INELASTIC SCATTERING IN URANIUM

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ABSTRACT

The distribution curves in paraffin for neutrons from Ra-Be and Ra-B sources as modified by passing through 4.7 cm of uranium have been analyzed. The results indicate an upper limit to the inelastic scattering cross section of Ra-Be neutrons in uranium of 2.5 barns and a degradation of energy to an average of about 200 Kev. The corresponding qualities for Ra-B neutrons are 2.4 barns and 100 Kev.
Two separate measurements have been made of the distribution of iridium resonance neutrons in paraffin with a central cavity of 12 cm radius containing the source. These measurements were made with and without a 4.7 cm thick uranium sphere surrounding the source in order to determine the fast fission effect. However, the shape of the curves suggests that two fairly distinct groups of neutrons are present: those which have lost no energy in passing through the uranium and those which have been inelastically scattered. From these data it is possible to secure information concerning the amount of inelastic scattering and the energy loss involved.

The nature of the slowing-down process in hydrogenous materials is such that if a source emits two groups of neutrons of appreciably different average energy, the density of resonance or thermal neutrons at a large distance from the source is determined almost completely by the higher-energy group. It therefore follows that if the introduction of material around a source produces a new source with these characteristics, it is possible to analyze the distribution curves corresponding to the two groups.

B. Feld et al, CP-1177.

2) See curves $A_u$ of Fig. 1.
In a material in which there is neither capture nor production of neutrons the effect of surrounding a source with such a material is simply that a fraction $f$ of the source neutrons escapes without energy loss, while the fraction, $I = 1 - f$ is inelastically scattered. If the energy loss in inelastic scattering is sufficient so that the range of these neutrons in the slowing medium is considerably less than that of the primary neutrons, then the ordinates of the distribution curve with material present are simply reduced by the factor $f$ at large distances. Specifically, if

$$A_0(r) = \text{activity due to source alone}$$
$$A_u(r) = \text{activity with material around source}$$
$$A_i(r) = \text{activity attributed to inelastically scattered neutrons}$$

then

$$\int A_0 r^2 \, dr = \int A_u r^2 \, dr = \int A_i r^2 \, dr + f \int A_0 r^2 \, dr$$

or, dividing by the left hand term

$$1 = I + f$$

where $I = \frac{\int A_i r^2 \, dr}{\int A_0 r^2 \, dr}$

In the case of capture, $f$ as determined from the activity at large distances is unaltered and this is also true in the case of production providing the new neutrons have an energy less than that of the primaries so that they do not contribute to $A_u$ at large distances. However, it is no longer true in either case that

$$\int A_0 r^2 \, dr = \int A_u r^2 \, dr$$
but I is still the fraction which has made collisions leading to inelastic scattering, capture, fission or combinations of these.

In the present case of normal uranium and a fast neutron source, capture is negligible and the production small. The production may be measured by

$$E = \int A_{\text{ur}}^2 dr \int A_{\sigma r}^2 dr$$

and the activity due to fission neutrons denoted as $A_F$ in order to separate it in discussion from $A_I$. Consideration of the shape of the difference distribution $A_I + A_F = A_U - fA_o$ is perturbed by fission neutrons (and capture in producing fission) but not to the full extent of $A_F$ since the fission neutrons are also inelastically scattered. In the cases to be considered this results in a negligible perturbation since $E$ is not much greater than one. Consequently invalid conclusions from the difference distribution can be drawn essentially by neglecting the small term, $A_F$.

The procedure is to determine $f$ from $A_U/A_o$ at large $r$ from which $I$ and the inelastic scattering cross section follow. The difference distribution $A_U - fA_o$ is then examined to obtain estimates of the energy of the inelastically scattered neutrons. This analysis is applied to the data of CP-1177.

There are two methods which may be applied to the difference distribution in order to obtain an estimate of the energy of the inelastically scattered neutrons:

1. Beyond a transition distance, $\log A_{r^2}$ is linear with $r$ and the slope, $\alpha_s$, is a function of the neutron energy. This function has been
determined with sources of neutrons of different energy and is given in CF-209.
The calibration applies to a point source in water whereas the present data pertain to the distribution about a cavity in paraffin. However, these data include the distribution for a Ra-Be source and the α therefrom is identical with that obtained from a similar source completely surrounded with water. In this case the cavity has compensated the greater hydrogen density of paraffin and consequently the curve of CF-209 has been used directly.

(2) The $r^2$ of a distribution in hydrogenous material is also a measure of the energy. The relation is given in CP-1251, again for a point source in water. Serber (LAA $u=5$) has computed the effect of a cavity in the form:

$$x' = x + 1/2 - 1/3 \left[ a^x = \frac{2}{\sqrt{\pi}} x^{1/2} F(1,3/2,x) \right]$$

where

$$x' = \frac{r^2}{6a^2} \quad r^2 \text{ as measured with cavity}$$

$$a = \text{radius of cavity}$$

and

$$x = \frac{r_0^2}{6a^2} \quad r_0^2 \text{ as measured without cavity}$$

This expression is then used to determine $r_0^2$ in paraffin and this value corrected to water in order to use the curves relating $r_0^2$ to energy given in CP-1251.

Ra-Be Data

For $r > 20$ cm each measured point for $A_u$ can be fitted within less than 3% by

$$A_u r^2 = Be^{-0.268r} + 0.60 A e^{-0.105r}$$
The last term is an accurate representation of $0.60 A_{or}^2$ in this range; that is,

$$A_{or}^2 = A_1 r^2 + fA_{or}^2$$

The $A_1 r^2$ curve shown in Fig. 1 illustrates the very good exponential character at large $r$. Therefore, $f = 0.60$ and $I = 0.40$. From this value of $f$ an upper limit of the cross section $\sigma$ can be determined using the thickness of the sphere $R$:

$$I = 1 - e^{-\sigma R}$$

from which $\sigma = 3.0 \text{ b}$. This cross section is for all processes except elastic scattering, in this case, inelastic scattering, $\sigma_1$, plus fission $\sigma_f$. For both Ra-Be and Ra-B $\sigma_f = 0.5$, hence $\sigma_1 = 2.5$. This value is in reasonably good agreement with 2.7 obtained with Ra-Be neutrons and a fission detector. The result 2.3 has been obtained for the total inelastic scattering for 3 Mev neutrons. The agreement must be fortuitous since the effect of multiple scattering is not negligible and therefore 3.0 b must be considered as an upper limit.

The energy as determined by the slope method is obtained using $\alpha = 0.268$. The $r^2$ values have been calculated by integration of the corresponding distribution curves. The results are given in Table I.

**Ra-B Data**

These data have been treated in an identical fashion, but there is not such a sharp distinction between primary and degraded neutrons. This may be due to the shorter range of available data, but it is definitely suggested in
the shape of the original curves as well as in the difference curve of Fig. 1. As a result, the value of $f$ and the average energy of the inelastically scattered neutrons are less certain. The tabulation is given in Table II. The value of $\alpha$ in this table is that indicated by the dotted line of curve $B$, Fig. 1.

Discussion

The agreement between the $\alpha$ and $\bar{r}_o^2$ methods of energy determination is better than could be anticipated. The nature of the methods is such that the $\alpha$ procedure will give a higher energy than the true average. The energy from $\bar{r}_o^2$ also weights the high energy part of a distribution. The results by this method may be suspected of being high for an additional reason, that $\bar{r}_o^2$ for Ra-Be in Table I is 5-10% higher than the usual value obtained for a point source in water. Better agreement would not be expected from the approximations involved in the correction for the cavity. At first thought the change of $\bar{r}^2$ from 295 to 284 (Table I) due to a cavity with an $a^2$ of 139 might be considered unreasonable. However, if one considers the cavity effect as an extension of the first mean free path the fact that subsequent paths are distributed at random with respect to this initial direction suggests that a cavity is approximately three times as effective in $\bar{r}^2$ as the slowing material. This concept is applicable for a small cavity and is evident from the approximation given by Serber for small $a$

$$\bar{r}^2 \approx \bar{r}_o^2 + 3a^2 - 2\sqrt{6/\pi} a^3/r_0$$
For a large compared with $r_0$, he has given

$$r^2 = a^2 + \left(\frac{h}{6\pi}\right) a r_0$$

which may be noted is suggested by the simple physical concept for this case:

$$r^2 = (r_0 + a)^2 \quad \text{for} \quad r_0 \ll a$$

The case for the data analyzed above is an intermediate one and therefore the exact expression has been used.

It is evident from these data that normal uranium has a large inelastic scattering for Ra-Be and Ra-B neutrons and that such scattering is accompanied by a large energy loss. The rather definite distinction between primary and scattered neutrons suggests the absence of intervening levels from which inelastically scattered neutrons might be emitted. It would be of considerable interest to extend the method to sources of different energies and to other materials.

A. C. Graves and J. C. Hoogterp have assisted in the evaluation of the data, and several discussions with R. Sorber have been very helpful.
**TABLE I**

Ra-Be Data -- $f = 0.60$

$\alpha = 2.5$

$E = 1.13$

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>En Mev</th>
<th>$r^2$ cm$^2$</th>
<th>$r^2_o$ cm$^2$</th>
<th>$r^2_o$ cm$^2$ Water</th>
<th>En Mev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>0.105</td>
<td>4.4</td>
<td>584</td>
<td>238</td>
<td>295</td>
<td>4.1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.268</td>
<td>0.2</td>
<td>292</td>
<td>54.3</td>
<td>67.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**TABLE II**

Ra-B Data -- $f = 0.66$

$\alpha = 2.4$

$E = 1.053$

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>En Mev</th>
<th>$r^2$ cm$^2$</th>
<th>$r^2_o$ cm$^2$</th>
<th>$r^2_o$ cm$^2$ Water</th>
<th>En Mev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>0.1515</td>
<td>2.5</td>
<td>500</td>
<td>150</td>
<td>186</td>
<td>2.1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.315</td>
<td>0.10</td>
<td>260</td>
<td>37.6</td>
<td>46.6</td>
<td>0.08</td>
</tr>
</tbody>
</table>
DISTRIBUTION CURVES
in paraffin for
inelastically scattered
neutrons in uranium

- Re-Be Source
- Ra-B. Source

\[ \alpha = 0.288 \text{ cm}^{-1} \]
\[ \alpha = 0.315 \text{ cm}^{-1} \]
Attention has been called to a numerical error which reduces the value of $\sigma = 3.6 \, b$ on page 7.

The difficulty of obtaining a reasonably precise value lies in the discrepancy in CF-1177 concerning the sphere. On the basis of the stated density of 16 and a path length of $5.5 - 1.0 \, cm = 4.5 \, cm$,

- $\text{RbE} \quad \sigma = 2.7 \, b$
- $\text{RbS} \quad \sigma = 2.2 \, b$

However, CF-1177 also gives the radius of $5.5 \, cm$ and mass 11,200 grams or a density of only slightly over 16. Hence

- $\text{RbE} \quad \sigma = 2.5 \, b$
- $\text{RbS} \quad \sigma = 2.0 \, b$

One must therefore state a range for the inelastic cross section:

- $\text{RbE} \quad \sigma^1 = 2.0 \, \text{to} \, 2.2 \, b$
- $\text{RbS} \quad \sigma^1 = 1.5 \, \text{to} \, 1.7 \, b$. 

**Secret**