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THREE-VELOCITY NEUTRON DIFFUSION CALCULATIONS
FOR AN UNTAMPED ORALLOY SPHERE

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THREE-VELOCITY NEUTRON DIFFUSION CALCULATIONS FOR AN UNTAMPED ORALLOY SPHERE

The results of a series of neutron diffusion calculations relating to an untamped Oralloy sphere are presented in detail in this report. The three-velocity neutron transport theory was taker as the basis for the analytical work preceding the computations. This particular theory, also known as the transport approximation, is defined in LA-1271 and known to be quite accurate for assemblies primarily involving materials of large atomic weight. For a sphere of uniform density and atomic composition the transport theory has another advantage. It can readily be formulated in terms of simultaneous integral equations (in our case three), relatively simple in form, involving the collision densities $N_g(r)$ (coll/cm^{3}sec) and a set of parameter values describing the material. $N_g(r)$ is, as indicated, a function of the radial distance r (cm), $0 \leq r \leq a$, and the velocity index g, g = 1, 2, 3. The parameters, fifteen in number for the three-velocity theory, are comprised of the velocities v_g (cm/sh), the inverse mean free paths σ_g (1/cm), and the transfer coefficients c_{gh} (number of neutrons of velocity v_g emerging per collision of neutron of velocity v_h).

The simultaneous integral equations referred to above can be solved

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by rather elementary numberical methods based on dividing the interval (0,a) into L intervals and replacing the continuous function $N_g(r)$ by step-functions. This reduces in effect the three integral equations to the same number of matrix equations, i.e., to three groups of L simultaneous equations. One finds that the latter can be solved by standard iteration methods, and readily so with IBM machines to do the arithmetic. The procedure is described in detail in LA-1271 which also includes a section with illustrative numerical examples. For the purposes of this report L was taken to be four and the intervals of equal length. Preliminary calculation had shown that such radial resolution would, in fact, give results accurate to within a few per mil.

The calculations were carried out in parallel for two sets of parameter values, given in Tables I and II together with the data required for their computation. Most of the cross-sections in these tables were supplied by H. Bethe. Among these the inelastic cross-sections were considered the most uncertain element. The total scattering cross-section, regarded as the sum of two components, one elastic and one inelastic, was therefore split in two ways. The resulting two sets of parameters are referred to below as the <u>First Set</u> and the <u>Second Set</u>, respectively, the latter having the larger inelastic cross-sections. Note that only the transfer coefficients c_{gh} are different for the two sets.

The main purpose of the work reported on here was to obtain

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theoretical predictions for some of the experiments planned in connection with the untamped Oralloy reactor now under construction at the Pajarito Laboratory. It is hoped that a comparison of theory with experiment will indirectly lead to a Third Set (and possibly the final set) of parameters and cross-sections. One would then be in a better position to predict the behavior of more complicated systems involving U_{235} and U_{238}. Tables III, IV, and V give comparisons of results obtained theoretically with earlier measurements on untamped Oralloy spheres. From these comparisons it is evident that the Second Set gives the best agreement. The Oralloy reactor now under construction will have a material density of 18.8 gr/cm^3. When completed the following experiments and very likely others will be considered, some being similar to earlier ones:

(1) The determination of the so-called delayed critical mass m_o(kg).

(2) The measurement of $(\frac{d1/M}{dm})_{m=m_o}$, the rate of change near $m=m_o$ of the inverse of the neutron multiplication due to a central fission source.

(3) The measurement of $(\frac{d\alpha}{dm})_{m=m_o}$, the exponential decay rate near $m=m_o$ of the neutron population in the assembly.

(4) The measurement of 1/M for various values of m less than m_o.

(5) The determination of approximate neutron velocity spectra, both at the center of the sphere and outside the sphere, by means of fission detectors.

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(6) The determination of detailed velocity spectra by photographic plate and other techniques. Such spectra will, in particular, give us better average velocities v_g for our calculations.

Using the Second Set of parameters we obtain the following theoretical results for the new reactor:

(1) $m_o = 52.45$ kg.

(2) $(\frac{d1/M}{dm})_{m=m_o} = -.00546\ 1/cm^3$.

(3) $(\frac{d\alpha}{dm})_{m=m_o} = .00854\ 1/cm^3 sh$.

Taking the fission cross-sections for 25 and 28 from this report, and estimating average fission cross-sections for Pu, 23, and 37 with the aid of LA-140A and LA-520, we construct the following table:

Detector	Fission Cross-Sections in barns		
	g=1	g=2	g=3
28	-	.05	.50
Oy	1.4	1.17	1.15
Pu	1.95	1.94	1.94
23	2.7	2.2	1.85
37	-	1.1	1.45

Using this table and the neutron flux spectra given in Table VI we can calculate the relative fission rates per atom in 28, Oy, Pu, 23, and 37 fission detectors. Taking the fission rate in Oralloy due to a fission source to be unity we obtain the following relative fission rates:

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Detector	Relative Fission Rates	
	Center of Reactor	Outside Reactor
28	.14	.16
Oy	1.06	1.05
Pu	1.64	1.64
23	1.93	1.90
37	.66	.73

Further calculations, in particular revisions of the above, will be made when new measurements become available.

The one-velocity results quoted in Tables IV and V were obtained by the methods of LA-1273, p.p. 8-10.

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TABLE I THREE-VELOCITY CROSS-SECTIONS

(FIRST SET)

Uranium 235

Source: Memo by H. Bethe, January 17, 1951

g	v_g cm/shake	E Mev	Cross-Sections in barns (10^{-24}cm^2)				
			σ_a	σ_s^{tr}	σ_{in}	σ_f	σ_t^{tr}
1	6	0.0-0.4	.22	6.28	0.0	1.50	8.0
2	12	0.4-1.6	.06	2.69	1.0	1.25	5.0
3	24	1.6- ∞	.00	1.50	1.2	1.20	3.9

Supplementary data: A = 235 (atomic masses),

Relative fission yields: $\bar{\nu}_1 = .100$, $\bar{\nu}_2 = .425$, $\bar{\nu}_3 = .475$.

Fission yields: $\nu_1 = .250$, $\nu_2 = 1.0625$, $\nu_3 = 1.1875$, $\sum_g \nu_g = 2.50$.

Inelastic spectrum: $\lambda_{12} = 1.000$, $\lambda_{13} = .667$, $\lambda_{23} = .333$.

Avogadro's number: $A_o = .6026$ (atomic masses per 10^{-24}gr).

Uranium 238

Estimated from data in memo by Richtmyer, T-137 dated 9/28/49.

g	v_g cm/shake	E Mev	Cross-Sections in barns (10^{-24}cm^2)				
			σ_a	σ_s^{tr}	σ_{in}	σ_f	σ_t^{tr}
1	6	0.0-0.4	.20	7.80	0.0	.00	8.0
2	12	0.4-1.6	.10	3.35	1.5	.05	5.0
3	24	1.6- ∞	.05	1.65	1.7	.50	3.9

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TABLE I (Continued)

Supplementary data: A = 238, fission yields and inelastic spectrum as for Uranium 235.

Oralloy (.935 Uranium 235 and .065 Uranium 238).

Calculated from the cross-sections given above.

g	v_g cm/shake	E Mev	Cross-Sections in barns ($10^{-24}cm^2$)				
			σ_a	σ_s^{tr}	σ_{in}	σ_f	σ_t^{tr}
1	6	0.0-0.4	.219	6.379	-	1.402	8.0
2	12	0.4-1.6	.063	2.733	1.032	1.172	5.0
3	24	1.6- ∞	.003	1.510	1.233	1.154	3.9

Supplementary data A = .935 · 235 + .065 · 238 = 235.2. Fission yields and inelastic spectrum as for Uranium 235.

Oralloy Transfer Coefficients c_{gh} and Macroscopic Cross-Sections σ_g.

Calculated from Oralloy cross-sections by formulae given in LA-1271.

	c_{gh} Transfer from group h to g			
g \ h	1	2	3	σ_g
1	.8412	.2650	.2847	.3792
2	.1862	.7956	.4198	.2370
3	.2081	.2784	.7386	.1849

Conversion factor N_o for σ (barns to 1/cm):

$N_o = \rho A_o/A = .04740$ using $\rho = 18.5$ gr/cm^3.

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TABLE II THREE-VELOCITY CROSS-SECTIONS

(SECOND SET)

Uranium 235

σ_g^{tr} changed to 2.19 for g=2, and to 0.90 for g=3.

σ_{in} changed to 1.5 for g=2, and to 1.8 for g=3.

ν_g changed to .249, 1.05825, and 1.18275, with $\sum_g \nu_g = 2.49$.

All other data unchanged.

Uranium 238

σ_g^{tr} changed to 2.60 for g=2, and to 0.80 for g=3.

σ_{in} changed to 2.25 for g=2, and to 2.55 for g=3.

ν_g changed to .249, 1.05825, and 1.18275.

All other data unchanged.

Oralloy

σ_g^{tr} changed to 2.217 for g=2, and to .8935 for g=3.

σ_{in} changed to 1.548 for g=2, and to 1.8495 for g=3.

ν_g changed to .249, 1.05825, and 1.18275.

All other data unchanged.

Oralloy Transfer Coefficients c_{gh} and Macroscopic Cross-Section σ_g.

Calculated from Table I as modified above.

	c_{gh} Transfer from group h to g			
g \ h	1	2	3	σ_g
1	.8410	.3680	.3899	.3792
2	.1855	.6915	.4713	.2370
3	.2073	.2772	.5789	.1849

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TABLE III

COMPARISON OF FIRST AND SECOND SET WITH EXPERIMENT

AT CRITICAL MASS

	First	Second	Experiment
m_o(kg)	56.33	54.16	54.2
$\left(\frac{dl/M}{dm}\right)_{m=m_o}$	-.00508	-.00529	-.0056
$\left(\frac{d\alpha}{dm}\right)_{m=m_o}$	.00839	.00814	.0075
$\bar{v}$ (1)	10.524	9.749	~9.0 (4)
$\overline{v_e}$ (1)	11.235	10.407	-
$\bar{\sigma}$ (2)	.2633	.2754	-
$\bar{c}$ (3)	1.3176	1.3035	-

(1) $\bar{v}$ = average neutron velocity in the sphere, $\overline{v_e}$ = average neutron velocity outside the sphere; averaged using n_g and $(n_g)_e$ respectively as weight functions.

(2) $\bar{\sigma}$ = average inverse mean free path in the sphere, averaged using $v_g n_g$ as weight function.

(3) $\bar{c}$ = average number of neutrons per collision, averaged using $\sigma_g v_g n_g$ as weight function.

(4) Estimated by H. Bethe from experimental data.

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TABLE IV

COMPARISON WITH EXPERIMENT

(First Set)

m/m_o	1/M			v		
	1-Vel.(1)	3-Vel.	Experiment(2)	$\bar{v}$	$\bar{v}_e$	$\bar{\sigma}$
.0	1.0	1.0	1.0	-	13.913	-
.3	.3279	.3309	.370	10.551	11.466	.2628
.5	.1939	.1965	.235	10.536	11.344	.2631
.7	.1000	.1016	.115	10.530	11.283	.2632
.85	.04528	.04599	.055	10.526	11.254	.2633
.95	.01409	.01442	.018	10.525	11.241	.2633
.98	.00552	.00566	.007	10.525	11.237	.2633
1.0	.0	.0	.0	10.524	11.235	.2633
m_o(kg)	56.33	56.33	54.2(3)	-	-	-
$\left(\frac{d1/M}{dm}\right)_{m=m_o}$	-.00487	-.00508	-.0056±.0003(3)	-	-	-
$\left(\frac{d\alpha}{dm}\right)_{m=m_o}$	.00868	.00839	.0075(3)	-	-	-

(1) Calculated using σ = .2633, v = 10.524, f = .3126 (corresponding to m_o = 56.33), and the formulae of LA-1273.

(2) LA-1155, p. 26, by V. Josephson, L. Woodward, and R. Paine.

(3) LA-1209, H. Paxton and J. Orndorff.

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TABLE V

COMPARISON WITH EXPERIMENT

(Second Set)

m/m_o	1/M			v		
	1-Vel.[1]	3-Vel.	Experiment[2]	$\bar{v}$	$\bar{v}_e$	$\bar{\sigma}$
.0	1.0	1.0	1.0	-	13.913	-
.3	.3292	.3339	.370	9.762	10.666	.2751
.5	.1951	.1984	.235	9.753	10.527	.2753
.7	.1005	.1026	.115	9.750	10.458	.2754
.85	.04545	.04648	.055	9.748	10.428	.2754
.95	.01448	.01457	.018	9.749	10.413	.2754
.98	.00572	.00572	.007	9.748	10.409	.2754
1.0	.0	.0	.0	9.749	10.407	.2754
m_o(kg)	54.16	54.16	54.2[3]	-	-	-
$\left(\frac{d1/M}{dm}\right)_{m=m_o}$	-.00509	-.00529	-.0056±.0003[3]	-	-	-
$\left(\frac{d\alpha}{dm}\right)_{m=m_o}$	.00837	.00814	.0075[3]	-	-	-

(1) Calculated using $\sigma = .2754$, $v = 9.749$, $f = .2984$ (corresponding to $m_o = 54.16$), and the formulae of LA-1273.

(2) LA-1155, p. 26, by V. Josephson, L. Woodward, and R. Paine.

(3) LA-1209, H. Paxton and J. Orndorff.

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TABLE VI

σvn, vn, and n as functions of m/m_o and v.

(Second Set)

m/m_o	a(cm)	m(kg)	σvn			vn			n			$(vn)_e$		
			g=1	g=2	g=3	g=1	g=2	g=3	g=1	g=2	g=3	g=1	g=2	g=3
.0												.1	.425	.475
.3	5.941	16.25	.1419	.0796	.0536	.3743	.3357	.2900	.0624	.0280	.0121	.2998	.3508	.3494
.5	7.044	27.08	.1428	.0785	.0540	.3765	.3313	.2922	.0628	.0276	.0122	.3114	.3456	.3430
.7	7.880	37.91	.1432	.0779	.0543	.3777	.3286	.2937	.0629	.0274	.0122	.3173	.3430	.3397
.85	8.406	46.04	.1434	.0775	.0545	.3782	.3272	.2946	.0630	.0273	.0123	.3199	.3419	.3382
.95	8.724	51.45	.1435	.0774	.0546	.3785	.3264	.2951	.0631	.0272	.0123	.3212	.3413	.3375
.98	8.815	53.08	.1436	.0773	.0546	.3786	.3262	.2952	.0631	.0272	.0123	.3215	.3412	.3373
1.0	8.874	54.16	.1436	.0773	.0546	.3786	.3260	.2954	.0631	.0272	.0123	.3217	.3411	.3372

TABLE VII

vn as a function of r and v for $m/m_o = 1$.

r_i	First Set			Second Set		
	g=1	g=2	g=3	g=1	g=2	g=3
1/8 a_o	.00577	.00648	.00604	.00728	.00611	.00547
3/8 a_o	.00503	.00567	.00529	.00634	.00534	.00479
5/8 a_o	.00370	.00422	.00396	.00467	.00397	.00357
7/8 a_o	.00205	.00242	.00231	.00258	.00228	.00208
$\sum r_i^2 (vn)_i$	.3082	.3559	.3360	.3786	.3260	.2954

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TABLE VIII

vn as a function of m/m_o, r, and v.

(Second Set)

m/m_o	r_i	g=1	g=2	g=3
.3	1/8 a	.03472	.03278	.02627
	3/8 a	.01667	.01483	.01248
	5/8 a	.00977	.00861	.00744
	7/8 a	.00510	.00464	.00411
.5	1/8 a	.02064	.01884	.01550
	3/8 a	.01140	.00989	.00853
	5/8 a	.00712	.00617	.00543
	7/8 a	.00376	.00337	.00304
.7	1/8 a	.01348	.01193	.01010
	3/8 a	.00870	.00744	.00653
	5/8 a	.00580	.00497	.00443
	7/8 a	.00311	.00277	.00251
.85	1/8 a	.00995	.00859	.00745
	3/8 a	.00736	.00624	.00554
	5/8 a	.00515	.00439	.00394
	7/8 a	.00281	.00248	.00227
.95	1/8 a	.00810	.00686	.00607
	3/8 a	.00666	.00561	.00502
	5/8 a	.00481	.00410	.00369
	7/8 a	.00265	.00234	.00214
.98	1/8 a	.00760	.00640	.00570
	3/8 a	.00646	.00544	.00487
	5/8 a	.00472	.00402	.00362
	7/8 a	.00261	.00230	.00210
1.0	1/8 a_o	.00728	.00611	.00547
	3/8 a_o	.00634	.00534	.00479
	5/8 a_o	.00467	.00397	.00357
	7/8 a_o	.00258	.00228	.00208

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