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of
THE UNIVERSITY OF CALIFORNIA

July 15, 1951

LA- 1278

This document consists of 22 pages

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11/16/95

THE INHOUR EQUATION FOR THE TUBALLOY TAMPED ORALLOY SPHERE

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ABSTRACT

Exponential rise times, T, of neutron flux in an Oy-Tu assembly have been measured for a set of points essentially equally spaced reactivity-wise in the interval between delayed and prompt critical corresponding to $\infty > T > 5$ seconds. The relationship between positive period and reactivity thus obtained is used to decide (preferentially) between two currently used but divergent collections of delayed neutron data, specifically with respect to the relative abundances of delayed neutrons in the different decay periods. Results of other reactivity measurements supplementing the positive period data are also given.

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THE INHOUR EQUATION FOR THE TUBALLOY TAMPED ORALLOY SPHERE

I. INTRODUCTION

Reactivity changes associated with small perturbations of a critical assembly generally are measured in terms of change in control rod setting required to maintain criticality. Expression of the results in standard units, namely the change in reproduction factor K or the "cent" (1/100 of the reactivity change getween delayed and prompt critical), involves a calibration of control rod position against the positive period of the assembly and use of the inhour equation. This equation and hence the conversion of reactivity units contain as parameters the abundances and periods of the delayed neutrons. Although recent experiments agree quite well on the periods, considerable discrepancy remains as to the relative abundances.(1,2)

This report concerns itself with an experiment with the thick tuballoy tamped oralloy sphere in which are made a set of perturbations whose effect on reactivity is known within a constant factor. Measurement of the positive periods associ-

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(1) D. J. Hughes, J. Dabbs, A. Cahn and D. Hall. Phys Rev 73, 111-124 (1948).

(2) F. deHoffman, B. T. Feld, and P. R. Stein. Phys Rev 74, 1330-1337 (1948)

[redacted]

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ated with these perturbations permits (again except for a scale factor) the determination of the inhour equation in the positive period range $\infty > T > 5$ seconds. This range of measurements is too restricted to yield accurate values of the relative abundances of the delayed neutrons but does suffice to discriminate between the values given in Ref. 1 and Ref. 2 in favor of the former.

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II. DESCRIPTION OF THE EXPERIMENT

The reactor employed was the Pajarito Site "Topsy" assembly(3), a pseudospherical core constructed from small oralloy blocks and surrounded by an 8½" thick tuballoy tamper. For remotely controlled assembly and disassembly, approximately one half of the active material was mounted on a ram, thus making the equatorial plane of the core and tamper readily accessible for replacing small amounts of either oralloy or tuballoy by other materials. Figure 1 gives a schematic diagram of the ram surface. The lettered positions represent ½" cubes equally distant from the center and occupied by tuballoy in the normal or critical stacking. Because of the spherical symmetry, the substitution of a cube of Oy for Tu, henceforth designated (Oy-Tu), in any of these positions produces the same reactivity increase; and, because the volume occupied by the lettered positions is sufficiently small compared to the total core volume, multiple substitutions in these equivalent positions gives a directly proportionate reactivity increase. (See Appendix for a justification.) This is the basis of the experiment. Table I includes a listing of number, N, of equivalent (Oy-Tu) substitutions above critical with the associated positive periods of the

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(3) See LA-744, p. 7 etc., for Topsy description.

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resulting assemblies. In order to correct for reactivity change produced by temperature drift, control rod settings required to make the standard stacking critical were determined between each positive period run. The equivalence, 2.50±.01" control rod = 1 (Oy-Tu), permitted positive period measurements on assemblies not corresponding to an integral number of cube substitutions.

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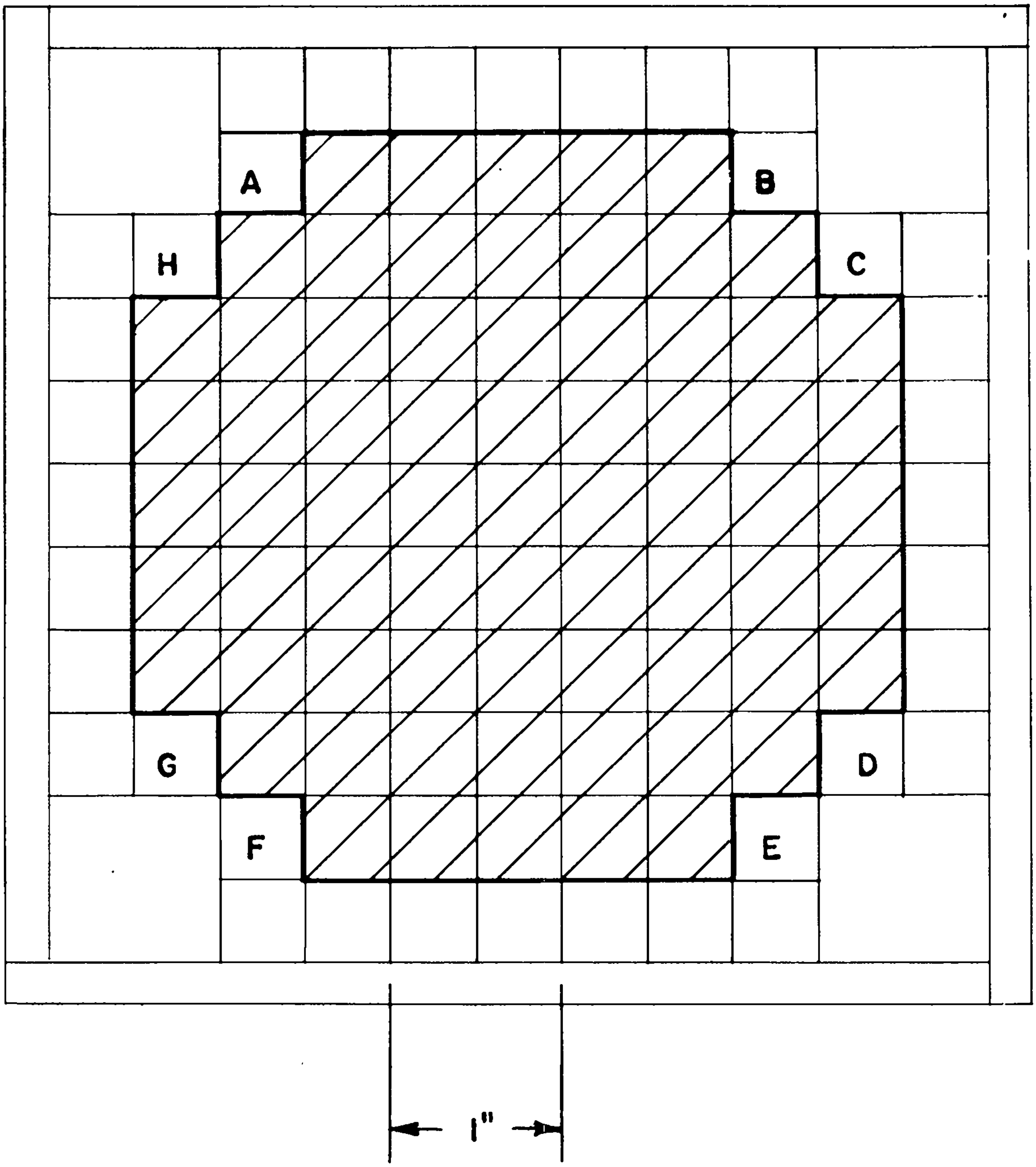

FIG. 1. Surface of Topsy ram ∿ the equatorial plane of the Oy pseudosphere. Equal changes in reactivity are obtained by substitution of Oy for Tu in any of the lettered positions.

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TABLE I. Summary of positive period data.

N (number of Oy-Tu cube replacements above critical)	Positive period (sec.)	ΔR_1 (cents)	ΔR_2 (cents)
0.25	595.0	1.97	2.67
0.31	509.5	2.31	3.09
0.61	234.0	4.73	6.23
1.05	121.0	8.23	10.78
1.41	82.2	11.13	14.45
1.68	65.1	13.21	17.05
1.80	57.0	14.49	18.66
2.00	50.5	15.75	20.22
3.00	25.8	23.82	30.02
3.54	19.3_4	27.94	34.76
3.98	15.3_4	31.46	38.74
4.00	15.3_0	31.49	38.77
4.00	15.2_8	31.51	38.79
4.47	12.2_7	35.06	42.66
4.52	11.8_2	35.68	43.33
4.96	9.5_0	39.39	47.25
5.99	6.2_0	46.94	54.94
6.40	5.2_5	49.94	57.89

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III. DISCUSSION OF RESULTS

In the positive period range under consideration, the inhour equation as given, for example, in LA-1033, is

$$\Delta R = 100 \sum a_i/(1 + \frac{T}{t_i})$$

where the reactivity change from delayed critical expressed in cent units, ΔR, is related to the positive period T through the lifetimes t_i and relative abundances a_i of the delayed neutrons. In turn, the reproduction factor K is obtained from $(K-1)/K = \gamma f \Delta R/100$ where γf is the effective fraction of neutrons delayed. Included with the positive period data in Table I are the predicted reactivity changes ΔR_1 and ΔR_2 associated respectively with the delayed neutron data of Ref. 1 and Ref. 2. That ΔR_1 is proportional and ΔR_2 is not proportional (within experimental limits) to the number N of equivalent cube replacements can be seen from Fig. 2, where the two functions $(\Delta R_1/C_1 - N)$ and $(\Delta R_2/C_2 - N)$ are plotted against N; the constants C_1 and C_2 being defined as

$$(\Delta R_1/N)_{N \to 0} = C_1 = 7.88¢ \text{ per (Oy-Tu)} \qquad (\Delta R_2/N)_{N \to 0} = C_2 = 10.23¢ \text{ per (Oy-Tu)}$$

Using $C_1 = 3.15¢$ and $C_2 = 4.09¢$ per inch of control rod

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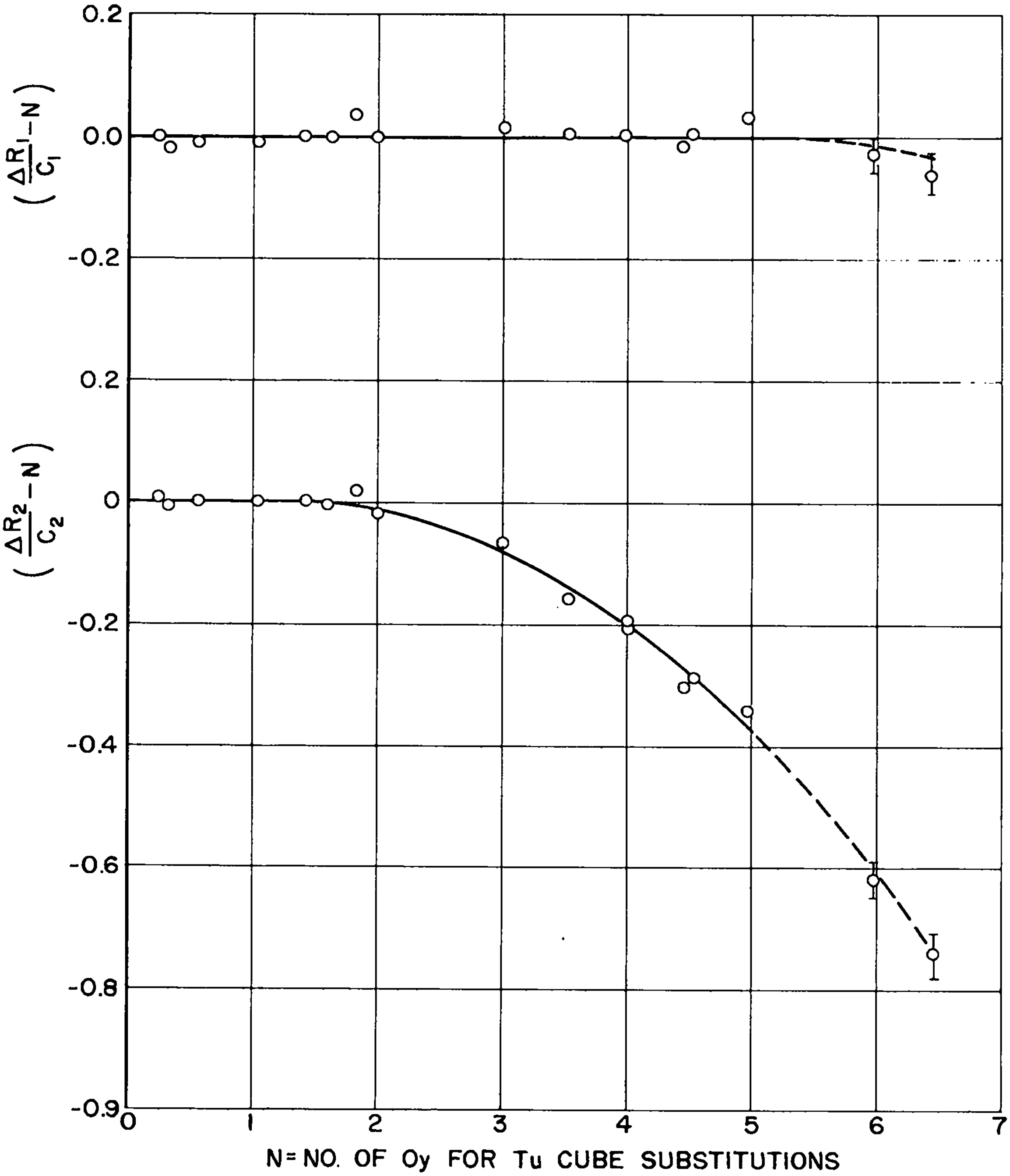

FIG. 2. Proportionality between number of equivalent cube substitutions and reactivity change, ΔR, predicted from the two inhour equations.

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as the comparable conversion factors, there are two more cases involving reactivity measurements which give preference to the delayed neutron values of Ref. 1, neither however being as direct as the above test of the inhour equation.

A. The mass difference, Δ m, between delayed and prompt critical

The measured reactivity change per mole substitution of Oy for Tu at the core-tamper interface is 17.6 inches of control rod. This gives

$$\Delta m_1 = (235/17.6) \times (100/3.15) \simeq 425 \text{ gms}$$

$$\Delta m_2 = (235/17.6) \times (100/4.09) \simeq 325 \text{ gms}$$

as compared to Δm (experimental) $\sim$ 420 gms based on Rossi α and $d\alpha/dm$ measurements (page 82, LA-744).

B. The effective fraction, γf, of neutrons delayed

From Equation VI of the Appendix, it is seen that the change in reproduction factor accompanying the substitution of Oy for Tu in the volume, v, is given very closely by

$$\Delta K = \frac{\Delta\left[(\nu-1-\alpha)\sigma_f\right] n_0^2(r)\, v}{\int \nu\, \sigma_f n_0^2 dv}$$

which leads, on summing over all v elements in the core, to the relation

$$\sum_{\text{core}} \Delta K = \Delta\left[(\nu-1-\alpha)\sigma_f\right]/\nu\,\sigma_f = \frac{\gamma f}{100} \sum_{\text{core}} \Delta R(\text{Oy}-\text{Tu})$$

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The table below gives the reactivity change per ½ inch cube, expressed in inches of control rod, corresponding to the substitution of Oy for air and Tu for air at the different radii, r.

r =	.56"	.78"	1.12"	1.58"	2.06"	2.55"
Δ R (Oy) =	9.48	9.20	8.33	7.02	5.36	3.46
Δ R (Tu) =	1.27	1.31	1.42	1.57	1.55	1.00

From these values, it follows that

$$\sum_{\text{core}} \Delta\ R(Oy-Tu) = 2120''\ \text{control rod}\ .$$

Using the approximate relation

$$\left[\frac{\Delta\ R(Oy)}{\Delta\ R(Tu)}\right]_{r=0} = \frac{(\nu-1-\alpha)\sigma_f(Oy)}{(\nu-1-\alpha)\sigma_f(Tu)}$$

and $(\nu-1-\alpha)/\nu = .60$ for Oy one obtains

$$\Delta\left[(\nu-1-\alpha)\sigma_f\right]/\nu\sigma_f = .53$$

thus leading to the two values

$$(\gamma f)_1 = (.53/2120)(100/3.15) \simeq .0079$$

$$(\gamma f)_2 = (.53/2120)(100/4.09) \simeq .0061$$

as compared to the measured value (Ref. 1) $f = .0075 \simeq \gamma f$.

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APPENDIX

The reactivity change caused by the substitution of Oy for Tu in the volume v at the radius r is, to first order, described in terms of the interaction between the unperturbed neutron flux at that radius and the changed neutron cross-sections in v. To this approximation, the reactivity change is directly proportional to the number of equivalent cube substitutions. The second order contribution to the reactivity change behaves as $(v)^2$, is proportional to the square of the number of cube substitutions, and contains a term depending on the spatial symmetry of these substitutions. It is the purpose of this appendix to estimate the importance of this second order correction by means of a perturbation calculation sufficiently simplified to be manageable yet containing the essential features of the cube substitutions.

Initial simplifications:

I. The use of a single energy group of neutrons. The first order change in reactivity due to the change in neutron energy spectrum produced by Oy-Tu substitution is a small fraction of the total and there is no reason to expect the fractional contribution to the $(v)^2$ term to be substantially different.

II. The use of $\sigma_{tr}(Oy) = \sigma_{tr}(Tu)$. This assumption,

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which is approximately correct, merely circumvents the need for introducing into the development the anisotropic components of the neutron flux and does not appreciably alter the relative contributions of the v and $(v)^2$ terms.

The integral equation governing the neutron flux distribution, $n_0(r)$, in a critical assembly is (see for example LA-53)

$$n_0(r) = \int dV' \, [\nu\sigma_f + \sigma_s](r') \, n_0(r') \, \frac{e^{-\sigma_t |r'-r|}}{4\pi |r'-r|^2} \tag{I}$$

If this assembly is perturbed by the substitution of foreign material such that $\sigma_f \rightarrow \sigma_f + \Delta\sigma_f$; $\sigma_s \rightarrow \sigma_s + \Delta\sigma_s$; $\sigma_t \rightarrow \sigma_t$, one may associate with this perturbed assembly a similar equation

$$n^p(r) = \int dV' \left[\nu\sigma_f + \sigma_s + \Delta(\nu\sigma_f + \sigma_s)\right](r') \, n^p(r') \, \frac{e^{-\sigma_t |r'-r|}}{4\pi |r'-r|^2} \tag{II}$$

Multiplying Equation I by $(\nu\sigma_f + \sigma_s)n^p(r)$ and Equation II by $(\nu\sigma_f + \sigma_s)n_0(r)$, then subtracting the second from the first and integrating over the volume leaves

$$0 = \int dV\,dV' \, (\nu\sigma_f + \sigma_s)(r) \, (\nu\sigma_f + \sigma_s)(r') \, n_0(r) \, n^p(r') \, \frac{e^{-\sigma_t |r'-r|}}{4\pi |r'-r|^2}$$

$$- \int dV\,dV' \left[\nu\sigma_f + \sigma_s + \Delta(\nu\sigma_f + \sigma_s)\right](r') \left[\nu\sigma_f + \sigma_s\right](r) \, n_0(r) \, n^p(r') \, \frac{e^{-\sigma_t |r'-r|}}{4\pi |r'-r|^2}$$

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or, after partial integration

$$-\Delta\nu \int dV\,[\sigma_f+\Delta\sigma_f]\,n^p n_0 = \int dV\,[\nu\Delta\sigma_f+\Delta\sigma_s]\,n^p n_0 \qquad \text{(III)}$$

where $\Delta\nu$ is that change in neutrons per fission which would be required to reduce the perturbed assembly to critical and is related to the actual change in the reproduction factor, K, by the equation $-\Delta\nu/\nu = (K-1)/K$ (this follows from the fact that $K = \nu P$, where P, the average probability that a neutron will produce fission, does not depend on ν). The above use of the formal eigenvalue nature of ν is a convenient way for introducing K, which however requires that

$$\Delta\sigma_f \equiv \left[(\nu\sigma_f)_x-(\nu\sigma_f)_y\right]/\,\nu_{Oy}$$

for an (x-y) replacement. Thus one obtains the following relation between the reactivity change and the perturbed cross sections (see KAPL-98 for a more general development)

$$\frac{K-1}{K}\int dV\,\nu\,[\sigma_f+\Delta\sigma_f]\,n^p n_0 = \int dV\,[\nu\Delta\sigma_f+\Delta\sigma_s]\,n^p n_0 \qquad \text{(IV)}$$

The "first order" solution results on placing $n^p = n_0$ and neglecting the term involving $(K-1)\Delta\sigma_f$, viz.

$$\frac{K-1}{K} \cong \frac{\int dV\,[\nu\Delta\sigma_f+\Delta\sigma_s]\,n_0^2}{\int dV\,\nu\sigma_f\,n_0^2} \equiv \Delta K_1 \qquad \text{(V)}$$

For our case of substituting Oy for Tu at a number, N, of equivalent spaces, v_k, Equation V may be re-expressed (remem-

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bering that the assumption of equal transport cross sections requires that $\Delta\sigma_f + \Delta\sigma_s + \Delta\sigma_a = 0$ and using the customary notation $\sigma_a/\sigma_f = \alpha$)

$$\Delta K_1 = \frac{\left[(\nu-1-\alpha)\,\sigma_f(Oy) - (\nu-1-\alpha)\,\sigma_f(Tu) \right] n_0^2 \sum v_R}{\int dV\, \nu\, \sigma_f\, n_0^2} \qquad \text{(VI)}$$

and to this approximation, the reactivity change is proportional to the number of these equivalent replacements.

For a better solution to Equation IV, it is necessary to determine the perturbed flux distribution more closely, and this is done conveniently by means of an expansion of n^p in terms of the normal flux modes of the unperturbed assembly

$$n^p = n_0 + \sum_1^\infty \alpha_i n_i \qquad \text{(VII)}$$

where the α_i are constants and the normal modes, n_i, have the definitions

$$\int dV' \left[\nu\sigma_f + \sigma_s\right] \frac{e^{-\sigma_t |r'-r|}}{4\pi |r'-r|^2}\, n_i(r') = f_i\, n_i(r) \qquad ;\ f_i \text{ a const.} < 1 \qquad \text{(VIII)}$$

and the consequent properties

$$\int dV \left[\nu\sigma_f + \sigma_s\right] n_i n_j = 0 \qquad \text{for } i \neq j \qquad \text{(IX)}$$

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The connection between α_i and the perturbed cross sections is obtained by multiplying Equation II by $(\nu\sigma_f + \sigma_s)n_i$ and integrating.

$$\alpha_i \int dV \, [\nu\sigma_f + \sigma_s] n_i^2$$

$$= \int dV\, dV' \, [\nu\sigma_f + \sigma_s](r) \, [\nu\sigma_f + \sigma_s + \Delta(\nu\sigma_f + \sigma_s)](r') \, n_i(r) \, [n_o(r') + \sum \alpha_j n_j(r')] \frac{e^{-\sigma_t|r'-r|}}{4\pi |r'-r|^2}$$

or, after partial integration and the neglect of higher order terms

$$\alpha_i \cong \frac{f_i}{1-f_i} \; \frac{\int dV [\nu\Delta\sigma_f + \Delta\sigma_s] n_o n_i}{\int dV [\nu\sigma_f + \sigma_s] n_i^2} \tag{X}$$

Equation IV may now be expressed as

$$\frac{K-1}{K} \left\{ \int dV \, \nu(\sigma_f + \Delta\sigma_f) n_o^2 + \sum \alpha_i \int dV \, \nu\sigma_f \, n_o n_i \right\}$$

$$= \int dV \, [\nu\Delta\sigma_f + \Delta\sigma_s] n_o^2 + \sum \alpha_i \int dV \, [\nu\Delta\sigma_f + \Delta\sigma_s] n_o n_i$$

or, separating the first and second order terms, as

$$\frac{K-1}{K} \cong \frac{\int dV [\nu\Delta\sigma_f + \Delta\sigma_s] n_o^2}{\int dV \, \nu\sigma_f n_o^2} - \frac{\int dV [\nu\Delta\sigma_f + \Delta\sigma_s] n_o^2 \int dV' \nu\Delta\sigma_f n_o^2}{\left[\int dV \nu\sigma_f n_o^2 \right]^2}$$

$$+ \frac{\sum \alpha_i \int dV [\nu\Delta\sigma_f + \Delta\sigma_s] n_o n_i}{\int dV \, \nu\sigma_f n_o^2} - \frac{\sum \alpha_i \int dV [\nu\Delta\sigma_f + \Delta\sigma_s] n_o^2 \int dV' \nu\sigma_f n_o n_i}{\left[\int dV \, \nu\sigma_f n_o^2 \right]^2} \tag{XI}$$

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It may be noted that for uniform dilution of Oy by Tu in the core, the two terms in Equation XI involving summation over i cancel and thus greatly simplify this equation. However, for the case of interest where the perturbation is confined to a particular radial position, the first of these terms is the larger. That the second order terms can be approximated quite well using relatively few of the higher modes follows not only from the fact that the eigenvalues, f_i, entering through the α_i, are decreasing functions of the order, i, of the normal modes, but by reason of the n_o and n_i becoming orthogonal over a cube volume for large i values. For the modes to be considered, this latter effect is not appreciable, and the integral over a cube volume of the product $n_o n_i$ will be taken as $v n_o(r) n_i(r,\theta,\phi)$ where r,θ,ϕ designates the position of the cube center. With this simplification, Equation XI can be expressed as

$$\frac{K-1}{K} \cong \frac{n_o^2 \sum V_K \Delta\left[(\nu-1-\alpha)\sigma_f\right]}{\int dv\, \nu\sigma_f n_o^2}\left[1 - \frac{\nu\Delta\sigma_f n_o^2 \sum V_K}{\int dv\, \nu\sigma_f n_o^2}\right.$$

$$\left. + \sum_i \frac{f_i}{1-f_i}\Delta\left[(\nu-1-\alpha)\sigma_f\right]\left\{\frac{\left[\sum V_K n_i(r,\theta_K,\phi_K)\right]^2}{\sum V_K \int dv(\nu\sigma_f+\sigma_s)n_i^2} - \frac{\sum V_K n_o n_i(r,\theta_K,\phi_K)\int dv\,\nu\sigma_f n_o n_i}{\int dv(\nu\sigma_f+\sigma_s)n_i^2 \int dv'\,\nu\sigma_f n_o^2}\right\}\right] \tag{XII}$$

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The term outside the main bracket is the "first order" contribution to the reactivity change, ΔK_1, and hence the deviation from unity of the sum of the enclosed terms gives the fractional contribution of the "second order" terms. The second term inside the bracket is seen to be equal to $(\nu\Delta\sigma_f/\Delta[\nu-1-\alpha]\sigma_f)\,\Delta K_1$. Thus for a reactivity change of 50¢, this term has the approximate value 1.7 x 0.5 γf = .006. An estimate of the magnitude of the remaining terms inside the bracket can be obtained by using the normal modes associated with the bare critical Oy sphere (which are readily calculable ala the extrapolated end-point method).

The following list of normal modes are used; they depend on the extrapolated end-point method and the values $\sigma_{tr} = .25\ \text{cm}^{-1}$ and $(\nu\sigma_f + \sigma_s) = 1.35\ \sigma_{tr}$.

A) Spherically symmetric modes: $n_{0i} = \sin k_{0i}r/k_{0i}r$.

$n_0 = n_{00} \rightarrow f_{00} = 1,\quad k_{00} = .2895\ \text{cm}^{-1}$

$(r_{\text{critical}} = 8.75\ \text{cm})$

$n_{01} \rightarrow f_{01} = .646,\quad k_{01} = .6216$

$n_{02} \rightarrow f_{02} = .459,\quad k_{02} = .9701$

$n_{03} \rightarrow f_{03} = .353,\quad k_{03} = 1.324$

$n_{04} \rightarrow f_{04} = .286,\quad k_{04} = 1.680$

B) "Dipole" modes; $n_{1i} = (\sin k_{1i}r/k_{1i}^2r^2 - \cos k_{1i}r/k_{1i}r)\cos\Theta$

$n_{10} \rightarrow f_{10} = .820,\quad k_{10} = .4296$

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$n_{11} \rightarrow f_{11} = .545, \quad k_{11} = .781$

$n_{12} \rightarrow f_{12} = .402, \quad k_{12} = 1.136$

C) "Quadrupole" modes: $n_{2i} = (-3 \cos k_{2i}r/k_{2i}^2 r^2 + 3 \sin k_{2i}r/k_{2i}^3 r^3 - \sin k_{2i}r/k_{2i}r) \times (\cos^2 \Theta - 1/3)$

$n_{20} \rightarrow f_{20} = .688, \quad k_{20} = .566$

$n_{21} \rightarrow f_{21} = .473, \quad k_{21} = .934$

$n_{22} \rightarrow f_{22} = .357, \quad k_{22} = 1.308$

Three representative cases are given below in which the values of the bracketed terms in Equation XII are computed at r = 4 cm and r = 6 cm. (The radius in the bare sphere at which the substitution of a ½ inch cube of Oy for Tu gives a reactivity increase of 10¢ is approximately at 5 cm.)

Case I. Single cube substitution.

[XII] = 1.008 at r = 4 cm

[XII] = 1.003 at r = 6 cm

Case II. Two cube substitutions along opposing radii.

[XII] = 1.004 at r = 4 cm

[XII] = 1.003 at r = 6 cm

Case III. N cube substitutions spaced 360°/N apart on an equatorial plane.

[XII] = 1 - .00010N at r = 4 cm

[XII] = 1 - .00005N at r = 6 cm

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These three cases would indicate that the second order contributions to the reactivity change is in the neighborhood of one-half of one percent of the total change for the perturbations in question. As the experimental accuracy is nowhere greater than this, the assumed linearity between reactivity change and number of equivalent cube replacements is justified.

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