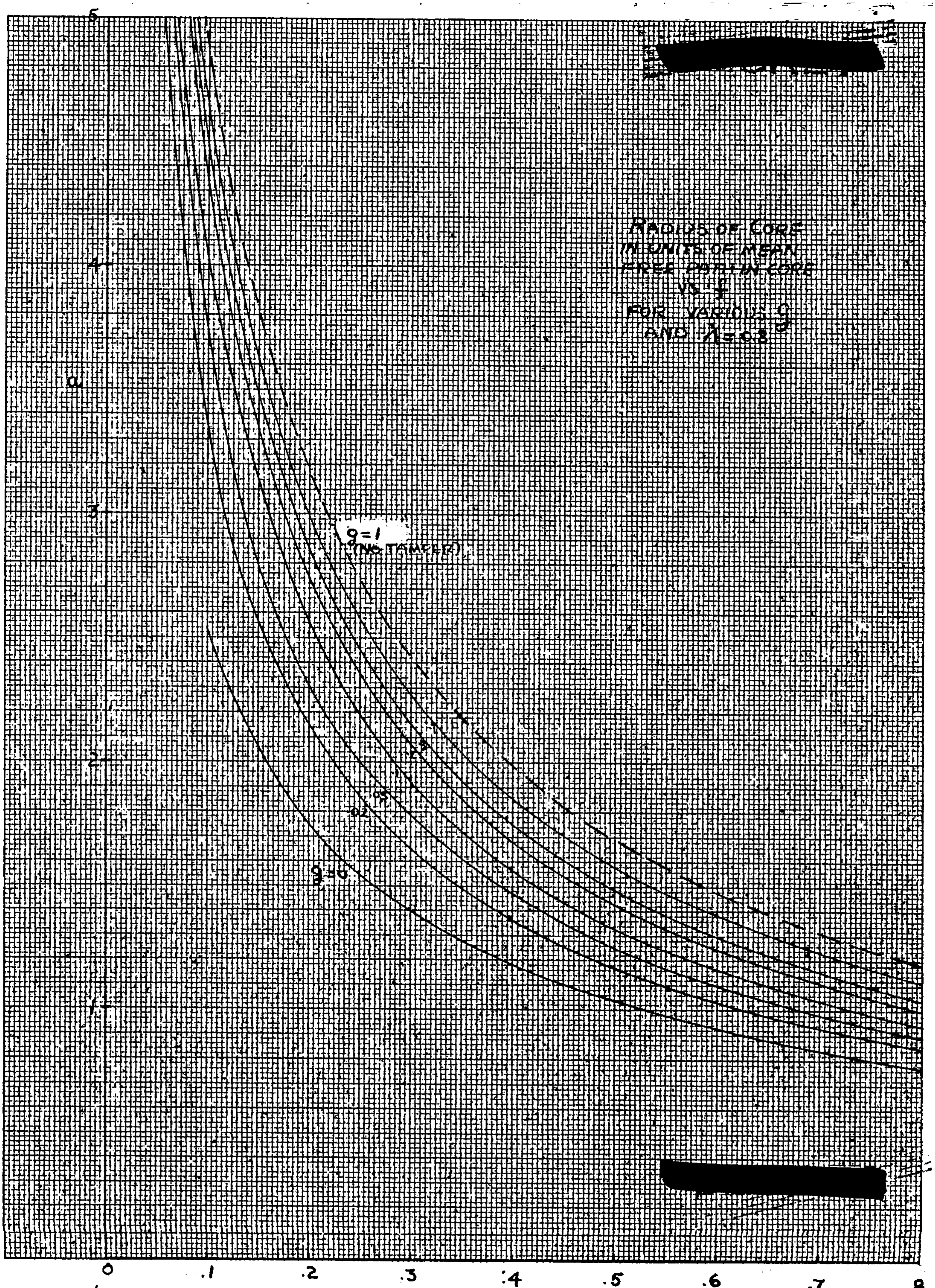

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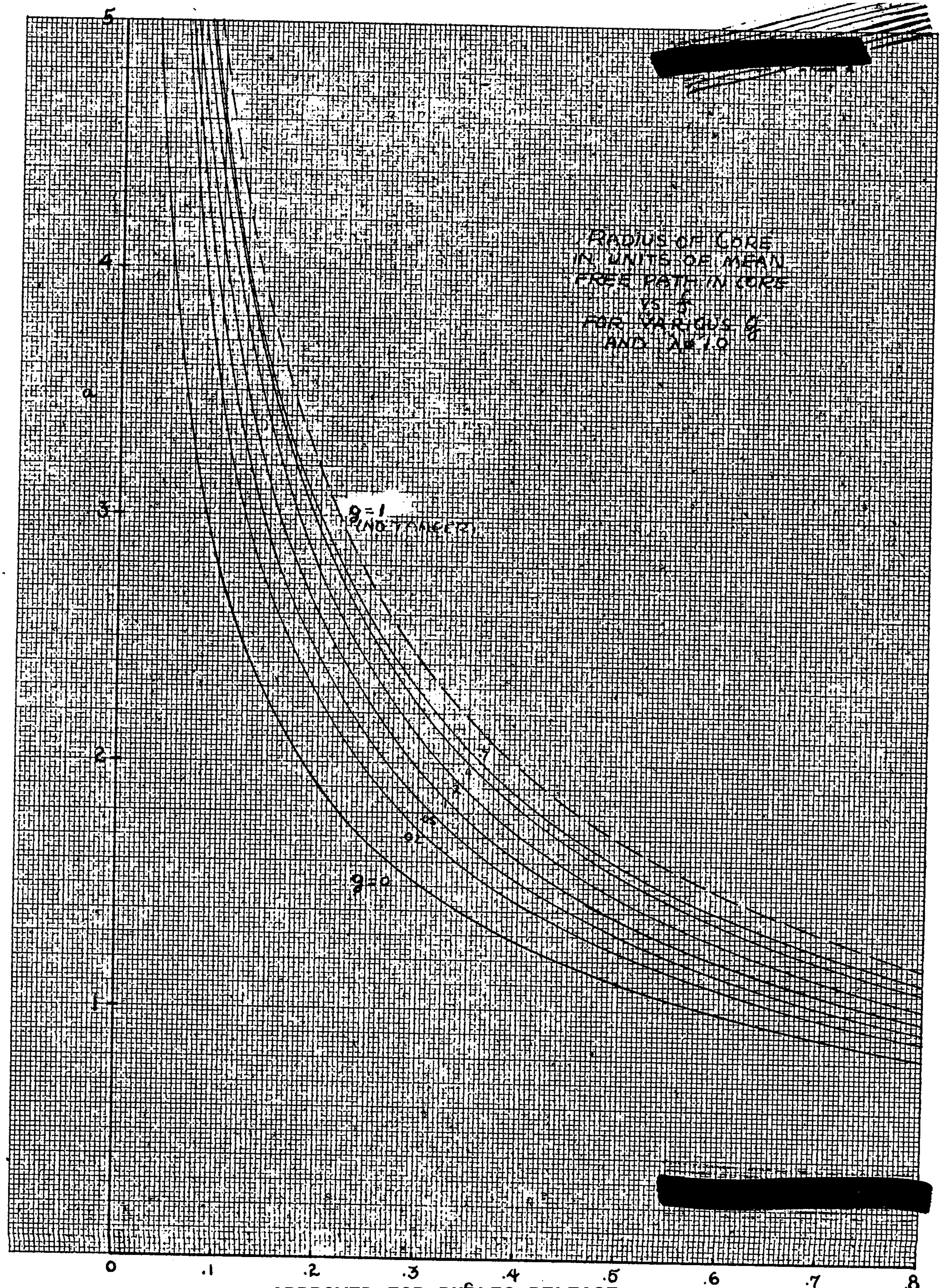
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RADIUS OF CORE
IN UNITS OF MEAN
FREE PATH IN CORE
VS. f
FOR VARIOUS g
AND λ = 1.0
a
g = 1
(NO TAMPER)
g = 0
5
4
3
2
1
0
.1
.2
.3
.4
.5
.6
.7
.8
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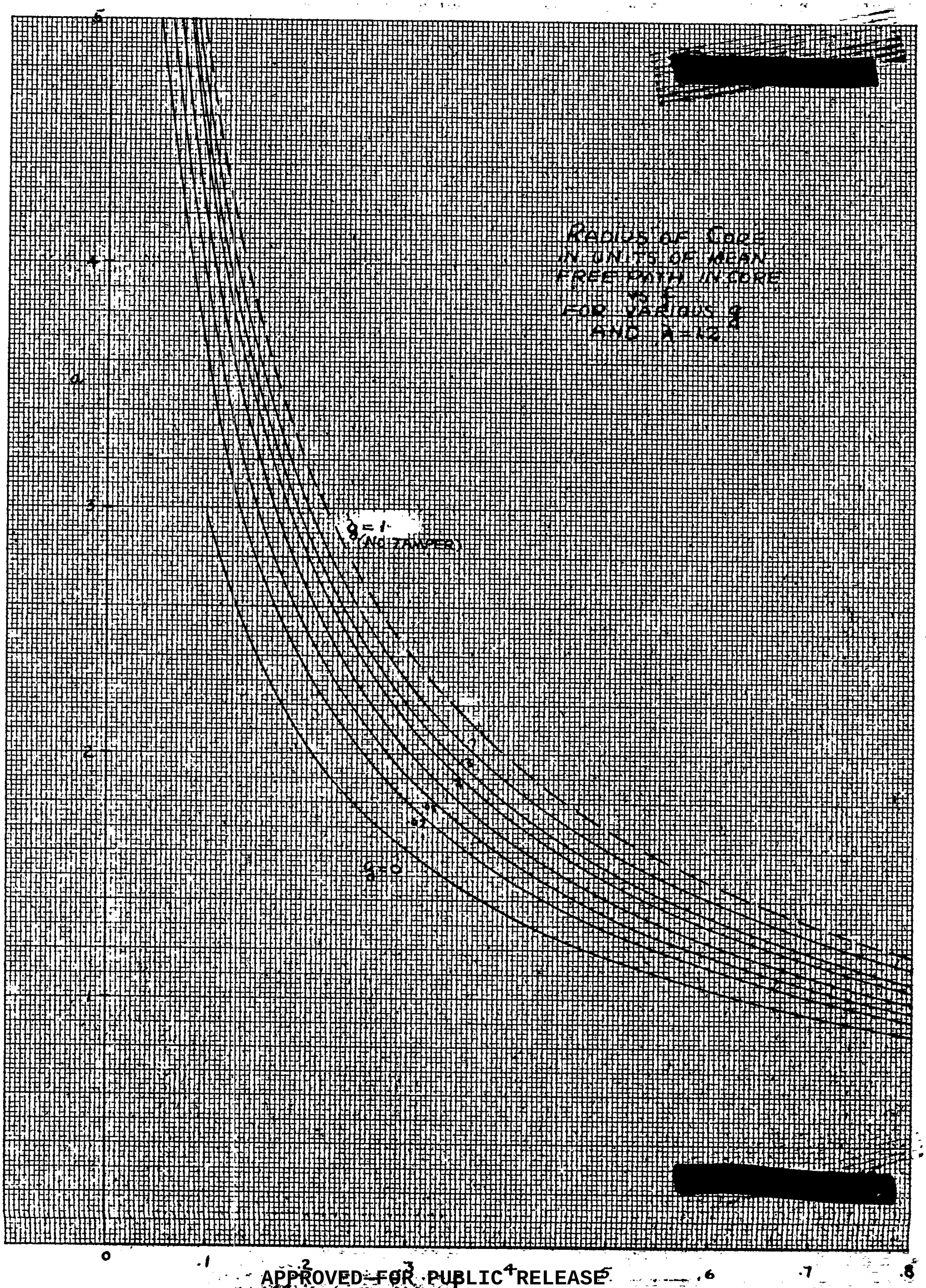

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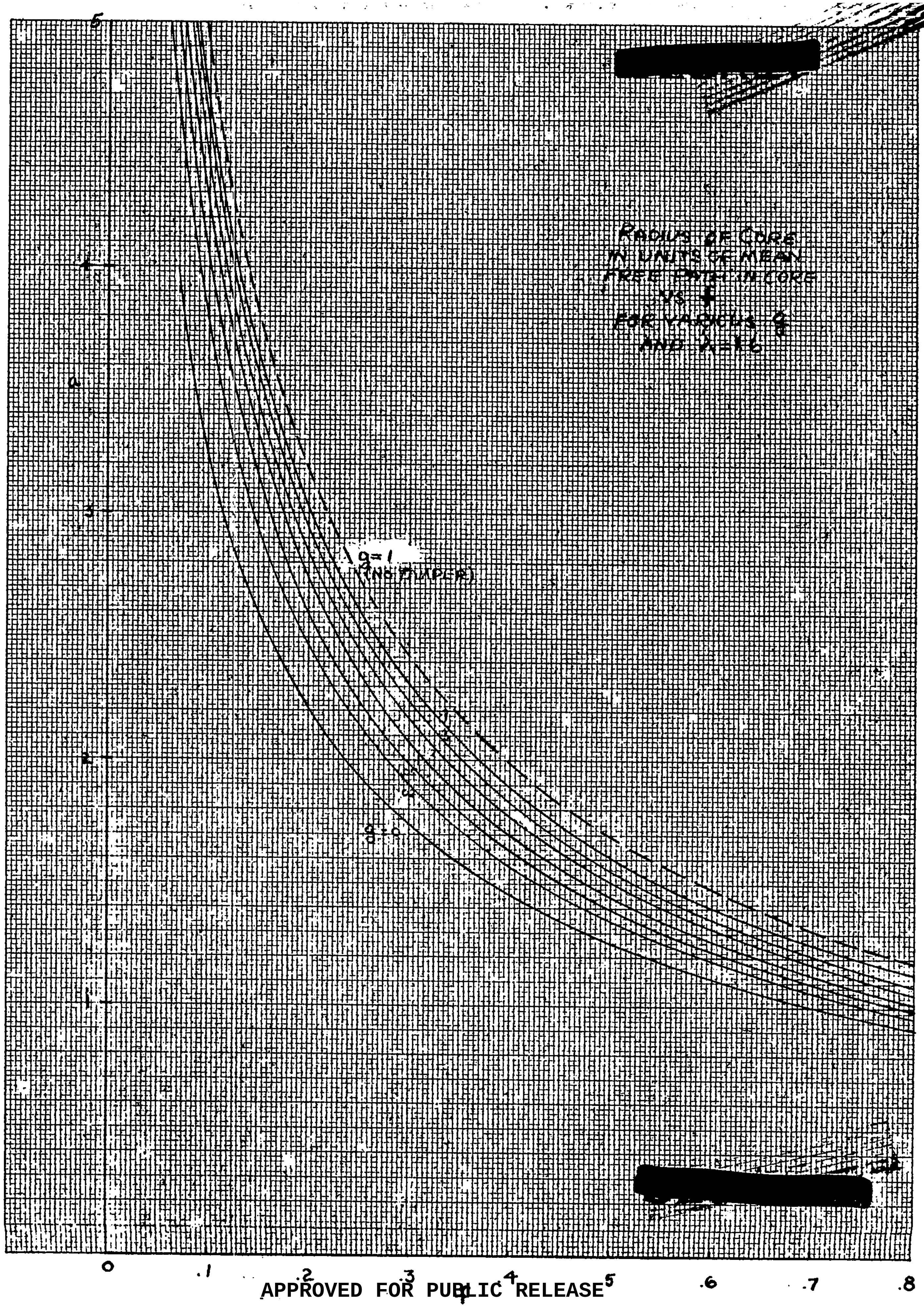

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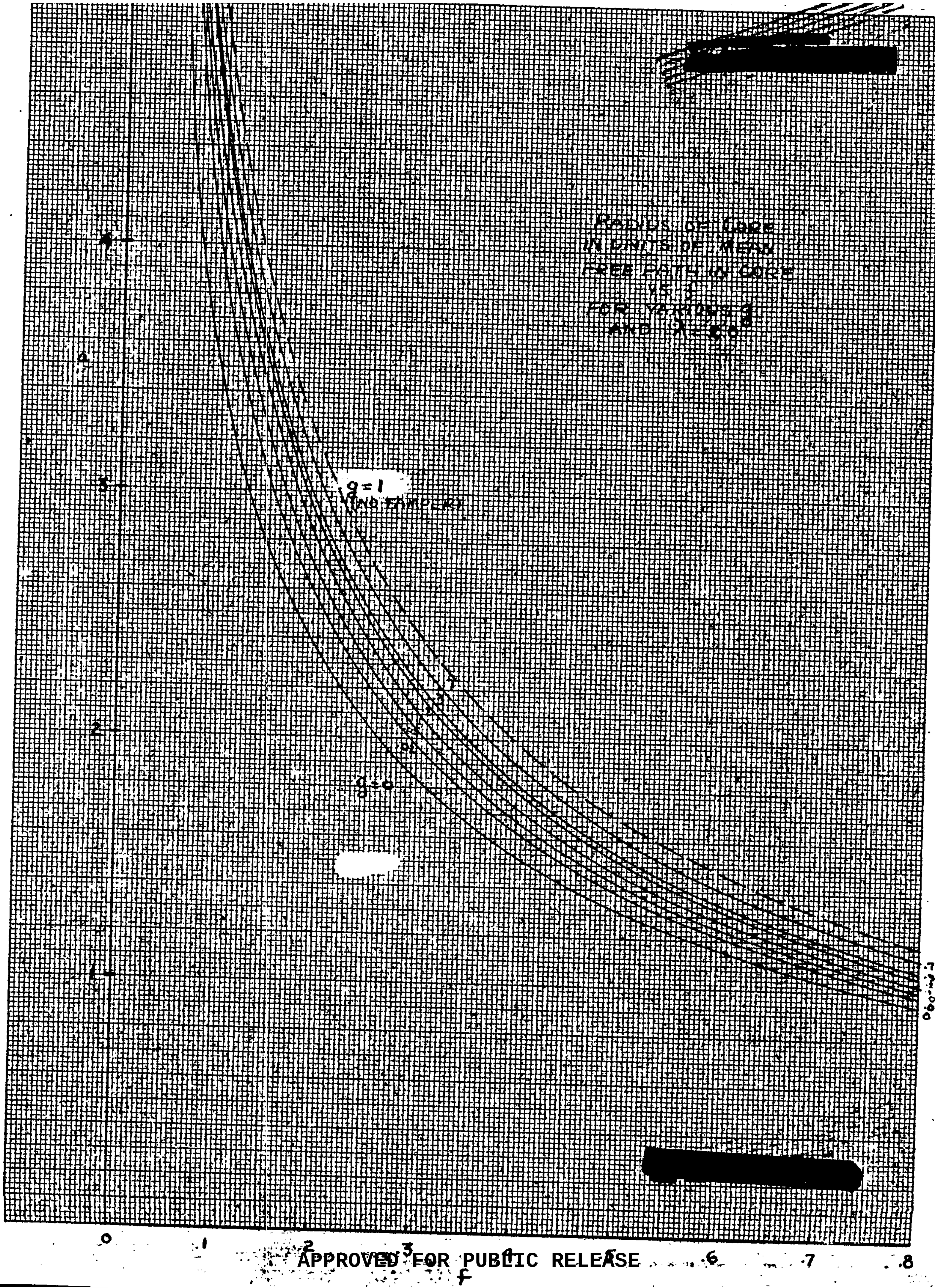

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