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TAYLOR INSTABILITY IN SHOCK ACCELERATION OF COMPRESSIBLE FLUIDS

by

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ABSTRACT

The initial growth of irregularities on an interface between two compressible fluids is studied for impulsive (i.e., shock) acceleration. It was found that the ultimate rate of growth is roughly the same as that given by the incompressible theory, if the initial compression of the irregularities and of the fluids is taken into account.

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Chapter I -- INTRODUCTION

G. I. Taylor developed a theory of the growth of irregularities on the interface between two fluids of different densities when they are in accelerated motion. The fluids are assumed incompressible and the interface to have sinusoidal corrugations, so that the position of the interface in a suitably-oriented cartesian coordinate system is given by

$$z = a \cos kx \tag{1}$$

at some instant, where a and k are constants and where

$$k\,a \ll 1. \tag{2}$$

Then if there is an acceleration of the system as a whole and if $g(t)$ represents the z-component of acceleration, the growth or decay of the amplitude $a = a(t)$ of the corrugations satisfies the equation,

$$\frac{d^2}{dt^2}\,a(t) = k\,g(t)\,a(t)\,\frac{\rho_{(+)} - \rho_{(-)}}{\rho_{(+)} + \rho_{(-)}}\,, \tag{3}$$

where $\rho_{(+)}$ and $\rho_{(-)}$ are the densities of the fluids, on the +z and -z sides of the interface, respectively.

If viscosity, surface tension, compressibility are absent, (3) is rigorous so long as (2) continues to hold. If the irregularity consists of a superposition of sinusoidal corrugations, each satisfying (2), then

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each of them satisfies also (3) with its appropriate value of k. For constant acceleration directed toward the denser fluid, (3) gives an exponential growth of a(t). It is also known that after ka has reached about 1, from there on the increase of a is more leisurely, at about a constant rate (the shape is no longer sinusoidal). It is therefore generally assumed that (3) describes fairly well the growth of long-wave-length irregularities of small amplitude ($ka \ll 1$), even though irregularities of very short wavelength have gone out of the linear range near the beginning of the acceleration, since the fuzziness of the interface caused by the short-wave-length irregularities may extend only over distances small compared to the amplitudes of the longer-wave-length irregularities. (3) has been used for calculating the growth of irregularities for practical purposes.

In some practical cases, part or all of the acceleration is impulsive; i.e., g(t) is very large during a very short time interval and zero or small outside that interval. Let v be the increment of velocity imparted by this acceleration, $= \int g(t)\,dt$; then if the situation before the acceleration is

$$a = a_o, \qquad \frac{da}{dt} = 0, \qquad \text{(before)}, \tag{4}$$

the situation immediately after is:

$$a = a_o, \quad \frac{da}{dt} = k\,v\,a_o\,\frac{\rho_{(+)} - \rho_{(-)}}{\rho_{(+)} + \rho_{(-)}} \text{ (immediately after)} \tag{5}$$

as can be seen by integrating (3).

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The limiting case of impulsive acceleration is acceleration by a shock. In this case the compressibility of the fluids certainly cannot be neglected, so that (3) cannot be used. The object of study reported here is to compute the growth of irregularities when a shock sweeps across a corrugated interface from a less dense to a more dense fluid.

In Fig. 1 are schematic before-and-after pictures, showing the incident shock, which is assumed plane, and the transmitted and reflected shocks, which are corrugated. Before the arrival of the shock, the materials are at rest, in accordance with the initial conditions (4). Pressure is assumed constant behind the incident shock.

It is clear that immediately after the passage of the shock, as depicted schematically in Fig. 1, the conditions will not be as given by (5), above. Instead, the amplitude a will be somewhat less than a_o, because of the overall compression of the fluids, and furthermore $\frac{da}{dt}$ will be zero, because the forward velocity imparted by the shock to the crest of the corrugation will be the same as that imparted to the trough, and $\frac{da}{dt}$ is simply one-half the difference of these. Until there has been time for communication, by sound signals, of effects over distances comparable with $1/k$, there can be no difference in behavior between crest and trough. (In the incompressible theory, effects are transmitted instantaneously by fluid pressure, so that $\frac{da}{dt}$ can acquire a non-vanishing value immediately, as stated in (5) above.)

But as time goes on, the amplitude of the corrugation of the

[redacted]

interface begins to grow, because near the crests, where the heavy fluid protrudes farthest into the light, the transmitted shock is slightly converging and the reflected shock slightly diverging; this produces a slight pressure excess in the heavy fluid and deficiency in the lighter, whereas at the troughs of the corrugations, the reverse situation holds; and these pressure perturbations are in such directions as to hold back the crests but accelerate the troughs into the heavy fluid. Our aim is to calculate these pressure perturbations and from them the motion of the interface.

One may surmise that by the time the shocks have moved to distances several times $1/k$ from the interface, the net result will be qualitatively as for the incompressible case; namely, the amplitude will have acquired a rate of change $\frac{da}{dt}$ of the same order of magnitude as that given by (5), and thereafter $a(t)$ will increase practically linearly with time. If the limiting value of $\frac{da}{dt}$ should be much greater or much less than for the incompressible case, this would perhaps be of practical importance in some problems, and would indicate the extent to which equation (3) is invalidated by effects of compressibility.

Chapter II - BASIC EQUATIONS.

Sec. 1. General. Referring to Fig. 1, we take a comoving coordinate system, after the primary shock has crossed the interface, in which the mean position of the interface is $z = 0$. Let w_1 and w_2

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be the mean speeds of the transmitted and reflected shocks. We represent the positions of the transmitted shock, interface, and reflected shock by

$$z = - w_1 t + a_1(t)\, e^{ikx} \qquad (6)$$

$$z = a_o(t)\, e^{ikx} \qquad (7)$$

$$z = w_2 t + a_2(t)\, e^{ikx} \qquad (8)$$

We call region 0 that of the undisturbed fluid to the left of the transmitted shock, region 1 that of the shocked heavy fluid between the transmitted shock and the interface, region 2 that of the twice-shocked light fluid between the interface and the reflected shock, and region 3 that of the once-shocked light fluid to the right of the reflected shock. Following A. E. Roberts*, we write e^{ikx} and $-ie^{ikx}$ as abbreviations for cos kx and sin kx, respectively.

We let:

$\vec{u} = (u_x, u_y)$ = material velocity,

p = pressure,

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*Many of our equations are similar to those used by A. E. Roberts (LA-299 Stability of a Steady Plane Shock, June 8, 1945) in his discussion of the stability of shocks.

v = specific volume,

e = specific internal energy,

s = specific entropy.

These are functions of x, z, and t. Then, to first order of small quantities,

$$u_z = \left\{ \begin{array}{lll} u_o^o & (>0) & \text{in region 0} \\ u_z^1(z,t)e^{ikx} & & \text{in region 1 and 2} \\ u_3^o & (<0) & \text{in region 3} \end{array} \right\}, \tag{9}$$

$$u_x = \left\{ \begin{array}{ll} 0 & \text{in regions 0 and 3} \\ u_x^1(z,t)e^{ikx} & \text{in regions 1 and 2} \end{array} \right\}, \tag{10}$$

$$p = \left\{ \begin{array}{ll} p_o^o & \text{in region 0} \\ p_1^o + p^1(z,t)\, e^{ikx} & \text{in region 1} \\ p_2^o + p^1(z,t)\, e^{ikx} & \text{in region 2} \\ p_3^o & \text{in region 3} \end{array} \right\}, \tag{11}$$

[redacted]

$$v = \left\{ \begin{array}{ll} v_o^{\,o} & \text{in region 0} \\ v_1^{\,o} + v^1(z,t)\, e^{ikx} & \text{in region 1} \\ v_2^{\,o} + v^1(z,t)\, e^{ikx} & \text{in region 2} \\ v_3^{\,o} & \text{in region 3} \end{array} \right\} , \tag{12}$$

$$s = \left\{ \begin{array}{ll} s_o^{\,o} & \text{in region 0} \\ s_1^{\,o} + s^1(z)\, e^{ikx} & \text{in region 1} \\ s_2^{\,o} + s^1(z)\, e^{ikx} & \text{in region 2} \\ s_3^{\,o} & \text{in region 3} \end{array} \right\} \tag{13}$$

This entire theory is a first-order theory. The first-order quantities are assumed small in comparison with the corresponding zero-order quantities, and higher-order quantities are neglected. Thus, e.g.; $p^1(z,t) \ll p_1^{\,o}$, $u_z^{\,1}(z,t) \ll w_1$, also $a_o(t) \ll \frac{1}{k}$ during the period of time covered by the calculation.

Note that $s^1(z)$ does not depend on t.

Sec. 2. Equations of Hugoniot. If for either of the shocks shown in Figure 2, we let S_o, P_o, V_o represent the initial specific entropy, pressure, and specific volume, and if p and e are known as functions of v and s, then the Hugoniot equation,

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$$\frac{p(v,s) + P_o}{2} (V_o - v) = e(v,s) - e(V_o,S_o), \qquad (14)$$

determines s (implicitly) as a function of v, and therefore also p as a function of v. Let these functions be denoted by:

	light fluid	heavy fluid	
$p =$	$\varphi_\ell(v)$,	$\varphi_h(v)$,	(15)
$s =$	$\psi_\ell(v)$,	$\psi_h(v)$.	(16)

Note that $\varphi_\ell(v)$ and $\psi_\ell(v)$ are Hugoniot functions starting from the condition in which the light fluid was left by the primary shock. According to the Hugoniot theory, the relations among the zero-order quantities are

$$p_1^{\circ} = p_2^{\circ} = \varphi_\ell(v_2^{\circ}) = \varphi_h(v_1^{\circ}) , \qquad (17)$$

$$s_1^{\circ} = \psi_h(v_1^{\circ}), \qquad (18)$$

$$s_2^{\circ} = \psi_\ell(v_2^{\circ}), \qquad (19)$$

$$\left(\frac{w_1}{v_1^{\circ}}\right)^2 = \frac{p_1^{\circ} - p_o^{\circ}}{v_o^{\circ} - v_1^{\circ}} = \left(\frac{w_1 + u_o^{\circ}}{v_o^{\circ}}\right)^2 , \qquad (20)$$

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$$\left(\frac{w_2}{v_2^{\circ}}\right)^2 = \frac{p_2^{\circ} - p_3^{\circ}}{v_3^{\circ} - v_2^{\circ}} = \left(\frac{w_2 - u_3^{\circ}}{v_3^{\circ}}\right)^2. \tag{21}$$

Sec. 3. The Pressure Perturbation. The first-order quantities satisfy partial differential equations in the regions between the shocks; and boundary conditions at $z = -w_1 t$, at $z = 0$, and at $z = w_2 t$. We discuss the partial differential equations first.

For region 1, if $p = f_1(v,s)$ is the equation of state of the heavy fluid; then to first order,

$$\left.\begin{aligned} p^1(z,t) &= v^1(z,t)\left[\frac{\partial f_1(v,s)}{\partial v}\right]_{\substack{v=v_1^{\circ} \\ s=s_1^{\circ}}} + s^1(z)\left[\frac{\partial f_1(v,s)}{\partial s}\right]_{\substack{v=v_1^{\circ} \\ s=s_1^{\circ}}} \\ &= -\left(\frac{c_1}{v_1^{\circ}}\right)^2 \hat{v}(z,t). \end{aligned}\right\} \tag{22}$$

where we define $\hat{v}(z,t)$ by:

$$\hat{v}(z,t) = v^1(z,t) + s^1(z)\left[\frac{\partial f_1}{\partial s}\bigg/\frac{\partial f_1}{\partial v}\right]_{\substack{v=v_1^{\circ} \\ s=s_1^{\circ}}}, \tag{23}$$

and where the sound speed is given by

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$$\left(\frac{c_1}{v_1^{\,o}}\right)^2 = -\left[\frac{\partial f_1}{\partial v}\right]_{\substack{v=v_1^{\,o} \\ s=s_1^{\,o}}} . \tag{24}$$

The equation of continuity is

$$v_1^{\,o}\left(\frac{\partial u_z^{\,1}}{\partial z} + i\,k\,u_x^{\,1}\right) = \frac{\partial v^1}{\partial t} = \frac{\partial \hat{v}}{\partial t} = -\left(\frac{v_1^{\,o}}{c_1}\right)^2 \frac{\partial p^1}{\partial t} \tag{25}$$

and the equations of motion are:

$$-\frac{\partial p^1}{\partial z} = \frac{1}{v_1^{\,o}}\,\frac{\partial u_z^{\,1}}{\partial t}, \tag{26}$$

$$-\,i\,k\,p^1 = \frac{1}{v_1^{\,o}}\,\frac{\partial u_x^{\,1}}{\partial t} \tag{27}$$

By elimination, we find that p^1 satisfies the equation

$$\frac{\partial^2 p^1}{\partial t^2} = c_1^{\,2}\left(\frac{\partial^2 p^1}{\partial z^2} - k^2\,p^1\right), \tag{28}$$

which is no great surprise. In region 2 the same equation holds with $c_1^{\,2}$ replaced by $c_2^{\,2}$.

By the introduction of the effective specific volume $\hat{v}$ defined by (23) we have got rid of entropy. It is not necessary to assume an equation of state independent of entropy, as was done by Roberts. Such an assumption does not simplify (in fact does not alter) the form of the equations -- it only simplifies the calculation of certain constants entering the boundary conditions.

Equations similar to (23), (24), and (28) hold for region 2.

Sec. 4. Boundary conditions at the shocks. The shock conditions at the reflected shock are

$$\left.\begin{aligned} &\frac{w_2 + \dot{a}_2(t)\, e^{ikx} - u_z^{\,1}(w_2 t, t)\, e^{ikx}}{v_2^{\,o} + v^1(w_2 t, t)\, e^{ikx}} = \frac{w_2 + \dot{a}_2(t) e^{ikx} - u_3^{\,o}}{v_3^{\,o}} = \\ &= \sqrt{\frac{p_2^{\,o} + p^1(w_2 t, t) e^{ikx} - p_3^{\,o}}{v_3^{\,o} - v_2^{\,o} - v^1(w_2 t, t) e^{ikx}}} \;. \end{aligned}\right\} \qquad (29)$$

We expand each member of this equation in powers of first-order small quantities and retain only zeroth and first powers. Actually, (29) is only correct to first order because if the obliquity of the shock were taken into account, the cosines of small angles would enter the first two members, and these cosines differ from unity by quantities of the second order, because the angles are themselves small quantities of the first order. The zero-order terms from (29) give

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$$\frac{w_2}{v_2^o} = \frac{w_2 - u_3^o}{v_3^o} = \sqrt{\frac{p_2^o - p_3^o}{v_3^o - v_2^o}}\ ,$$

in agreement with equation (21) above. By equating the first-order terms, we obtain the equations:

$$\frac{1}{v_2^o}\left\{\dot{a}_2 - u_z^1(w_2t,t)\right\} - \frac{w_2}{(v_2^o)^2}\, v^1(w_2t,t) = \sqrt{\frac{p_2^o - p_3^o}{v_3^o - v_2^o}}\ \cdot$$

$$\left\{ \frac{1}{2}\,\frac{p^1(w_2t,t)}{p_2^o - p_3^o} + \frac{1}{2}\,\frac{v^1(w_2t,t)}{v_3^o - v_2^o} \right\} \tag{30}$$

$$\frac{1}{v_3^o}\,\dot{a}_2 = \text{same} \tag{31}$$

Similar equations hold for the transmitted shock. We can express the volume increment, v^1, at the shock in terms of the pressure increment, p^1, there, by noting that from (15),

$$p^1(w_2t,t) = \left[\frac{d\,\varphi_{\ell}(v)}{dv}\right]_{v=v_2^o} v^1(w_2t,t). \tag{32}$$

This suggests that we introduce the dimensionless quantities

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$$K_2 = - \frac{\left[\dfrac{d\varphi_\ell(v)}{dv}\right]_{v=v_2{}^o}}{(c_2/v_2{}^o)^2} \tag{33}$$

$$K_1 = - \frac{\left[\dfrac{d\varphi_h(v)}{dv}\right]_{v=v_1{}^o}}{(c_1/v_1{}^o)^2} \tag{34}$$

Then (31) and its analogue for the transmitted shock give us for one set of boundary conditions,

$$\dot{a}_2 = \frac{1}{2} \frac{v_3{}^o \, v_2{}^o}{v_3{}^o - v_2{}^o} \left\{\frac{1}{w_2} - \frac{w_2}{K_2 \, c_2{}^2}\right\} p^1(w_2t,t), \tag{35}$$

$$\dot{a}_1 = - \frac{1}{2} \frac{v_o{}^o \, v_1{}^o}{v_o{}^o - v_1{}^o} \left\{\frac{1}{w_1} - \frac{w_1}{K_1 \, c_1{}^2}\right\} p^1(-w_1t,t). \tag{36}$$

The factor multiplying p^1 is in each case a constant that is evaluated at the beginning of the calculation.

Eliminating $\dot{a}_2$ from (30) then gives:

$$u_z{}^1(w_2t,t) = \frac{1}{2} \, v_2{}^o \left\{\frac{1}{w_2} + \frac{w_2}{K_2 \, c_2{}^2}\right\} p^1(w_2t,t). \tag{37}$$

(37) is an identity in t, and therefore,

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$$\left(\frac{\partial}{\partial t} + w_2 \frac{\partial}{\partial z}\right) u_z^1 = \frac{1}{2} v_2^{\circ} \left\{ \frac{1}{w_2} + \frac{w_2}{K_2 c_2^2} \right\} \left(\frac{\partial}{\partial t} + w_2 \frac{\partial}{\partial z}\right) p^1 \quad \text{at } z = w_2 t. \tag{38}$$

The tangential component of the fluid velocity, $\vec{u}$, is continuous at the shock. To first order of small quantities, this is $u_x^{\,1}(w_2\, t, t)\, e^{ikx}$ just behind the shock, and $u_3^{\circ} \tan\alpha$ just ahead, where $\tan\alpha$ is the inclination of the shock front to the x axis and is given by

$$\tan\alpha = \frac{\partial}{\partial x} \left\{ w_2 t + a_2(t)\, e^{ikx} \right\} = ika_2\, e^{ikx}, \tag{39}$$

according to equation (8) for the form of the shock front. Therefore,

$$u_x^1(w_2 t, t) = iku_3^{\circ}\, a_2(t),$$

or

$$ikw_2 u_x^1 = -k^2 u_3^{\circ}\, w_2\, a_2 \quad \text{at} \quad z = w_2 t. \tag{40}$$

If we add (38) and (40), the material velocity components, u_x^1 and u_z^1 may be eliminated by use of the equation of continuity, and the equation of motion, similar to (25) and (27) for region 2. It is convenient to introduce a notation for the shock pressures:

$$\left.\begin{aligned} p(s)_1(t) &= p^1(-w_1 t, t) \\ p(s)_2(t) &= p^1(\ w_2 t, t) \end{aligned}\right\}. \tag{41}$$

The result is:

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$$\left\{ w_2 + \frac{c_2^2}{2w_2} + \frac{w_2}{2K_2} \right\} \frac{dp(s)_2}{dt} = (w_2^2 - c_2^2)\left[\frac{\partial p^1}{\partial z}\right]_{z=w_2 t} + \frac{k^2 c_2^2 u_3^o w_2}{v_2^o}\, a_2(t),$$

$$\left\{ w_1 + \frac{c_1^2}{2w_1} + \frac{w_1}{2K_1} \right\} \frac{dp(s)_1}{dt} = (w_1^2 - c_1^2)\left[-\frac{\partial p^1}{\partial z}\right]_{z=-w_1 t} + \frac{k^2 c_1^2 u_o^o w_1}{v_1^o}\, a_1(t). \tag{42}$$

These boundary conditions give the rate of change of the shock pressures. The first term on the right arises from pressure gradient behind the shock and the second term from convergence or divergence.

Sec. 5. Boundary conditions at the interface. From continuity of the normal component of fluid velocity, and hence of acceleration, across the interface;

$$\ddot{a}_o(t) = \left[-v_1^o \frac{\partial p^1}{\partial z}\right]_{z=-0} = \left[-v_2^o \frac{\partial p^1}{\partial z}\right]_{z=+0}. \tag{43}$$

The equality of the second and third members of this equation is what is needed by way of boundary condition for the partial differential equation (28) for the pressure; the first member is used to determine the function $a_o(t)$, which is the principal objective of the calculation.

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Sec. 6. Oblique shock transmission. In preparation for writing down the initial conditions, we consider the transmission and reflection of a plane shock crossing a plane interface as depicted in Fig. 3. We look for solutions of the hydrodynamical equations such that conditions are constant in each of the five regions shown in that figure. For the purpose of this discussion we suppose the fluids initially at rest, and we later transform to the comoving coordinate system used in connection with Fig. 2.

One Hugoniot relation says that the effect of a shock on the material velocity $\vec{u}$ is to impart to it an increment $\Delta\vec{u}$ whose direction is normal to the shock front and whose magnitude is $\sqrt{\Delta P \Delta V}$. Therefore (see Fig. 3 for the notation used in this section; the angles indicated there are regarded as positive):

$$\left.\begin{aligned}
u_{1x} &= 0, & u_{1z} &= -\sqrt{(P_1-P_o)(V_o-V_1)}\,, \\
u_{2x} &= \sqrt{(P_2-P_1)(V_1-V_2)}\,\sin\alpha_2, & u_{2z} &= u_{1z} + \sqrt{(P_2-P_1)(V_1-V_2)}\,\cos\alpha_2, \\
u_{3x} &= \sqrt{(P_3-P_o)(V_o^1-V_3)}\,\sin\alpha_4, & u_{3z} &= -\sqrt{(P_3-P_o)(V_o^1-V_3)}\,\cos\alpha_4.
\end{aligned}\right\} \quad (44)$$

Let U_o, U_1, U_2, U_3 be the velocities of the various planes as indicated in Fig. 2, measured in directions normal to these planes. In a time Δt the intersection moves a distance $U_o\Delta t/\sin\alpha_1$ along the interface. Therefore, the reflected shock moves a distance $\frac{U_o\Delta t}{\sin\alpha_1}\sin(\alpha_2 - \alpha_1)$ normal to itself. From these and similar relations,

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$$\left.\begin{aligned} U_1 &= U_o \frac{\sin(\alpha_2 - \alpha_1)}{\sin \alpha_1}, \\ U_2 &= U_o \frac{\sin(\alpha_1 - \alpha_4)}{\sin \alpha_1}, \\ U_3 &= U_o \frac{\sin(\alpha_1 - \alpha_3)}{\sin \alpha_1}. \end{aligned}\right\} \quad (45)$$

The Hugoniot equation for the velocity of a shock relative to the material ahead of it gives, for the three shocks;

$$\left.\begin{aligned} U_o &= V_o\sqrt{\frac{P_1 - P_o}{V_o - V_1}}, \\ U_1 - (u_{1z}\cos\alpha_2 + u_{1x}\sin\alpha_2) &= V_1\sqrt{\frac{P_2 - P_1}{V_1 - V_2}}, \\ U_2 &= V_o^1\sqrt{\frac{P_3 - P_o}{V_o^1 - V_3}}. \end{aligned}\right\} \quad (46)$$

The velocity of the interface must agree with the normal component of fluid velocity on either side, or;

$$U_3 = -(u_{2z}\cos\alpha_3 + u_{2x}\sin\alpha_3) = -(u_{3z}\cos\alpha_3 + u_{3x}\sin\alpha_3). \quad (47)$$

Lastly, since the fluids are assumed to have no acceleration except by shocks,

$$P_2 = P_3. \quad (48)$$

From equations (44) through (48), we can draw the following conclusions (details are omitted):

1) The component of the fluid velocity $\vec{u}$ tangential to the interface is not in general the same on the two sides. That is,

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there is slipping along the interface. This effect will not concern us directly.

2) We may define the <u>compression</u> of the interface as the amount of purely geometrical compression in the z-direction that would be required to turn the interface from its initial orientation into its final orientation. This is $\tan\alpha_1/\tan\alpha_3$: It gives the initial compression by the shock of the corrugations in our problem. It can be calculated from the above equations, by means of a little algebra, in the limiting case in which all angles are $\ll 1$ so that we can approximate by writing $\sin\alpha \approx \alpha$, $\cos\alpha \approx 1$. It turns out to depend on the ratios $\frac{V_o}{V_2}$, $\frac{V^1_o}{V_3}$, and $\frac{V_o}{V_1}$.

We might have hoped to be able to express the compression, α_1/α_3, of the interface, in terms of the compressions, $\frac{V_o}{V_2}$ and $\frac{V^1_o}{V_3}$ of the fluids on either side of it, but the initial compression, $\frac{V_o}{V_1}$, of the light fluid by the incident shock, also enters. Of course, V_1 is not arbitrary: (P_1, V_1) is a point in the state space of the light fluid that must lie on the Hugoniot starting from (P_o, V_o); and, in turn the Hugoniot starting from (P_1, V_1) must pass through (P_2, V_2). Thus the point (P_1, V_1) is determined as the intersection of two Hugoniot curves. But a consideration of the thermodynamic properties of the fluids is required to fix this point, and we must despair of computing the compression of the interface in terms of purely kinematic quantities.

These two conclusions are mentioned only incidentally. Below, we shall derive initial conditions for our problem from the considerations of this section. The reasoning is as follows: the amplitudes a_1, a_o, a_2 appearing in equations (6), (7), and (8), are understood to be $\ll \frac{1}{k}$. At sufficiently early times after impact, Fig. 2 is as shown schematically on an enlarged scale in Fig. 3. The separations of the surfaces are small compared to the wavelength of the corrugations, and sound signals have only been able to travel distances small compared to a wavelength since the beginning of the impact. Therefore, a region, R, sufficiently small that the surfaces are very nearly plane in it, is still unaffected by the fact that the tangent planes have slightly different positions in some other region, such as R'. Therefore, in either of these regions the collision problem reduces to that of Fig. 2.

Sec. 7. Initial condition. Each of the inclined planes in Fig. 2 is to be thought of as a very small piece of one of the corrugated surfaces in our problem; the tangent of the angle of inclination is, therefore, proportional to the amplitude of the corresponding corrugation. Specifically

$$\tan\alpha_1 : \tan\alpha_2 : \tan\alpha_3 : \tan\alpha_4 = a_o(-0) : a_2(+0) : a_o(+0) : a_1(+0) \tag{49}$$

where $t = +0$ and -0 refer to just after, and just before the shock crosses the interface. The correspondence in notation between Sec. 6 and the preceding sections is:

Notation of Sec. 6	Notation of Previous Sections
U_1	$w_2 - u_o^o$
U_2	$w_1 + u_o^o$
U_3	u_o^o

This correspondence is correct down to small quantities of first order. Therefore, if we assume $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ all $\ll 1$, (45) gives

$$\left.\begin{aligned} u_o^o - w_2 &= U_o\left[1 - \frac{a_2(+0)}{a_o(-0)}\right] \\ u_o^o + w_1 &= U_o\left[1 - \frac{a_1(+0)}{a_o(-0)}\right] \\ u_o^o &= U_o\left[1 - \frac{a_o(+0)}{a_o(-0)}\right] \end{aligned}\right\} \tag{50}$$

Furthermore, the pressure P_2 of Sec. 6 is the pressure $p_2^o + p^1(z, t)\, e^{ikx}$. Clearly this is an even function of the angles in Fig. 2 and, therefore, P_2 is independent of the angles to first order of small quantities, so

$$p^1(0,0) = 0. \tag{51}$$

As noted in the introduction, the forward speeds imparted by the shock to crest and trough of the corrugations are equal, so

$$\dot{a}_o(0) = 0. \tag{52}$$

Lastly, we need initial values of the pressure gradients, because these are needed to compute the rate of change of shock pressures by (42). These initial values will be denoted by:

$$\left.\begin{aligned}\beta_1 &= \lim_{t\to 0+} -\left[\frac{\partial p'}{\partial z}\right]_{z=-w_1 t}, \\ \beta_2 &= \lim_{t\to 0+} \left[\frac{\partial p'}{\partial z}\right]_{z=w_2 t}.\end{aligned}\right\} \tag{53}$$

In (42), we write out the total derivatives; e. g.

$$\left[\frac{dp(s)_2(t)}{dt}\right]_{t\,=\,0} = \left[\frac{\partial p'}{\partial t} + w_2 \frac{\partial p'}{\partial z}\right]_{\substack{z=0\\ t=0}} \tag{54}$$

and eliminate $\frac{\partial p'}{\partial t}$ to give

$$\left.\begin{aligned}&\frac{\left(\frac{3c_2^2}{2w_2} + \frac{w_2}{2K_2}\right)\beta_2 - \frac{k^2\, c_2^2\, u_3^o\, w_2\, a_2(+0)}{v_2^o}}{w_2 + \frac{c_2^2}{2w_2} + \frac{w_2}{2K_2}} = \\ &\qquad\qquad \frac{\left(\frac{3c_1^2}{2w_1} + \frac{w_1}{2K_1}\right)\beta_1 - \frac{k^2\, c_1^2\, u_o^o\, w_1\, a_1(+0)}{v_1^o}}{w_1 + \frac{c_1^2}{2w_1} + \frac{w_1}{2K_1}}\end{aligned}\right\} \tag{55}$$

(43) gives

$$v_1^o\, \beta_1 + v_2^o\, \beta_2 = 0 \tag{56}$$

(55) and (56) can be solved for the constants β_1 and β_2.

[redacted]

Sec. 8. Statement of problem. The problem to be solved is that of the differential equation (28) in the wedge $t \geqslant 0$, $-w_1 t \leqslant z \leqslant 0$; the same differential equation with c_1 replaced by c_2 in the wedge $t \geqslant 0$, $0 \leqslant z \leqslant w_2 t$; subject to boundary conditions (35), (36), (42), and (43) and to initial conditions (50), (51), (52),(55), and (56). Among desired results is a graph of $\dot{a}_o$ vs. t carried far enough that an approximate asymptotic value of $\dot{a}_o$ is evident. Parameters in the problem are: the thermodynamic properties of the fluids, their initial states (e. g., densities - their initial pressures are, of course, equal), and the strength of the incident shock. The wave number, k, is not really a parameter, because the problem is invariant to multiplication of all lengths and times by a common constant, so that k can be taken equal to unity without loss of generality.

This is a linear initial-boundary value problem and can possibly be solved by analytic methods. Consideration of the work of A. E. Roberts (op. cit.) on a similar but considerably simpler problem suggests that the analytic solution is likely to be very complicated; therefore, we chose to use numerical integration.

Chapter III -- FINITE DIFFERENCE METHODS

The time increment is Δt; we allow for different space increments $\Delta_1 z$ and $\Delta_2 z$ is the regions 1 and 2,but these are assumed so chosen that

$$w_1 \Delta t = y_1 \Delta_1 z, \quad w_2 \Delta t = y_2 \Delta_2 z \tag{57}$$

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where $\frac{1}{y_1}$ and $\frac{1}{y_2}$ are positive integers - then the transmitted (reflected) shock arrives at a mesh point exactly every $\frac{1}{y_1}$ cycles (every $\frac{1}{y_2}$ cycles) which simplifies the procedures for adding mesh points. At cycle n let $j_1 = j_1(n)$ denote the greatest integer less than $y_1 n$ and similarly for j_2 so that the number of mesh points between the shocks is $j_1 + j_2 + 1$.

Calling

$$q = \frac{\partial p^1}{\partial t} , \tag{58}$$

replacing p^1 by simply p, and adopting the convention that for any function $f(z, t)$, f_j^n denotes $f(j\Delta_1 z, n\Delta t)$ for $j \leqslant 0$ and $f(j\Delta_2 z, n\Delta t)$ for $j \geqslant 0$, we can write for the <u>partial differential equation and the boundary condition at $z = 0$</u>:

$$\left.\begin{aligned} q_j^{n+1/2} &= q_j^{n-1/2} + \begin{Bmatrix} K_{13} \\ \text{or} \\ K_{14} \end{Bmatrix} \left(p_{j+1}^n - 2p_j^n + p_{j-1}^n \right) - \begin{Bmatrix} K_{15} \\ \text{or} \\ K_{16} \end{Bmatrix} p_j^n \\ &\qquad \text{for} \begin{Bmatrix} -j_1 < j < 0 \\ \text{or} \\ 0 < j < j_2 \end{Bmatrix} \end{aligned}\right] \tag{59}$$

(The expressions for the constants, such as $K_{13}, \ldots, K_{16}$, are collected in Table II). For the point on the interface,

$$q_o^{n+1/2} = q_o^{n-1/2} + K_{17}(p_{-1}^n - p_o^n) + K_{18}(p_1^n - p_o^n) - K_{19}\, p_o^n. \tag{60}$$

(This equation was obtained by writing:

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[redacted]

$$\left[\frac{\partial^2 p}{\partial t^2}\right]_{z=0} = c_1^2 \left\{ \left[\frac{1}{v_1^o} \frac{\partial}{\partial z} v_1^o \frac{\partial p}{\partial z}\right]_{z=-0} - k^2\, p(0,\, t) \right\} =$$

$$= c_2^2 \left\{ \left[\frac{1}{v_2^o} \frac{\partial}{\partial z} v_2^o \frac{\partial p}{\partial z}\right]_{z=+0} - k^2\, p(0,\, t) \right\};$$

by replacing the second space derivatives by expressions of the form

$$\frac{1}{v} \frac{1}{\Delta z/2} \left[v \frac{p_{\pm 1}^n - p_o^n}{\Delta z} - \left(v \frac{\partial p}{\partial z}\right)_{z=\pm 0} \right];$$

and by eliminating the $\left(v \dfrac{\partial p}{\partial z}\right)_{z=\pm 0}$ by use of equation (43)).

The last mesh point in either direction has a mesh point on one side of it and a shock on the other, and generally the spatial intervals are different on the two sides of it. Pressure gradient is centered at the midpoint of the intervals, the difference equation will be centered halfway between these two midpoints, rather than at the last mesh point: therefore, a correction is applied to compute $q_j^{n+1/2}$ at $j = -j_1$ or $j = j_2$. Call

$$\frac{p_{(s)1}^n - p_{-j_1}^n}{y_1 n - j_1} = \overset{*}{\Delta}_1\, p^n, \qquad \frac{p_{(s)2}^n - p_{j_2}^n}{y_2 n - j_2} = \overset{*}{\Delta}_2\, p^n. \tag{61}$$

The pressure halfway between the two midpoints is

$$\left.\begin{aligned} \overset{*}{p}{}_{-j_1}^n &= \left[\frac{3}{4} + \frac{1}{4}(y_1 n - j_1)\right] p_{-j_1}^n + \left[\frac{1}{4}(j_1 + 1 - y_1 n)\right] p_{-j_1+1}^n \\ &\text{or} \\ \overset{*}{p}{}_{j_2}^n &= \left[\frac{3}{4} + \frac{1}{4}(y_2 n - j_2)\right] p_{j_2}^n + \left[\frac{1}{4}(j_2 + 1 - y_2 n)\right] p_{j_2-1}^n \end{aligned}\right\} \tag{62}$$

and the temporal increment of q halfway between the two midpoints is

$$\left.\begin{aligned} \overset{*}{\Delta} q_{-j_1} &= \frac{2K_{13}(\overset{*}{\Delta}_1 p - p^n_{-j_1} + p^n_{-j_1+1})}{1 + y_1 n - j_1} - K_{15}\, \overset{*}{p}{}^n_{-j_1} \\ \text{or} \qquad \overset{*}{\Delta} q_{j_2} &= \frac{2K_{14}(\overset{*}{\Delta}_2 p - p^n_{j_2} + p^n_{j_2-1})}{1 + y_2 n - j_2} - K_{16}\, \overset{*}{p}{}^n_{j_2} \end{aligned}\right\} \qquad (63)$$

Therefore, by linear extrapolation of the temporal increment of q to the last mesh point,

$$\left.\begin{aligned} q^{n+1/2}_{j_1} &= q^{n-1/2}_{j_1} + \frac{\overset{*}{\Delta} q_{-j_1} - \frac{1}{4}\left(j_1+1 - y_1 n\right)\left(q^{n+1/2}_{-j_1+1} - q^{n-1/2}_{-j_1+1}\right)}{\frac{3}{4} + \frac{1}{4}(y_1 n - j_1)} \\ q^{n+1/2}_{j_2} &= q^{n-1/2}_{j_2} + \frac{\overset{*}{\Delta} q_{j_2} - \frac{1}{4}\left(j_2+1 - y_2 n\right)\left(q^{n+1/2}_{j_2-1} - q^{n-1/2}_{j_2-1}\right)}{\frac{3}{4} + \frac{1}{4}(y_2 n - j_2)} \end{aligned}\right\} \qquad (64)$$

Now that the new q is known for each mesh point, the new p is found by simply

$$p^{n+1}_j = p^n_j + q^{n+1/2}_j \Delta t \quad \text{for} \quad -j_1 \leqslant j \leqslant j_2. \qquad (65)$$

The <u>boundary conditions at the shocks</u> are put in the form of the simultaneous linear equation pair:

$$\left.\begin{aligned} p^{n+1}_{(s)1} &= p^n_{(s)1} + K_5(a^{n+1}_1 + a^n_1) + K_9\left\{ - \frac{\partial p^{n+1}}{\partial z}\bigg|_{(s)1} - \frac{\partial p^n}{\partial z}\bigg|_{(s)1} \right\} \\ \text{or} \qquad p^{n+1}_{(s)2} &= p^n_{(s)2} + K_6(a^{n+1}_2 + a^n_2) + K_{10}\left\{ \frac{\partial p^{n+1}}{\partial z}\bigg|_{(s)2} + \frac{\partial p^n}{\partial z}\bigg|_{(s)2} \right\} \end{aligned}\right\} \qquad (66)$$

[redacted]

and

$$\left.\begin{aligned} a_1^{n+1} &= a_1^n + K_7\left(p_{(s)1}^{n+1} + p_{(s)1}^n\right) \\ \text{or} \quad & \\ a_2^{n+1} &= a_2^n + K_8\left(p_{(s)2}^{n+1} + p_{(s)2}^n\right) \end{aligned}\right\} \tag{67}$$

where

$$-\left.\frac{\partial p^n}{\partial z}\right|_{(s)1} = \frac{\dfrac{p_{(s)1} - p_{-j_1}}{y_1 n - j_1}\left(y_1 n - j_1 + \frac{1}{2}\right) - \left(p_{-j_1} - p_{-j_1+1}\right)\dfrac{y_1 n - j_1}{2}}{\left(\frac{1}{2} + \dfrac{y_1 n - j_1}{2}\right)\Delta_1 z}$$

$$+\left.\frac{\partial p^n}{\partial z}\right|_{(s)2} = \frac{\dfrac{p_{(s)2} - p_{j_2}}{y_2 n - j_2}\left(y_2 n - j_2 + \frac{1}{2}\right) - \left(p_{j_2} - p_{j_2-1}\right)\dfrac{y_2 n - j_2}{2}}{\left(\frac{1}{2} + \dfrac{y_2 n - j_2}{2}\right)\Delta_2 z} \tag{68}$$

The procedure for adding mesh points is as follows: if $ny_1 = j_1+1$, the transmitted shock has just reached the (j_1+1)st mesh point so that this point must be included in the mesh henceforth. This is done as follows: at the end of the cycle, a value of $q_{-j_1-1}^{n+1/2}$ is supplied by the equations

$$\left.\begin{aligned} q_{(s)1}^n &= \frac{2K_5}{\Delta t}\, a_1^n + K_{11}\left[-\frac{\partial p^n}{\partial z}\right]_{(s)1} \\ q_{(s)1}^{n+1} &= \frac{2K_5}{\Delta t}\, a_1^{n+1} + K_{11}\left[-\frac{\partial p^{n+1}}{\partial z}\right]_{(s)1} \\ q_{-(j_1+1)}^{n+1/2} &= \frac{\left(\frac{1}{2}\, q_{(s)1}^{n+1} + \frac{1}{2}\, q_{(s)1}^n\right) + \dfrac{y_1}{2}\, q_{-j_1}^{n+1/2}}{1 + \dfrac{y_1}{2}} \end{aligned}\right\} \tag{69}$$

[redacted]

the quantity $p^{n+1}_{-(j_1+1)} = p^{n}_{(s)1} + \Delta t \, q^{n+1/2}_{-(j_1+1)}$ is also supplied and then the substitution $j_1+1 \rightarrow j_1$ is made. A similar procedure is used for adding mesh points behind the reflected shock.

Finally, the quantity $a_o(t)$, which is the principal object of the study, is calculated from the equation

$$a_o^{n+1} = 2a_o^n - a_o^{n-1} + K_{21}(p^n_{-1} - p^n_o) + K_{22}(p^n_1 - p^n_o) + K_{23}\, p^n_o \tag{70}$$

The calculation as described here was coded for the Los Alamos MANIAC, and is called problem L-36. The code requires an input data tape giving values of Δt, $\Delta_1 z$, $\Delta_2 z$, y_1, y_2, β_1, β_2, $a_1(+0)$, $a_2(+0)$ and most of the K's, from K_5 through K_{23}

CHAPTER IV -- RESULTS FOR γ-LAW GASES

The code described in the preceding chapter places no restriction on the equation of state, provided only that the values of several constants are known. But the only calculations performed with this code so far were for γ-law gases, initially cold, both having the same value of γ.

By suitable choice of units (note that we are free to choose different units of length for infinitesimal lengths like a_o, a_1, a_2 and for macroscopic lengths like z, $\frac{2\pi}{k}$) we can make U_o, $a_o(-0)$, V_o, $\frac{2\pi}{k}$ equal to any given value, like 2^{-10}, for convenience. Then the dimensionless parameters characterizing a problem are γ, and R, the initial density ratio at the interface. The further parameters Δt, y_1, y_2 fix the method of integration used.

A separate routine (called L-36 set-up) was coded which calculates the various constants needed in the main calculation for this case, and punches out a data tape for L-36.

The input to the set-up consists simply of placing a number in R_4 after each of six special stop instructions, as follows:

SPECIAL STOP INSTRUCTION	NUMBER TO BE PLACED IN R_4
1 1 1 1 1	Problem Number
1 1 1 1 2	$(\gamma-1)\ (\gamma+1)$
1 1 1 1 3	R
1 1 1 1 4	Δt
1 1 1 1 5	y_1
1 1 1 1 6	y_2

Therefore, a problem can be started without having prepared a data tape in advance.

The problems run are summarized in Table III and Figs. 4, 5, 6, 7. Problems 1C and 1E are the same except that 1C has twice as fine a space-time mesh for the numerical calculation. 1E is presumably less accurate but could be carried further before the numerical storage requirement exceeded the capacity of the MANIAC. Comparison of the solid and dashed curves of Fig. 4 suggests that the truncation error is not large.

[redacted]

From Fig. 4 one can read off an approximate asymptotic value of $\dot{a}_o$ with some confidence: in decimal notation this value is $\approx 0.164 \cdot 10^{-3}$. We wish to compare this value with one given by equation (5), but there is a certain ambiguity in the interpretation of that equation because in our compressible case the amplitude $a_o(t)$ and the densities $\rho_{(+)}$ and $\rho_{(-)}$ change discontinuously at $t = 0$. If we substitute values appropriate to $t = -0$ into (5), we find

$$\dot{a}_o(\infty) \approx u_o^o \frac{1-R}{1+R} \, k a_o(-0) \tag{71}$$

but if we substitute values appropriate to $t = +0$, we find

$$\dot{a}_o(\infty) \approx u_o^o \frac{v_2^o - v_1^o}{v_2^o + v_1^o} \, k \, a_o(+0). \tag{72}$$

Values obtained from these formulas are $0.347 \cdot 10^{-3}$ and $0.151 \cdot 10^{-3}$, so that equation (72) is in rough agreement with our result, but (71) is wrong by about a factor 2. As seen in Table III, similar results were obtained for the other problems.

The conclusion is that if the initial compression of the interface and of the fluids is taken into account, the ultimate rate of growth of the corrugation is about the same as that given by the incompressible theory.

The waves in the curves of Figs. 4, 5, 6, and 7 are real and are related to the phenomenon discovered by Roberts (op. cit.), namely that corrugations in a shock are superstable -- they oscillate in a damped fashion. The two shocks in our problem are executing such oscillations

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with different natural frequencies and with complications due to the coupling, and these oscillations influence the motion of the interface.

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TABLE I -- SUMMARY OF NOTATION

Region 0	1	2	3
$u_x = 0$	$u_x^1(z, t)e^{ikx}$	$u_x^1(z, t)e^{ikx}$	0
$u_z = u_o^o > 0$	$u_z^1(z, t)e^{ikx}$	$u_z^1(z, t)e^{ikx}$	$u_3^o < 0$
$v = v_o^o$	$v_1^o + v^1(z, t)e^{ikx}$	$v_2^o + v^1(z, t)e^{ikx}$	v_3^o
$p = p_o^o$	$p_1^o + p^1(z, t)e^{ikx}$	$p_2^o + p^1(z,t)e^{ikx}$	p_3^o
		$(p_1^o = p_2^o)$	
$s = s_o^o$	$s_1^o + s^1(z)e^{ikx}$	$s_2^o + s^1(z)e^{ikx}$	s_3^o
(eq. of state)	$p = f_1(v, s)$	$p = f_2(v, s)$	
	$\left(\frac{c_1}{v_1^o}\right)^2 = -\left(\frac{\partial f_1}{\partial v}\right)_1^o$	$\left(\frac{c_2}{v_2^o}\right)^2 = -\left(\frac{\partial f_2}{\partial v}\right)_2^o$	
(Hugoniot)	$p = \phi_h(v)$	$p = \phi_\ell(v)$	
	$s = \psi_h(v)$	$s = \psi_\ell(v)$	
	$p_{s1} = p^1(-w_1 t, t)$	$p_{s2} = p^2(w_2 t, t)$	

[redacted]

TABLE II -- CONSTANTS IN THE FINITE-DIFFERENCE FORMULAS

$$K_3 = w_1 + \frac{c_1^2}{2w_1} + \frac{w_1}{2K_1} \qquad K_4 = w_2 + \frac{c_2^2}{2w_2} + \frac{w_2}{2K_2}$$

(see Eq. (34) for K_1) (see Eq. (33) for K_2

$$K_5 = \frac{k^2 c_1^2 w_1 u_o^o}{v_1^o} \frac{\Delta t}{2K_3} \qquad K_6 = \frac{k^2 c_2^2 w_2 u_3^o}{v_2^o} \frac{\Delta t}{2K_4}$$

(See Eqs. (17)---(21) for u_o^o and u_3^o)

$$K_7 = -\frac{w_1 v_1^o v_o^o}{2c_1^2(v_o^o - v_1^o)}\left[\left(\frac{c_1}{w_1}\right)^2 - \frac{1}{K_1}\right]\frac{\Delta t}{2} \qquad K_8 = \frac{w_2 v_2^o v_3^o}{2c_2^2(v_3^o - v_2^o)}\left[\left(\frac{c_2}{w_2}\right)^2 - \frac{1}{K_2}\right]\frac{\Delta t}{2}$$

$$K_9 = (w_1^2 - c_1^2)\frac{\Delta t}{2K_3} \qquad K_{10} = (w_2^2 - c_2^2)\frac{\Delta t}{2K_4}$$

$$K_{11} = \frac{2}{\Delta t} K_9 - w_1 \qquad K_{12} = \frac{2}{\Delta t} K_{10} - w_2$$

$$K_{13} = \left(\frac{c_1}{\Delta_1 z}\right)^2 \Delta t \qquad K_{14} = \left(\frac{c_2}{\Delta_2 z}\right)^2 \Delta t$$

$$K_{15} = (c_1 k)^2 \Delta t \qquad K_{16} = (c_2 k)^2 \Delta t$$

$$K_{17} = \frac{v_1^o}{\Delta_1 z} \frac{\Delta t}{\dfrac{v_1^o \Delta_1 z}{2c_1^2} + \dfrac{v_2^o \Delta_2 z}{2c_2^2}} \qquad K_{18} = \frac{v_2^o}{\Delta_2 z} \frac{\Delta t}{\dfrac{v_1^o \Delta_1 z}{2c_1^2} + \dfrac{v_2^o \Delta_2 z}{2c_2^2}}$$

$$K_{19} = \frac{v_1^o \Delta_1 z + v_2^o \Delta_2 z}{\dfrac{v_1^o \Delta_1 z}{c_1^2} + \dfrac{v_2^o \Delta_2 z}{c_2^2}} k^2 \Delta t \qquad K_{20} = \frac{c_2^2}{v_2^o \Delta_2 z} + \frac{c_1^2}{v_1^o \Delta_1 z}$$

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Table II (Continued)

$$K_{21} = \frac{(c_1 \Delta t / \Delta_1 z)^2}{K_{20}} \qquad K_{22} = - \frac{(c_2 \Delta t / \Delta_2 z)^2}{K_{20}}$$

$$K_{23} = \frac{c_2^2 - c_1^2}{2K_{20}} (k \Delta t)^2 .$$

TABLE III -- SUMMARY OF PROBLEMS RUN

PROBLEM NUMBER	1C	1E	1D	2A	2B
γ	3/2	3/2	3/2	9/7	9/7
R	1/8	1/8	1/16	1/16	1/16
Δt	1/16	1/8	1/16	3/64	1/8
$10^3\ \dot{a}_o$, asymptotic	0.164		0.168	0.164	
$10^3\ u_o^o \frac{1-R}{1+R} k\ a_o(-0)$	0.347		0.312	0.371	
$10^3\ u_o^o \frac{v_2^o - v_1^o}{v_2^o + v_1^o} k\ a_o(+0)$	0.151		0.173	0.170	

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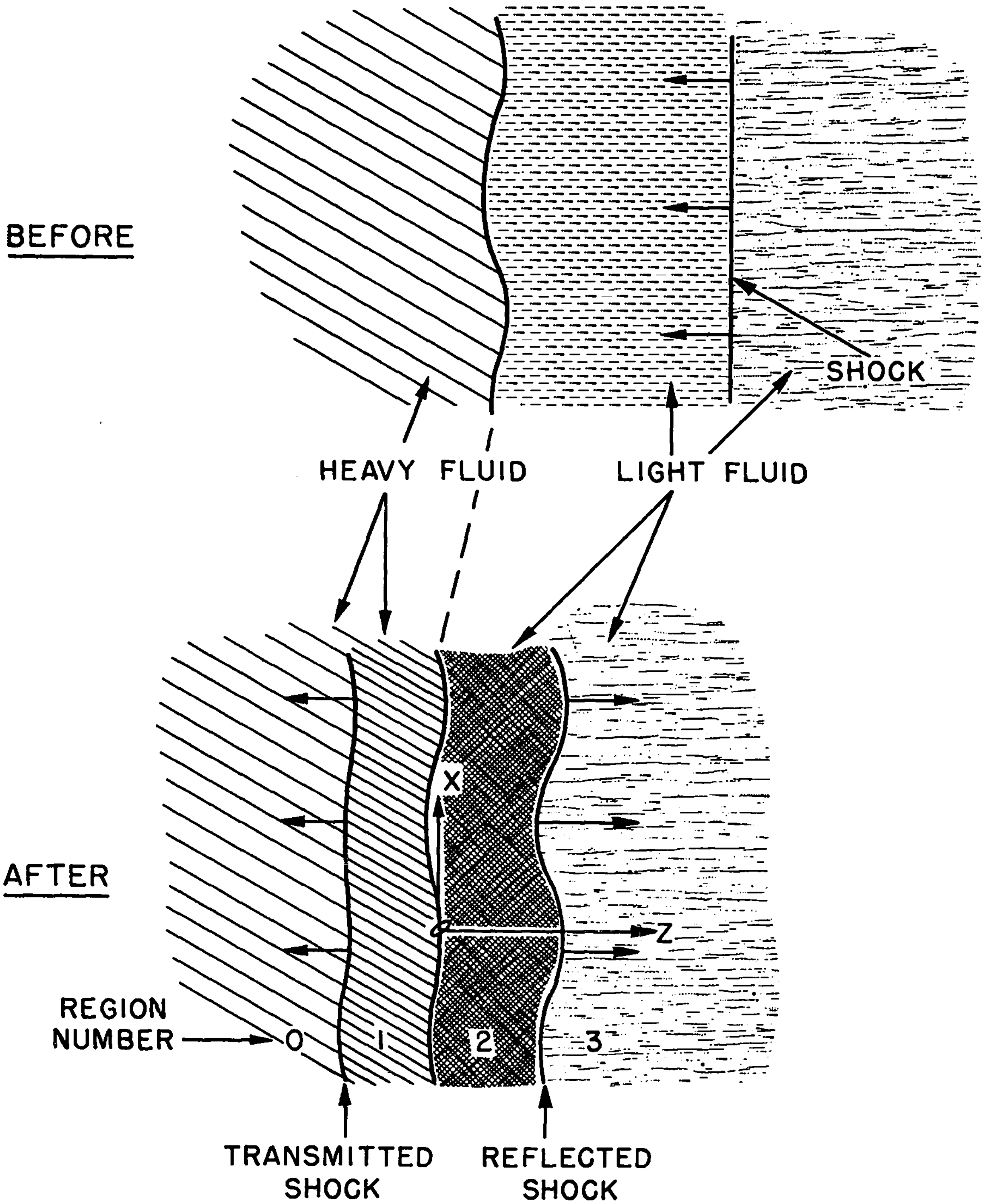

Fig. 1

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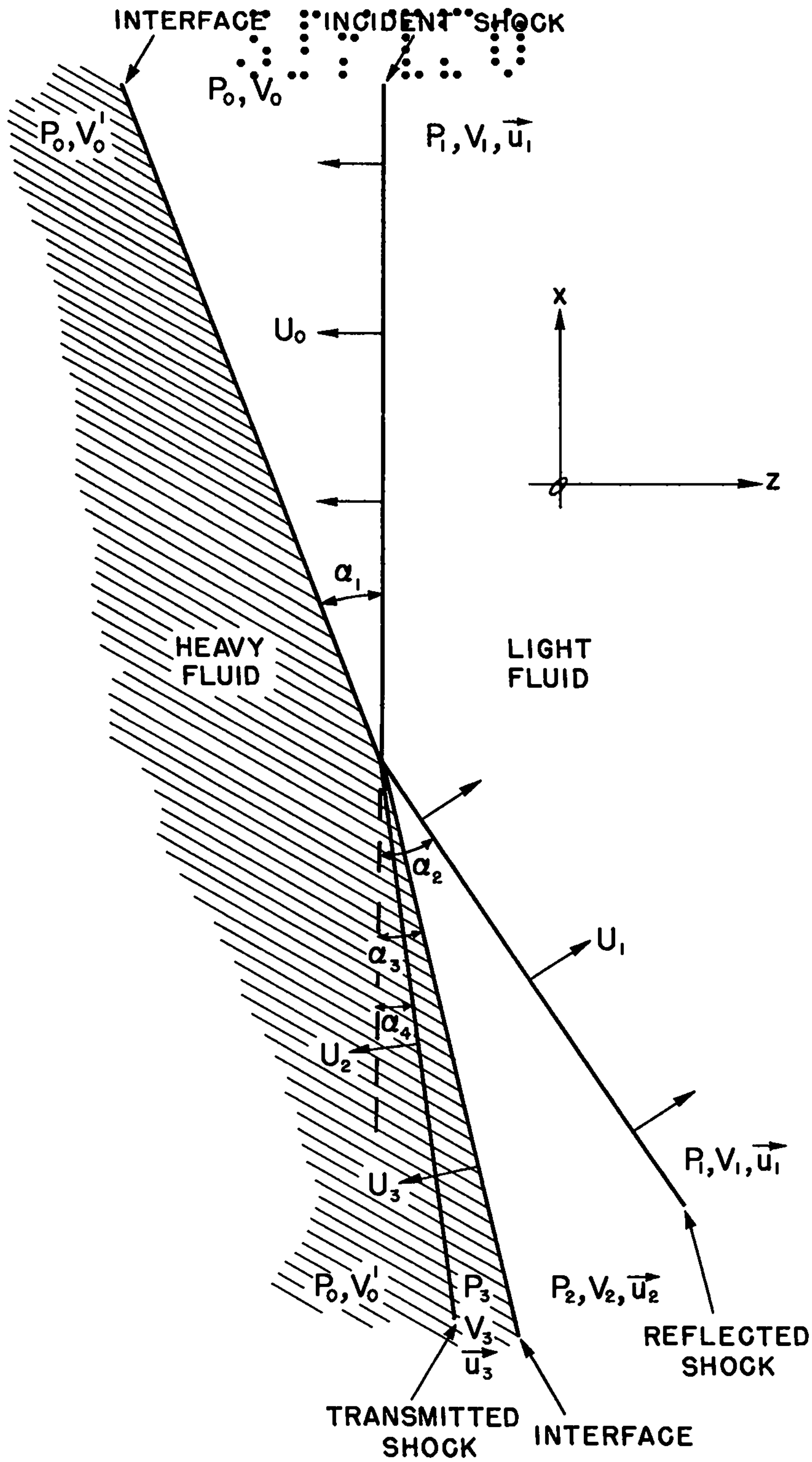
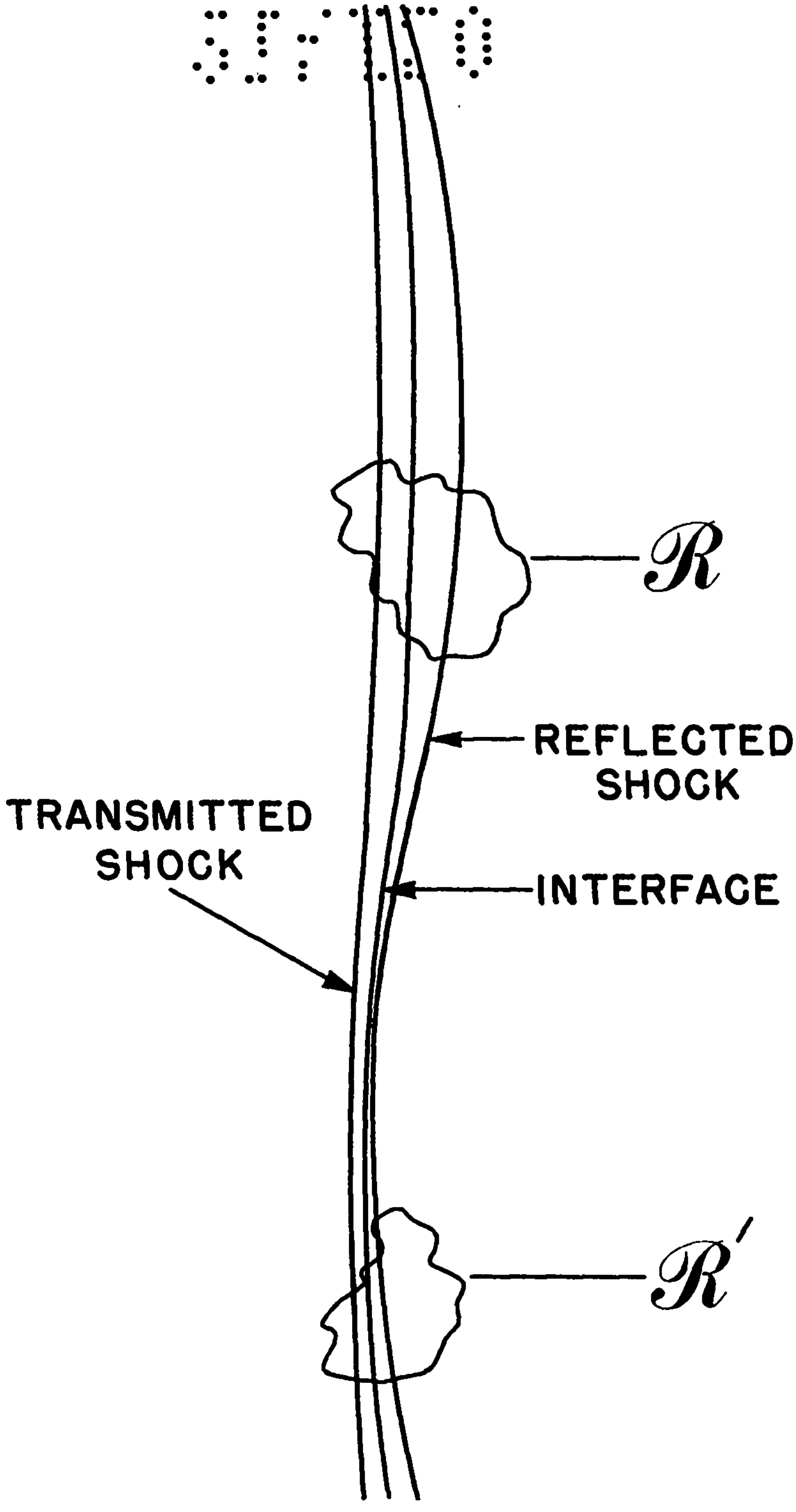


Fig. 2

[redacted]



$\mathscr{R}$
REFLECTED SHOCK
TRANSMITTED SHOCK
INTERFACE
$\mathscr{R}'$

Fig. 3

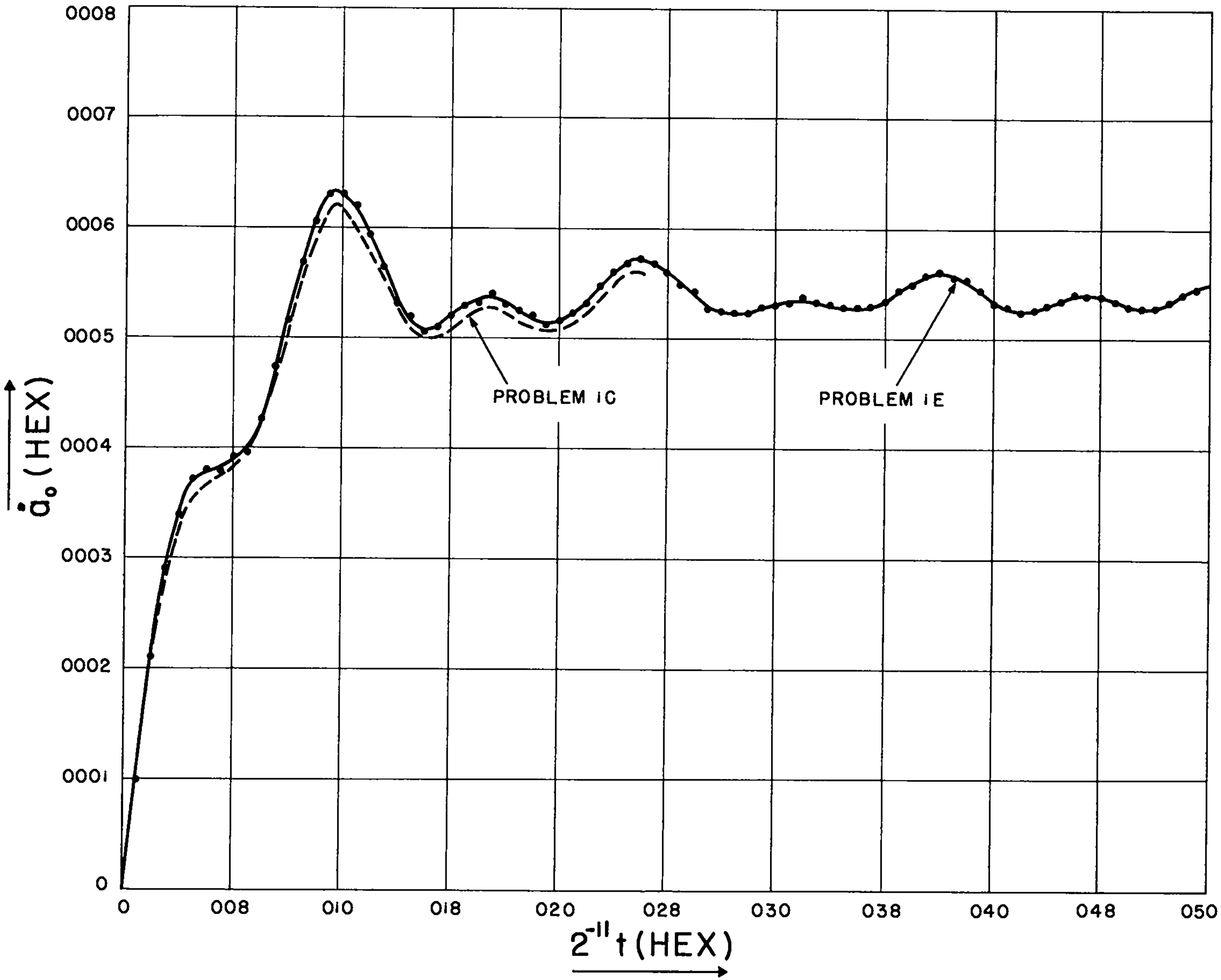

0008
0007
0006
0005
0004
0003
0002
0001
0
$\bar{a}_o$ (HEX)
0
008
010
018
020
028
030
038
040
048
050
$2^{-11}t$ (HEX)
PROBLEM IC
PROBLEM IE

Fig. 4

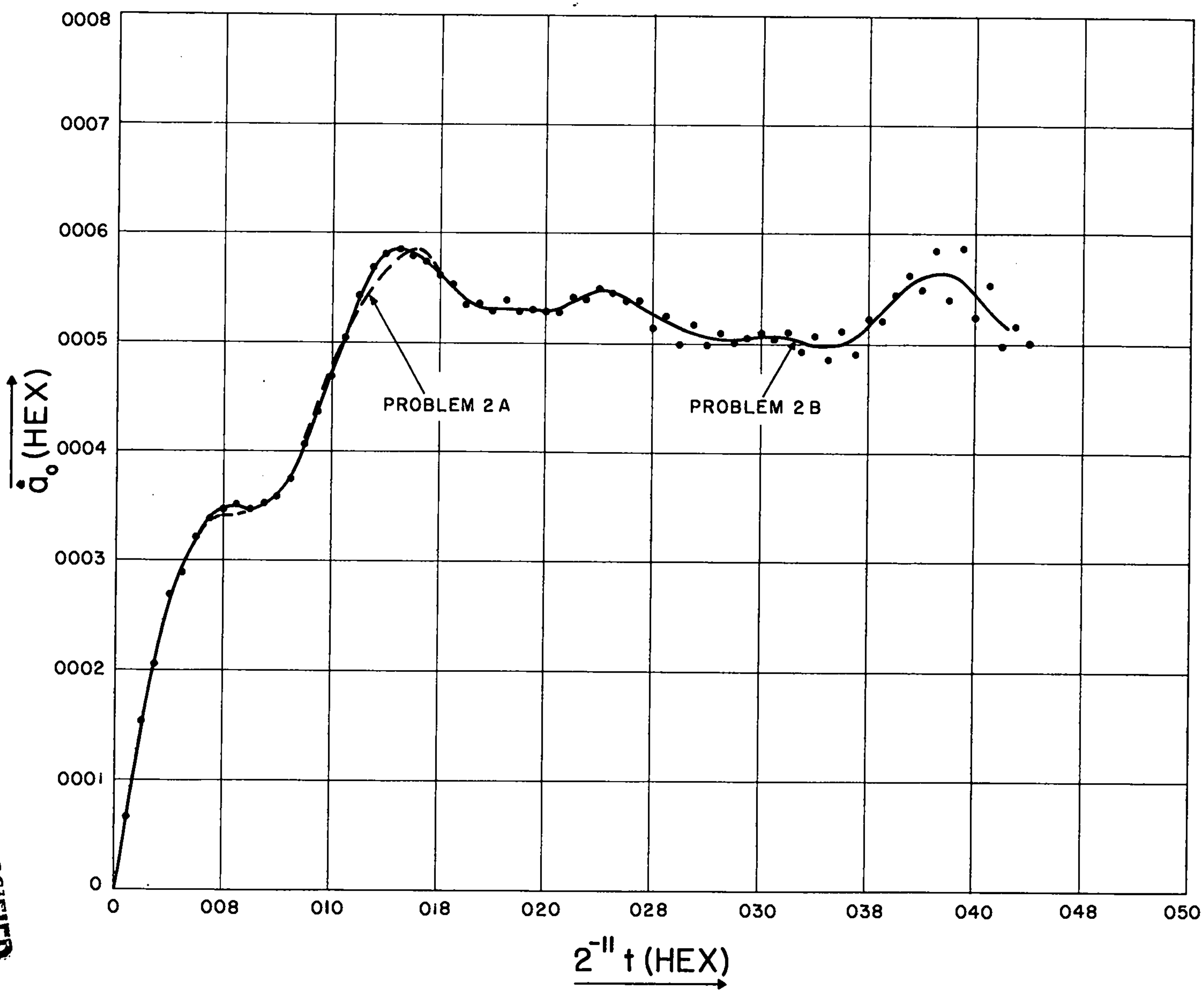

0008
0007
0006
0005
0004
0003
0002
0001
0
$\bar{a}_o$ (HEX)
PROBLEM 2 A
PROBLEM 2 B
0
008
010
018
020
028
030
038
040
048
050
2^{-11} t (HEX)

Fig. 5

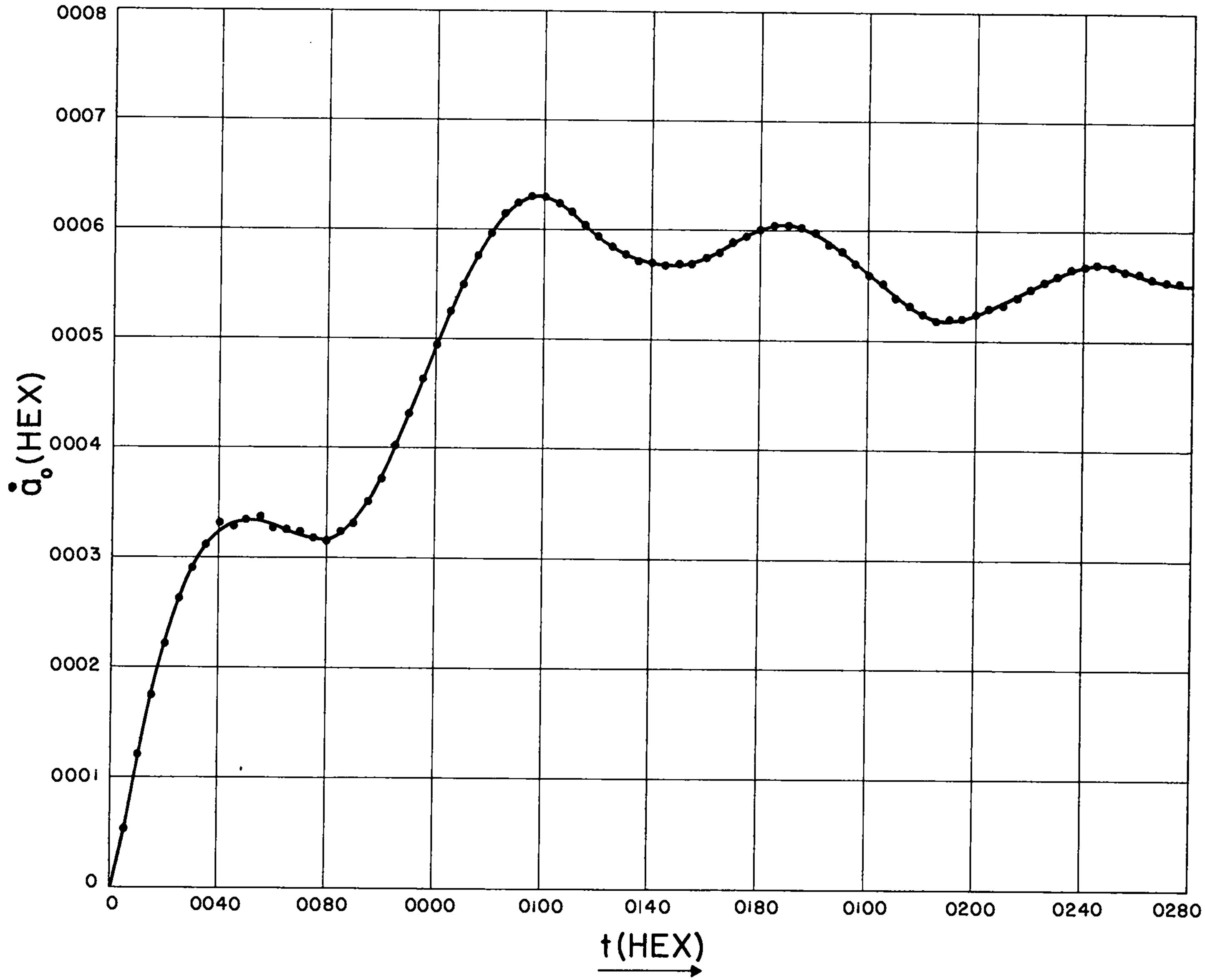


Fig. 6

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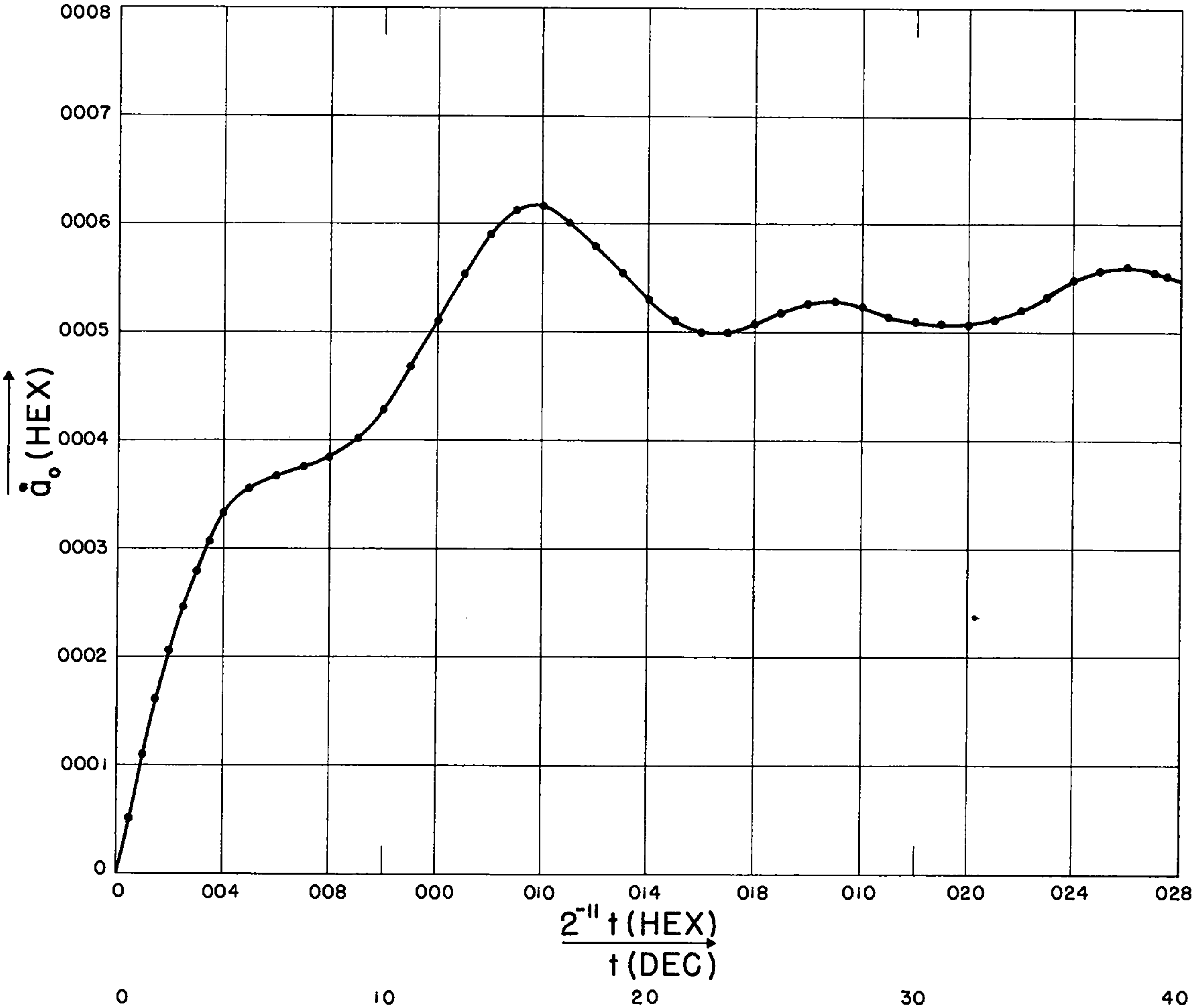
0008
0007
0006
0005
0004
0003
0002
0001
0
$\bar{\ddot{a}}_o$ (HEX)
0
004
008
00C
010
014
018
01C
020
024
028
2^{-11} t (HEX)
t (DEC)
0
10
20
30
40

Fig. 7