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DISTRIBUTION ARISING FROM A POINT SOURCE OF FAST NEUTRONS BETWEEN TWO SLOWING-DOWN MEDIA

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ABSTRACT

The effect of the soil on neutron measurements at Trinity may be estimated from Tables I, II, and III as soon as the constitution of the soil (in particular, the hydrogen content) and the scattering cross sections of air and soil are known as functions of energy. Tables I and III give the effect of the soil on neutron measurements on and above ground, respectively, assuming that the soil is merely compressed air. Table II takes into account the different chemical composition of the two although it is still based on the assumption that the scattering cross sections vary the same way with energy in both air and soil.

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DISTRIBUTION ARISING FROM A POINT SOURCE OF FAST NEUTRONS BETWEEN TWO SLOWING-DOWN MEDIA

If neutron measurements are carried out at Trinity, it will be necessary to know the corrections which must be introduced because of the presence of the soil. The gadget will be a source of delayed fast neutrons, and it is contemplated to measure with cadmium detectors the low-energy neutrons slowed down by the atmosphere (and earth) at various distances from the explosion. It is clear qualitatively that near the source of fast neutrons, the effect of the dense earth (relative to air) will be to enhance the neutron density above the cadmium cutoff, since the fast neutrons "age" much more rapidly in the earth and slower neutrons will give a greater contribution to the density, while farther out the greater "aging" in the earth will bring a greater percentage of neutrons below the cadmium cutoff and reduce the neutron density above the cadmium cutoff as compared to the neutron density without soil. However, it is useful to have more quantitative information on these points and in this paper we obtain some information in this direction by solving several two-media problems on the basis of age-velocity theory.

Let us consider, therefore, a point source of monoenergetic fast neutrons situated at the interface between two semi-infinite media (Cf. Fig. 1) of different slowing-down properties. We assume that the age-velocity theory holds for both media; this is certainly true as long as measurements are not made more than several slowing-down lengths from the source and as long as there is not an appreciable admixture of hydrogen in either medium. The equations for the slowing-down densities, $\chi_1(\rho,z,u)$ and $\chi_2(\rho,z,u)$ in the two media are thus (it is convenient to use cylindrical coordinates):

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$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\chi_1}{\partial\rho}\right)+\frac{\partial^2\chi_1}{\partial z^2}=\frac{\partial\chi_1}{\partial\tau_1}-\frac{\delta(r)\delta(\tau_1)}{4\pi r^2} \qquad (1)$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\chi_2}{\partial\rho}\right)+\frac{\partial^2\chi_2}{\partial z^2}=\frac{\partial\chi_2}{\partial\tau_2}-\frac{\delta(r)\delta(\tau_2)}{4\pi r^2} \qquad (2)$$

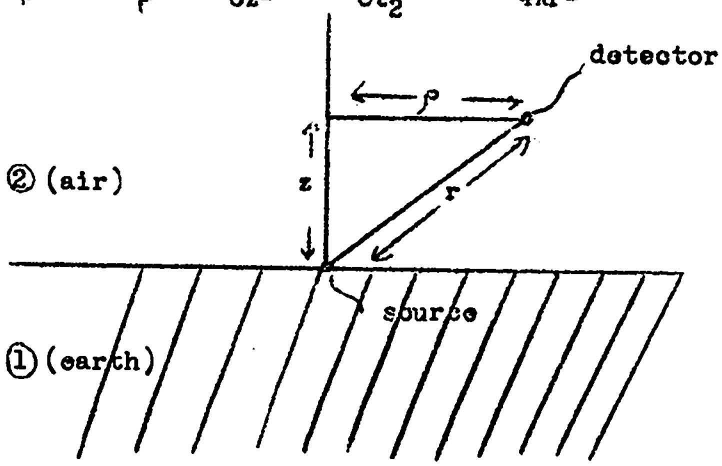

Fig. 1

where χ is defined as the number of neutrons per cc, per second reaching the age ~~τ and~~ τ is defined as

$$(1/3)\int_0^u \frac{\lambda^2(u')\,du'}{a(u')\left[1-b(u')\right]} \quad ,$$

λ is the mean free path for scattering, a is the average logarithmic energy loss, b is the average of the cosine of the angle of deflection in one collision and finally $u = \log(E_0/E)$, with E_0 the initial energy and E the energy of interest.

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Eqs. (1) and (2) are to be solved subject to the boundary conditions:

$$\frac{\lambda_1(u)}{a_1(u)} \chi_1(\rho, o, u) = \frac{\lambda_2(u)}{a_2(u)} \chi_2(\rho, o, u) \qquad \text{(A)}$$

(continuity of neutron density on plane $z = 0$ for all ρ and u).

$$\frac{\lambda_1^{\,2}}{a_1(1-b_1)} \frac{\partial \chi_1}{\partial z} (\rho, o, u) = \frac{\lambda_2^{\,2}}{a_2(1 - b_2)} \frac{\partial \chi_2}{\partial z} (\rho, o, u) \qquad \text{(B)}$$

(continuity of normal neutron current on plane $z = 0$ for all ρ and u).

We now make two further simplifying assumptions: we assume that the a's and b's are constants independent of the energy - which is generally true for mixtures which do not contain an appreciable abundance of hydrogen - though we permit them to be different in the two media, and we assume that the scattering mean free paths of the two media vary with energy in an identical fashion. The seriousness of the latter assumption is difficult to judge at present since we do not know the scattering cross sections of nitrogen and Trinity earth as a function of energy. With the above assumptions we may rewrite (A) and (B):

$$\chi_1(\rho, o, u) = \sqrt{Da} \quad \chi_2(\rho, o, u) \qquad \text{(A')}$$

$$\frac{\partial \chi_1}{\partial z} (\rho, o, u) = D \frac{\partial \chi_2}{\partial z} (\rho, o, u)$$ [redacted]

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where D and α are constants and defined by:

$$D = \frac{\lambda_2^{\ 2}}{a_2(1-b_2)} \bigg/ \frac{\lambda_1^{\ 2}}{a_1(1-b_1)} \tag{3}$$

$$\alpha = \frac{(1-b_2)}{a_2} \bigg/ \frac{(1-b_1)}{a_1} \tag{4}$$

Moreover it follows that $\tau_2 = D\tau_1$ so that eq. (1) becomes:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\chi_1}{\partial\rho}\right) + \frac{\partial^2\chi_1}{\partial z^2} = D\frac{\partial\chi_1}{\partial\tau_2} - D\frac{\delta(r)\delta(\tau_2)}{4\pi r^2} \tag{5}$$

Applying a Laplace transformation with respect to τ_2 to eqs. (1) and (5), we obtain:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi_1}{\partial\rho}\right) + \frac{\partial^2\phi_1}{\partial z^2} = D\gamma\phi_1 - \frac{D\delta(r)}{4\pi r^2} \tag{6}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi_2}{\partial\rho}\right) + \frac{\partial^2\phi_2}{\partial z^2} = \gamma\phi_2 - \frac{\delta(r)}{4\pi r^2} \tag{7}$$

where

$$\phi_{1,\,2} = \mathcal{L}\chi_{1,\,2} = \int_0^\infty e^{-\gamma\tau_2}\,\chi_{1,\,2}\,d\tau_2$$

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The boundary conditions (A') and (B') yield:

$$\phi_1(\rho,o,\eta) = \sqrt{Da}\quad \phi_2(\rho,o,\eta) \tag{C}$$

$$\frac{\partial\phi_1}{\partial z}(\rho,o,\eta) = D\,\frac{\partial\phi_2}{\partial z}(\rho,o,\eta) \tag{D}$$

The solutions of (6) and (7) are:[1]

$$\phi_1(\rho,z,\eta) = \frac{D}{4\pi r}\, e^{-\sqrt{D\eta}\,r} + \int_0^\infty f_1(\lambda)J_o(\lambda\rho)e^{\sqrt{\lambda^2+D\eta}}\,d\lambda \quad (z < o) \tag{8}$$

$$\phi_2(\rho,z,\eta) = \frac{1}{4\pi r}\, e^{-\sqrt{\eta}\,r} + \int_0^\infty f_2(\lambda)J_o(\lambda\rho)e^{-\sqrt{\lambda^2+\eta}\,z}\,d\lambda \quad (z > o) \tag{9}$$

The functions f_1 and f_2 are abritrary amplitudes to be determined by the boundary conditions (C) and (D). This is most easily done by expressing the singular terms in (8) and (9) in terms of Bessel functions, namely:

$$\frac{e^{-\sqrt{D\eta}\,r}}{r} = \int_0^\infty \frac{J_o(\lambda\rho)e^{\sqrt{\lambda^2+D\eta}\,z}}{\sqrt{\lambda^2+D\eta}}\,\lambda d\lambda \quad (z < o) \tag{10}$$

$$\frac{e^{-\sqrt{\eta}\,r}}{r} = \int_0^\infty \frac{J_o(\lambda\rho)\,e^{-\sqrt{\lambda^2+\eta}\,z}}{\sqrt{\lambda^2+\eta}}\,\lambda d\lambda \quad (z > o) \tag{11}$$

1) Cf. J.A. Stratton - "Electromagnetic Theory" [redacted]

[redacted]

Making use of (C) and (D) and the Fourier-Bessel theorem, we get:

$$f_1 + \frac{D}{4\pi} \frac{\lambda}{\sqrt{\lambda^2 + D\eta}} = \sqrt{D\alpha} \left(f_2 + 1/4\pi \frac{\lambda}{\sqrt{\lambda^2 + \eta}} \right) \qquad (12)$$

$$\sqrt{\lambda^2 + D\eta}\; f_1 = - D \sqrt{\lambda^2 + \eta}\; f_2 \qquad (13)$$

Eqs. (12) and (13) yield for f_2 :

$$f_2 = - \frac{\lambda}{4\pi \sqrt{\lambda^2 + \eta}} \frac{\left[\sqrt{\alpha} - \sqrt{\frac{D(\lambda^2+\eta)}{(\lambda^2+D\eta)}} \right]}{\left[\sqrt{\alpha} + \sqrt{\frac{D(\lambda^2+\eta)}{(\lambda^2+D\eta)}} \right]} \qquad (14)$$

and for ϕ_2 :

$$\phi_2(\rho, z, \eta) = \frac{\sqrt{D}}{2\pi} \int_0^\infty \frac{J_o(\lambda\rho) e^{-\sqrt{\lambda^2+\eta}\, z}\, \lambda d\lambda}{\left[\sqrt{\alpha(\lambda^2+D\eta)} + \sqrt{D(\lambda^2+\eta)} \right]} \qquad (15)$$

The quantity in which we are interested is the slowing-down density, χ_2, which is given as the Laplace inverse of ϕ_2, i.e. $\chi_2(\rho, z, \tau_2) = \mathcal{L}^{-1}\, \phi_2(\rho, z, \eta)$.

In principle, it is possible to obtain the Laplace inverse of (15) for arbitrary D, α, ρ and z. However, we have worked out the Laplace inverses for three interesting limiting cases although the extension to other cases is not much more complicated. The cases we treat are:

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(I) $\alpha = 1$, $z = 0$, D and ρ arbitrary

(II) $D = \infty$, $z = 0$, α and ρ arbitrary

(III) $D = \infty$, $\alpha = 1$, ρ and z arbitrary

<u>Case I:</u> <u>$\alpha = 1$, $z = 0$, D and ρ arbitrary</u>

$$\phi_2(\rho, 0, \eta) = \frac{\sqrt{D}}{2\pi} \int_0^\infty \frac{J_0(\lambda\rho)\,\lambda d\lambda}{\left[\sqrt{\lambda^2+D\eta} + \sqrt{D(\lambda^2+\eta)}\right]} \qquad (16)$$

We rationalize the denominator in (16) :

$$\phi_2(\rho, 0, \eta) = \frac{1}{2\pi(1-1/D)} \int_0^\infty \frac{J_0(\lambda\rho)}{\lambda} \left[\sqrt{\lambda^2+\eta} - \sqrt{\lambda^2/D+\eta}\right] d\lambda \qquad (17)$$

Now [2] (we drop the subscript on τ_2 until the final expression):

$$\frac{1}{\sqrt{\lambda^2+\eta}} = \mathcal{L}\left[\frac{1}{\sqrt{\pi}} \frac{e^{-\tau\lambda^2}}{\sqrt{\tau}}\right], \quad \frac{1}{\sqrt{\lambda^2/D+\eta}} = \mathcal{L}\left[\frac{1}{\sqrt{\pi}} \frac{e^{-\tau\lambda^2/D}}{\sqrt{\tau}}\right]; \qquad (18)$$

integrating each of these results between η and ∞, we find:

$$\left[2\sqrt{\lambda^2+\eta}\right]_\eta^\infty = \frac{1}{\sqrt{\pi}} \int_0^\infty \left[\frac{e^{-\eta\tau}}{-\tau}\right]_\eta^\infty \frac{e^{-\tau\lambda^2}}{\sqrt{\tau}}\, d\tau$$

$$\left[2\sqrt{\lambda^2/D+\eta}\right]_\eta^\infty = \frac{1}{\sqrt{\pi}} \int_0^\infty \left[\frac{e^{-\eta\tau}}{-\tau}\right]_\eta^\infty \frac{e^{-\tau\lambda^2/D}}{\sqrt{\tau}}\, d\tau$$

2) All Laplace transforms written down without derivation will be found in Doetsch - "Laplace Transformation", Whittaker and Watson - "Modern Analysis", or McLachlan and Humbert - "Formulaire pour Le Calcul Symbolique".

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Therefore:

$$\mathcal{L}^{-1}\left[(\lambda^2+\gamma) - \sqrt{\lambda^2/D+\gamma}\right] = \frac{1}{2\sqrt{\pi}\;\tau^{3/2}}\left[e^{-\tau\lambda^2/D} - e^{-\tau\lambda^2}\right] \tag{19}$$

Substituting (19) into (17), we get:

$$\chi_2 = \mathcal{L}^{-1}\phi_2 = \frac{1}{4(1-1/D)(\pi\tau_2)^{3/2}}\int_0^\infty \frac{J_0(\lambda\rho)}{\lambda}\left[e^{-\tau_2\lambda^2/D} - e^{-\tau_2\lambda^2}\right] d\lambda \tag{20}$$

But:

$$\int_0^\infty J_0(\lambda\rho)\, e^{-\tau_2\lambda^2}\,\lambda d\lambda = (1/2\tau_2)\;\; e^{-\rho^2/4\tau_2} \tag{21}$$

Integration of (21) with respect to τ_2 leads to the form appearing in (20) and hence to the final result:

$$\chi_2(\rho,0,\tau_2) = \frac{1}{(4\pi\tau_2)^{3/2}(1-1/D)}\left[\mathrm{Ei}(-\rho^2 D/4\tau_2) - \mathrm{Ei}(-\rho^2/4\tau_2)\right] \tag{22}$$

It is easy to show that as $D \longrightarrow 1$, (22) reduces to the well-known Gaussian solution:

$$\chi_2 = \frac{e^{-\rho^2/4\tau_2}}{(4\pi\tau_2)^{3/2}} \tag{22a}$$

For $D \rightarrow \infty$, (22) reduces to:

$$\chi_2 = \frac{1}{(4\pi\tau_2)^{3/2}}\left[-\mathrm{Ei}(-\rho^2/4\tau_2)\right] \tag{22b}$$

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Table I gives $R_1(D) = \frac{\chi_2(D)}{\chi_2(D=1)}$ as a function of $(\rho/2\sqrt{\tau_2})$ for D=2, 10, ∞.

Case II: $D = \infty$, $z = 0$, α and ρ arbitrary

$$\phi_2(\rho,0,\eta) = \frac{1}{2\pi(1-\alpha)}\int_0^\infty \frac{J_0(\lambda\rho)\lambda\left[\sqrt{\lambda^2+\eta}-\sqrt{\alpha\eta}\right]\,d\lambda}{\left[\lambda^2/(1-\alpha)+\eta\right]} \tag{23}$$

Now:

$$\frac{\sqrt{\lambda^2+\eta}}{\left[\lambda^2/(1-\alpha)+\eta\right]} = \mathcal{L}\left[\frac{2}{\sqrt{\pi}}\,\frac{\lambda^2\alpha}{(\alpha-1)}\,e^{\lambda^2\tau/(\alpha-1)}\int_0^{\sqrt{\tau}} e^{\lambda^2\alpha x^2/(1-\alpha)}dx + \frac{e^{-\tau\lambda^2}}{2\sqrt{\tau}}\right] \tag{24}$$

Thus:

$$\mathcal{L}^{-1}\left[\frac{1}{2\pi(1-\alpha)}\int_0^\infty \frac{J_0(\lambda\rho)\lambda\sqrt{\lambda^2+\eta}\,d\lambda}{\left[\lambda^2/(1-\alpha)+\eta\right]}\right]$$

$$= \frac{-\alpha}{\pi^{3/2}(\alpha-1)^2}\int_0^\infty J_0(\lambda\rho)\lambda^3 e^{\lambda^2\tau/(\alpha-1)}\left[\int_0^{\sqrt{\tau}} e^{\lambda^2\alpha x^2/(1-\alpha)}dx\right]d\lambda$$

$$+ \frac{1}{2\pi^{3/2}(1-\alpha)\sqrt{\tau}}\int_0^\infty J_0(\lambda\rho)\lambda e^{-\tau\lambda^2}d\lambda \tag{25}$$

The second integral is evaluable directly (cf. eq. (21))and leads to

$$\frac{1}{4(1-\alpha)\pi^{3/2}}\quad\frac{e^{-\rho^2/4\tau}}{\tau^{3/2}}$$

The first integral is reduced to (by letting $x^2 = u$ in x-integral):

$$\frac{-\alpha}{\pi^{3/2}(\alpha-1)^2}\int_0^{\tau}\frac{du}{2\sqrt{u}}\int_0^\infty J_0(\lambda\rho)\,\lambda^3\,e^{\lambda^2(\tau-\alpha u)/(\alpha-1)}$$ [redacted]

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It is easily seen by differentiation with respect to the parameter τ, and identification of a new τ that:

$$\int_0^\infty J_o(\lambda\rho)\lambda^3 \, e^{\lambda^2(\tau-au)/(a-1)} d\lambda = 1/2\left(\frac{a-1}{\tau-au}\right)^2\left[1+\frac{\rho^2(a-1)}{4(\tau-au)}\right] e^{\rho^2(a-1)/4(\tau-au)} \tag{26}$$

Hence

$$\mathcal{L}^{-1}\left[\frac{1}{2\pi(1-a)}\right]\int_0^\infty \frac{J_o(\lambda\rho)\lambda\sqrt{\lambda^2+\eta}\, d\lambda}{[\lambda^2/(1-a)+\eta]}$$

$$= -\frac{a}{4\pi^{3/2}}\int_0^\tau \frac{du}{\sqrt{u}}\left\{\frac{1}{(\tau-au)^2}\left[1+\frac{\rho^2(a-1)}{4(\tau-au)}\right] e^{\rho^2(a-1)/4(\tau-au)}\right\}$$

$$+ \frac{1}{4(1-a)\pi^{3/2}} \quad \frac{e^{-\rho^2/4\tau}}{\tau^{3/2}} \tag{27}$$

To evaluate the second part of (23) we make use of the fact that

$$\frac{\sqrt{\eta}}{[\lambda^2/(1-a)+\eta]} = \mathcal{L}\left[\frac{2\lambda^2}{\sqrt{\pi}(a-1)} \, e^{\lambda^2\tau/(a-1)}\int_0^{\sqrt{\tau}} e^{\lambda^2 x/(1-a)} dx + \frac{1}{2\sqrt{\tau}}\right] \tag{28}$$

and proceeding along the same lines as above, we find:

$$\mathcal{L}^{-1}\frac{1}{2\pi(1-a)}\int_0^\infty \frac{J_o(\lambda\rho)\lambda\sqrt{a\eta}\, d\lambda}{[\lambda^2/(1-a)+\eta]}$$

$$= \frac{\sqrt{a}}{4\pi^{3/2}}\int_0^\tau \frac{dv}{\sqrt{v}}\left\{\frac{1}{(\tau-v)^2}\left[1+\frac{\rho^2(a-1)}{4(\tau-v)}\right] e^{\rho^2(a-1)/4(\tau-v)}\right\} \tag{29}$$

 [redacted]

Finally:

$$\chi_2(\rho, o, \tau_2) = \frac{e^{-\rho^2/4\tau_2}}{4(1-a)(\pi\tau_2)^{3/2}} - \frac{\sqrt{a}}{4\pi^{3/2}} \int_{\tau_2}^{a\tau_2}$$

$$\frac{dv}{\sqrt{v}} \left\{ \frac{1}{(\tau_2 - v)^2} \left[1 + \frac{\rho^2(a-1)}{4(\tau_2 - v)} \right] e^{\rho^2(a-1)/4(\tau_2 - v)} \right] \qquad (30)$$

It is possible to show that as $a \longrightarrow 1$, (30) goes over into (22b).

Table II contains values of $R_2 = \dfrac{\chi_2(D = \infty,\ a)}{\chi_2(D = 1,\ a = 1)}$

for $a = 2,3$ and for various $(\rho/2\sqrt{\tau_2})$.

<u>Case III:</u> <u>$D = \infty$, $a = 1$, ρ and z arbitrary</u>

$$\phi_2(\rho, z, \eta) = \frac{1}{2\pi} \int_0^\infty \frac{J_o(\lambda\rho)}{\lambda} e^{-z\sqrt{\lambda^2+\eta}} \left[\sqrt{\lambda^2+\eta} - \sqrt{\eta} \right] d\lambda \qquad (31)$$

Now:

$$e^{-z\sqrt{\lambda^2+\eta}} = \mathcal{L}\left[\frac{z}{\sqrt{4\pi}} \quad \frac{e^{-\lambda^2\tau - z^2/4\tau}}{\tau^{3/2}} \right]$$

$$\frac{1}{\sqrt{a_1 + \eta}} = \mathcal{L}\left[\frac{1}{\sqrt{\pi}} \quad \frac{e^{-a_1\tau}}{\sqrt{\tau}} \right]$$

Integrating with respect to a_1 over (o, λ^2), we get:

$$\sqrt{\lambda^2 + \eta} - \sqrt{\eta} = \mathcal{L}\left[\frac{1}{\sqrt{4\pi\tau}} \quad \frac{(1 - e^{-\tau\lambda^2})}{\tau} \right]$$

Using the convolution theorem, we therefore obtain:

$$e^{-z\sqrt{\lambda^2+\gamma}}\left[\sqrt{\lambda^2+\gamma}-\sqrt{\gamma}\right]=$$

$$=\mathcal{L}\left\{\frac{z}{4\pi}\int_0^{\tau}\frac{e^{-z^2/4u}}{u^{3/2}}\frac{\left(e^{-\lambda^2 u}-e^{-\lambda^2\tau}\right)}{(\tau-u)^{3/2}}\,du\right\} \tag{33}$$

Consequently:

$$\mathcal{L}^{-1}\left\{\frac{1}{2\pi}\int_0^{\infty}\frac{J_0(\lambda\rho)e^{-z\sqrt{\lambda^2+\gamma}}}{\lambda}\left[\sqrt{\lambda^2+\gamma}-\sqrt{\gamma}\right]d\lambda\right.$$

$$=\frac{z}{8\pi^2}\int_0^{\tau}\frac{e^{-z^2/4u}}{u^{3/2}(\tau-u)^{3/2}}\left[\int_0^{\infty}\frac{J_0(\lambda\rho)}{\lambda}\left[e^{-\lambda^2 u}-e^{-\lambda^2\tau}\right]d\lambda\right]du$$

$$=\frac{z}{16\pi^2}\int_0^{\tau}\frac{e^{-z^2/4u}}{u^{3/2}(\tau-u)^{3/2}}\left[\int_{\rho^2/4\tau}^{\rho^2/4u}\frac{e^{-x}}{x}\,dx\right]du$$

or:

$$\chi_2(\rho,z,\tau_2)=\frac{z}{16\pi^2}\int_0^{\tau_2}\frac{e^{-z^2/4u}}{u^{3/2}(\tau_2-u)^{3/2}}\left[-Ei(-\rho^2/4\tau_2)+Ei\left(-\frac{\rho^2}{4u}\right)\right]du \tag{34}$$

It is readily verified that (34) approaches (22b) as $z \rightarrow 0$.

Table III lists $R_3 = \dfrac{\chi_2(D=\infty)}{\chi_2(D=1)}$

as a function of

$$(\rho/2\sqrt{\tau_2}) \quad \text{and} \quad (z/2\sqrt{\tau_2})$$

[redacted]

Table I: R_1 = Ratio of neutron density with adjacent medium of higher density to neutron density with adjacent medium of equal density, measured at interface between two media. (D represents square of density ratio)

$\longleftarrow R_1 \longrightarrow$

$\rho/2\sqrt{\tau_2}$	D = 2	D = 3.7 quad. interp.	D = 10	D = ∞	Ratio to ∞
.5	1.239	1.32	1.449	1.336	.99
1.0	.923	.82	.661	.595	1.38
1.5	.623	.53	.368	.331	1.60
2.0	.409	.34	.230	.209	1.63

Table II: R_2 = Ratio of neutron density with adjacent medium of infinite density but different chemical composition to neutron density with adjacent medium of same density and chemical composition, measured at interface between two media (α is defined in eq. (4)).

$\longleftarrow R_2 \longrightarrow$

$\rho/2\sqrt{\tau_2}$	α = 1	α = 1.7	α = 2	α = 3	α = 1.7 ratio to ∞ for D = 3.7
.5	1.336	1.1	1.084	.946	1.1
1.0	.595	.45	.397	.304	.62
1.5	.331	.23	.204	.147	.37

[redacted]

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Table III: R_3 = Ratio of neutron density with infinitely dense adjacent medium to neutron density with equally dense adjacent medium, measured at various distances above surface.

$\frac{\rho}{2\sqrt{\tau_2}}$	$\frac{z}{2\sqrt{\tau_2}}$	R_3	A Ratio to Surface	B Take $\frac{1-A}{1+\text{Ratio to } \infty \text{ on p 15 Table I}}$
0	.5	1.487		
0	1.0	1.182		
.5	0	1.336		
.5	.5	1.225	.92	.93
.5	1.0	1.108		
1.0	0	.595		
1.0	.5	.889	1.49	1.35
1.0	1.0	.970		
1.5	0	.331		
1.5	.5	.667	2.02	1.64
1.5	1.0	.820		

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