CIC-14 REPORT COLLECTION

REPRODUCTION
COPY

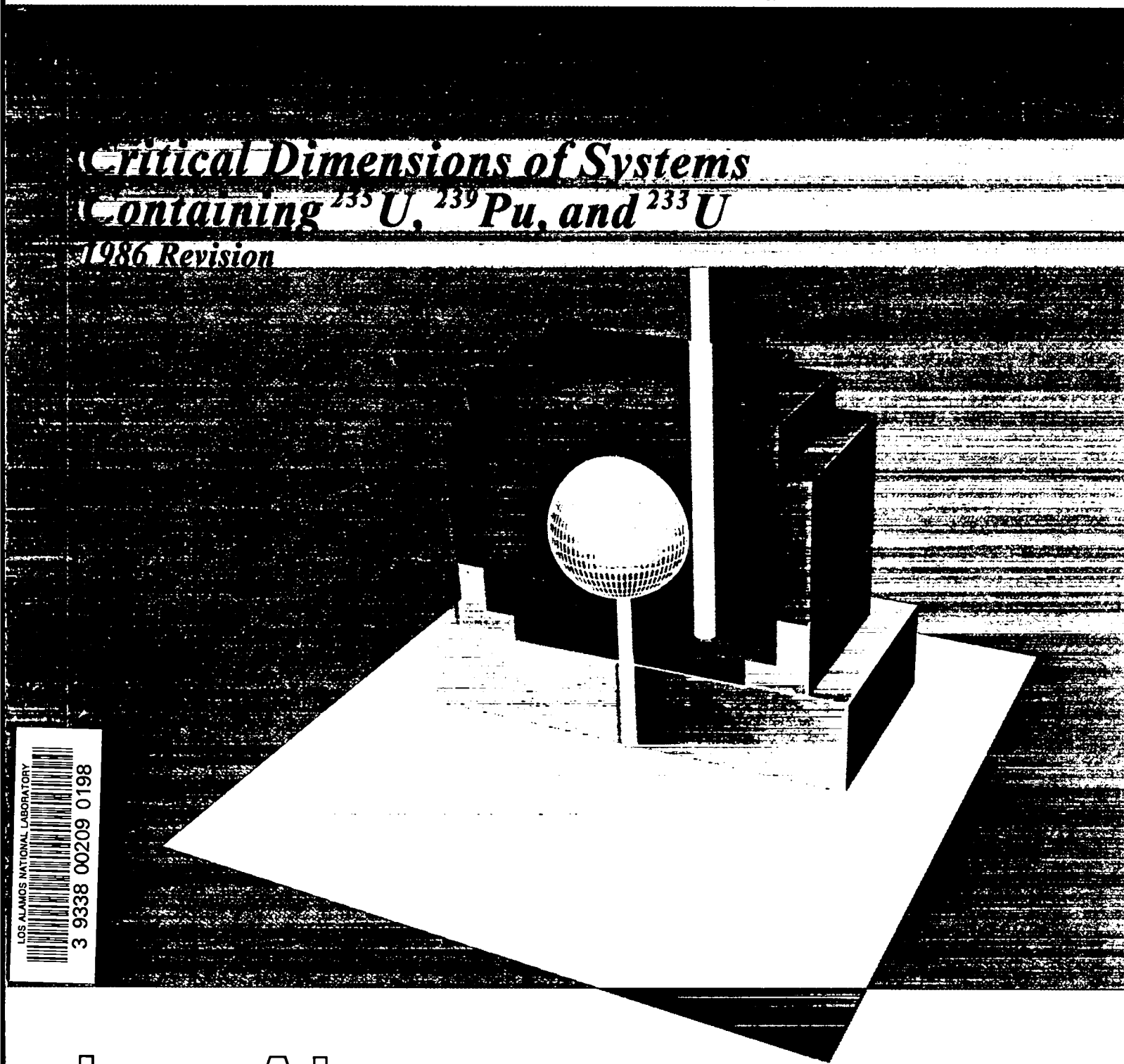
LA-10860-MS

C.3

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

Critical Dimensions of Systems Containing ^{235}U, ^{239}Pu, and ^{233}U

1986 Revision



LOS ALAMOS NATIONAL LABORATORY
3 9338 00209 0198

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

An Affirmative Action/Equal Opportunity Employer

This work was supported by the US Department of Energy, Office of Nuclear Safety.

Cover designed by Vicki Hartford and based on the Los Alamos water boiler LOPO

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

LA-10860-MS

UC-46
Issued: July 1987

Critical Dimensions of Systems Containing ^{235}U, ^{239}Pu, and ^{233}U

1986 Revision

H. C. Paxton
N. L. Pruvost

LOS ALAMOS NATL. LAB. LIBS.
3 9338 00209 0198

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

TABLE OF CONTENTS

PREFACE

This report is made possible by financial support of the Office of Nuclear Safety of the Department of Energy as the result of a proposal by D. R. Smith of the Los Alamos National Laboratory. The compilers of this version were assisted ably by an editorial advisory group consisting of E. B. Johnson, J. T. Thomas, and Dixon Callihan (all of whom, with H. C. Paxton, compiled the original report), D. R. Smith, E. D. Clayton of Pacific Northwest Laboratory and C. M. Hopper of Oak Ridge National Laboratory. The in-depth effort of E. B. Johnson was especially valuable. We also wish to acknowledge the outstanding skill of Ann Hopkins in preparation of this document.

H. C. Paxton

N. L. Pruvost

CRITICAL DIMENSIONS OF SYSTEMS CONTAINING ^{235}U, ^{239}Pu, AND ^{233}U

1986 REVISION

by
H. C. Paxton and N. L. Pruvost

ABSTRACT
This document is a revision of TID-7028, CRITICAL DIMENSIONS OF SYSTEMS CONTAINING U^{235}, Pu^{239}, AND U^{233} (Ref. 1).

INTRODUCTION

This report is primarily a compilation of critical data obtained from experiments performed in a number of laboratories during the period of 1945 through 1985. It supplements the Nuclear Safety Guide [Report TID-7016 (Rev. 2)][2] in presenting critical data on which recommendations of the Guide are based.

It must be emphasized that this report gives critical data without safety factors, so it is no substitute for the Guide or for the related document, The American National Standard for Nuclear Criticality Safety in Operations with Fissionable Materials Outside Reactors[3]. This standard is supported by publications of H. K. Clark that interpret criticality information for Pu, ^{233}U and ^{235}U systems.[4–6]

Information for guiding the safe design of equipment for handling the three common fissile materials appears to a certain extent in the Nuclear Safety Guide, and with greater detail in several handbooks.[7] Other publications specify conditions for unique processes with a particular fissionable material.[8]

Experiments of several types which contribute results applicable to nuclear design and to safety problems have been described by Callihan.[9] Critical measurements with materials of interest in desired configurations yield information of greatest usefulness and accuracy. Where it is not feasible to obtain the desired critical data, for example, as a result of safety restrictions, subcritical data may be directly applicable, and in some cases may be extrapolated to approximate critical conditions. Critical conditions also may be approximated from the distribution of neutrons introduced into a subcritical assembly. These "exponential experiments" may be the only alternative where the quantity of material required is too great for a critical experiment.

In some cases, calculated results are desirable to fill in experimental data or to extend them. Where they appear in this report, they are identified as calculations because of the uncertainty associated with them. In general, the 16-group Hansen-Roach cross-section set[10] is used with the one-dimensional transport code, DSN,[11] or its modern version, ONEDANT[12] in one dimension and TWODANT[13] in two dimensions. The reason for this choice is Stratton's extensive comparison of results of such calculations with experimental data.[14] Where this comparison is unfavorable, particularly for solution or hydrogeneous mixtures of few-percent ^{235}U-enriched uranium, the Los Alamos MCNP Monte Carlo code[15] and cross-sections based on ENDF-B/V* are used instead. This combination is also applied in the few cases where finite-cylinder calculations are desired.

Calculated extensions of experimental data are included to show the nature of trends, not to substitute for results of experiments. They should be used with caution.

A fundamental aim of this document is to illustrate relationships among critical data. The compilation and correlation of data for this purpose, from many measurements in a number of laboratories, require a certain amount of normalization or reduction to common terms. Frequently, for example, the effects of variations in geometry or density must be removed to show trends in data. The manner in which these alterations may be made is discussed in the early section RELATIONS FOR CONVERSION TO STANDARD CONDITIONS.

Reactor mockups and related critical assemblies are generally outside the scope of this document. Many of those that are appropriate to serve as computational models are fast-neutron systems that are of secondary interest in criticality safety matters.[16] Nevertheless, fast-reactor mockups which are used as benchmark assemblies (for comparison with calculations) are instructive, for they illustrate the value of data other than criticality specifications, namely, spectral information, neutron flux distributions, prompt-neutron lifetime, and reactivity coefficients. Unfortunately, data for critical assemblies suitable as safety benchmarks are usually limited to critical compositions, dimensions and masses without useful supplementary information.

* Robert C. Little, Los Alamos National Laboratory, Los Alamos, NM 87545, 1986.

PART I — INDIVIDUAL UNITS.

HISTORY OF EARLY CRITICAL EXPERIMENTS

EXPERIMENTS RELATED TO THE WEAPONS PROGRAM

The first critical assembly with enriched uranium* was the Los Alamos water boiler LOPO, operated initially in 1944.[17,18] This was a predecessor of HYPO that operated as a neutron source for many years. Of course, they were both preceded by reactor-oriented assemblies of natural uranium, namely the Chicago and Oak Ridge piles.[19] LOPO was a 14.95-liter sphere of $U(14.67)O_2SO_4$-H_2O** solution at $H/^{235}U=647$ in a 0.08-cm-thick container and reflected by a roughly 90-cm-diam stack of beryllium oxide (density 2.7 g/cm^3) on a graphite base.

About the same time, before U(>90) became available, Los Alamos critical experiments were directed toward weapon design. The critical assemblies of which descriptions have been declassified consisted of U(~80) metal cores reflected by natural uranium, tungsten carbide, or beryllium oxide; $U(\sim 74)H_{10}C_4$ cores reflected by natural uranium, tungsten carbide, beryllium oxide, or iron; and plutonium metal reflected by thin natural uranium within tungsten carbide.[20] The $UH_{10}C_4$ was an intimate mixture of UH_3 and Styrex ($CH_{1.75}$). Table 1 gives critical specifications of those assemblies with nearly regular geometries. Because uncertain geometric details cannot be clarified at this late date, these specifications are given for historical interest rather than scientific value.

EXPERIMENTS RELATED TO THE GASEOUS DIFFUSION PROCESS

In 1945, Oak Ridge undertook the first of a series of critical experiments directed toward criticality control of enriched uranium as UF_6, the working material of the gaseous diffusion plant.[21] Critical assemblies at that time came as close as practicable to simulating condensed $U(24)F_6$ with hydrogen moderation. One-inch cubes of an intimate mixture of $U(24)_3O_8$ and as much viscous fluorocarbon, reported to be $CF_{0.68}$, as would retain its shape were prepared in 0.013-cm-thick aluminum. The effective composition, $U(24)O_{2.83}C_{5.30}F_{3.60}Al_{0.4}$, did not approach the actual F/U atomic ratio 6. Because the available inventory, 9.5 kg ^{235}U, was subcritical without moderation, one-inch cubes of polyethylene ($CH_{1.87}$) were latticed among the uranium units to make some assemblies critical. Table 2 gives the reported description of these assemblies, with roughly cubical cores reflected by paraffin. Because of a question concerning the reported fluorocarbon formula,*** these assemblies, also, are described primarily for historical interest.

---

* Uranium enriched in the isotope ^{235}U.

** Numbers in parentheses indicate the ^{235}U content of the uranium in weight percent.

*** An expert in fluorocarbon chemistry believes that the formula $CF_{0.68}$ should represent a solid instead of a viscous fluid. The formula was provided by the laboratory of the Gaseous Diffusion Plant which produced the fluorocarbon.

TABLE 1. EARLY LOS ALAMOS METAL AND HYDRIDE CRITICAL ASSEMBLIES.

Core		Reflector		
Composition, Shape[a]	^{235}U or Pu Density (g/cm^3)	Material, Form	Density (g/cm^3)	Critical Mass (kg ^{235}U or Pu)
U(78.7) pseudosphere	17.8	U(natural) 48-cm-od sphere	19.0	21.9
U(78.5) pseudosphere	17.8	WC 35.6-cm cube	14.7	20.8
U(82.7) pseudosphere	17.8	BeO 61-cm cube	2.69	10.3
U(75.0)$H_{10}C_4$ pseudosphere	2.17	WC 41-cm cube	14.7	7.00
U(75.2)$H_{10}C_4$ near cube	2.18	BeO 48-cm cube	2.69	3.52
U(75.2)$H_{10}C_4$ near cube	2.18	BeO 79-cm cube	2.69	2.80
Pu(1.35 wt% ^{240}Pu)[b] sphere	15.6	U(natural) inside sphere, 1.14-cm-thick WC outside parallelepiped 32.4x32.4x27.0 cm	19.0 ~14.7	6.13

[a] All uranium cores built of 1/2-in. cubic units.
[b] Contained 1.0 wt% Ga, coated with ~0.013-cm-thick Ni.

TABLE 2. CRITICAL CUBIC LATTICES OF 1-IN. CUBES OF $U(24)O_{2.83}C_{5.30}F_{3.60}Al_{0.4}$ (DENSITY 3.50 g/cm^3) AND $CH_{1.87}$ (DENSITY 0.915 g/cm^3) REFLECTED BY >6-IN.-THICK PARAFFIN.

H Cubes/U Cubes	$H/^{235}U$ Atomic Ratio	Average Density(g/cm^3)	Critical Mass (kg ^{235}U)
3/4	46	2.14	8.3
1/1	63	1.95	6.1
2/1	129	1.67	3.9

In 1946 there was sufficient highly enriched uranium to extend the above experiments with one-inch cubes of intimately mixed UF_4 and polytetrafluoroethylene $(CF_2)_n$, effectively UF_6C. These were latticed with either one-inch-cubes or 1 x 1 x 0.5 in. half-cubes of polyethylene. A series of experiments with U(95.3) was conducted by Oak Ridge personnel at Los Alamos.[22] Roughly cubic cores with 18-cm-thick paraffin reflectors consisted of hydrogen-to-uranium cubes in the ratios $\frac{1}{2}$:1, 1:1, 2:1, 4:1, and 7:1. For the 1:1 ratio, heterogeneity was varied such that there was reasonable extrapolation to a homogeneous core, with the result included later in Table 10. Critical masses for all hydrogen-to-uranium combinations, uncorrected for heterogeneity, appear in Fig. 1 as the left-most curve. Approximate densities of ^{235}U in g/cm^3 may be obtained by dividing 14.8 by $H/^{235}U$.

Later that year, a similar series of experiments was performed at Oak Ridge with U(29.8) instead of highly enriched uranium.[23] Again, one composition could be corrected reasonably for heterogeneity, with the result included in Table 10. Uncorrected critical masses also appear in Fig. 1. The curves rise at large $H/^{235}U$ as a result of extreme heterogeneity. The expression relating ^{235}U density and $H/^{235}U$ given above for U(95.3) also applies for U(29.8).

Although not belonging to this very early period, the only known experiments with homogeneous hydrogen-moderated UF_6 are described here to complete the account of UF_6 criticality.[24] In this series of experiments, performed at Valduc, France,* the primary material was liquid $U(93)F_6$ to which various proportions of liquid HF were added while retaining homogeneity. These mixtures were contained in 0.4-cm-thick Monel spheres surrounded by an effectively infinite thickness of water.

Experimental critical data appear in Table 3 and in Table 4 after adjustment to spherical volumes and normalization to 75°C. Figures 2 and 3 show critical volumes, diameters, and masses as functions of $H/^{235}U$ atomic ratio and ^{235}U density. The available inventory of UF_6, 93.9 kg ^{235}U, could not be made critical without internal hydrogen. Consequently, the entry under $H/^{235}U = 0$ in Table 4 is the result of extrapolations shown in Figs. 2 and 3, supported by Monte Carlo and transport calculations.

* La Station de Criticalite de Valduc may be called simply Valduc, or Dijon, which is nearby.

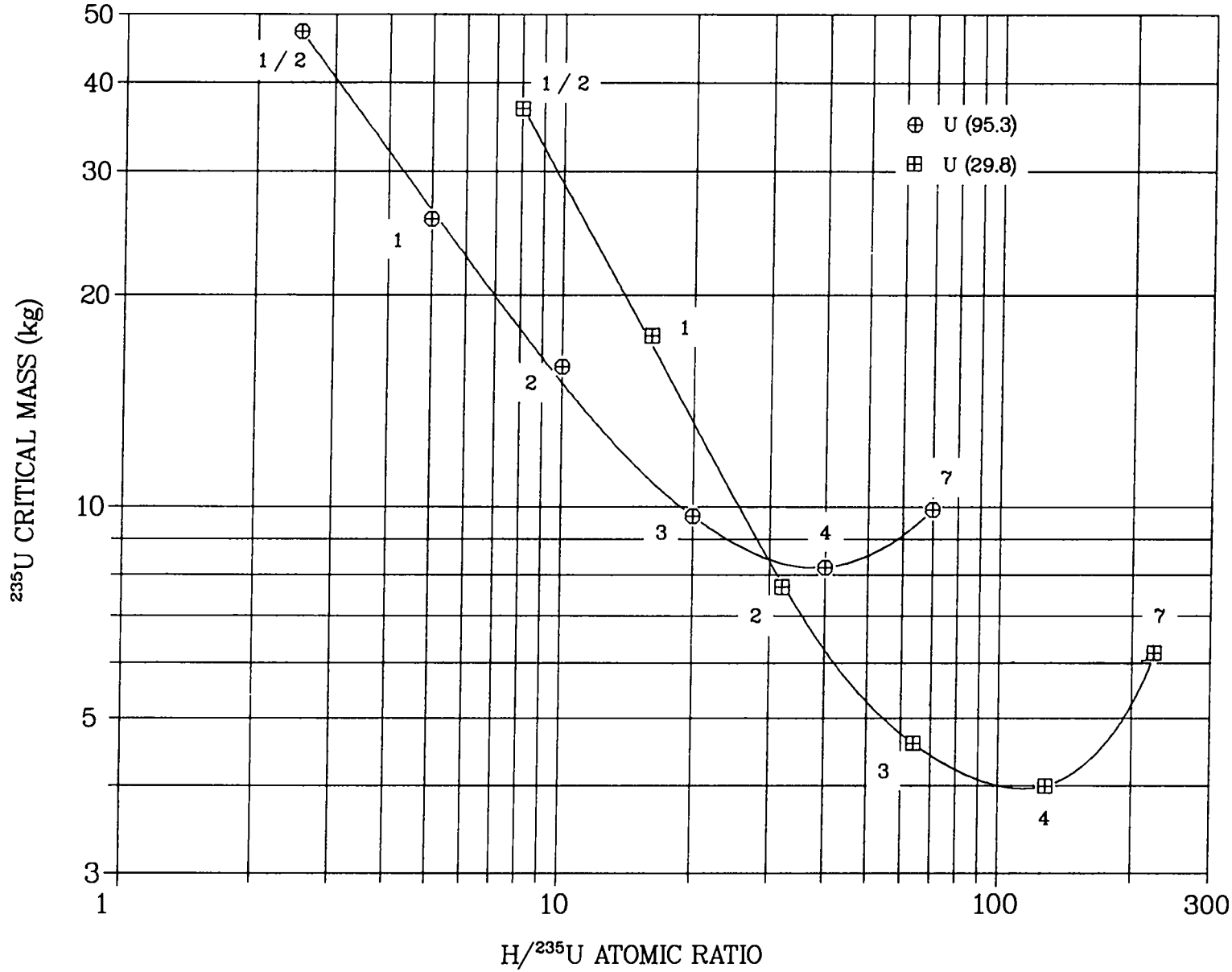


Fig. 1. Critical masses of paraffin-reflected cubes consisting of intermixed one-inch cubes of UF_6C and polyethylene, for U(95.3) and U(29.8). The ratios of polyethylene to UF_6C cubes are indicated.

TABLE 3. CRITICAL U(93)F_6-HF MIXTURES IN WATER-REFLECTED SPHERICAL CONTAINERS.

	Inside Diameter of Container (cm)										
	53.9	53.9	50.9	50.9	50.9	53.9	50.9	50.9	50.9	50.9	53.9
H/U	5.7	9.9	10.8	11.2	15.2	16.9	21.0	26.0	38	74	82
Δ(H/U)[a]	0.1	0.2	0.2	0.2	0.3	0.3	0.4	0.7	1	2	3
T(^{0}C)	89.0	75.0	85.8	75.1	74.8	94.3	74.3	75.0	75.6	75.1	75.3
ΔT(^{0}C)[a]	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Density											
ρ(g/cm^3)	1.92	1.71	1.61	1.63	1.467	1.336	1.321	1.245	1.149	0.990	0.973
$\Delta\rho$(g/cm^3)[a]	0.02	0.01	0.01	0.01	0.009	0.008	0.008	0.007	0.007	0.006	0.006
Liquid											
Height (cm)	full	47.3	full	full	full	51.0	46.6	46.2	46.1	full	46.1
Critical Volume											
V_C (L)	85.0	78.5	76.0[b]	71.3[b]	69.4[b]	81.3	67.3	67.1	67.0	71.3[b]	77.2

[a] A value designated by Δ represents the uncertainty in the quantity.
[b] Value of critical volume from extrapolated reciprocal multiplication curves.

TABLE 4. CRITICAL DIMENSIONS OF WATER-REFLECTED U(93)F_6-HF MIXTURES ADJUSTED TO SPHERICAL VOLUMES AT 75°C.[a]

$H/^{235}U$	0.0	6.1	10.6	11.6	12.0	16.3	18.2	22.6	28.0	41	88
$\rho(g/cm^3)$	3.63	1.99	1.71	1.65	1.63	1.465	1.413	1.317	1.245	1.140	0.973
^{235}U Concentration											
$C(g/cm^3)$	2.27	0.939	0.69	0.639	0.622	0.490	0.473	0.373	0.313	0.225	0.107
$\Delta C(g/cm^3)$[b]		0.014	0.01	0.008	0.008	0.007	0.007	0.006	0.007	0.009	0.004
Critical Diameter											
$d_C(cm)$	55[c]	53.74	52.8	52.8	52.1	51.04	51	50	50	50	53
$\Delta d_C(cm)$[b]		0.09	0.2	0.2	0.2	0.05	1	2	2	2	2
$V_C(L)$	88[c]	81.3	77	75.1	74.1	69.6	68.6	67.0	66.2	66.2	76.5
$\Delta V_C(L)$[b]		0.4	1	0.7	0.5	0.3	0.4	0.7	0.7	0.7	0.7
Critical Mass											
$M_C(kg^{235}U)$	200[c]	76	53.0	48.4	46.5	34.1	32.5	25.0	20.7	14.90	8.2
$\Delta M_C(kg^{235}U)$[b]		1.8	1.5	2.0	1.9	1.7	2.0	2.4	2.8	5.0	4.2

[a] Uncorrected for 0.4-cm-thick Monel container.
[b] Uncertainties represented by Δ(quantity);Δ(H/U) and $\Delta(\rho)$ appear in Table 3.
[c] Extrapolated value supported by calculations.

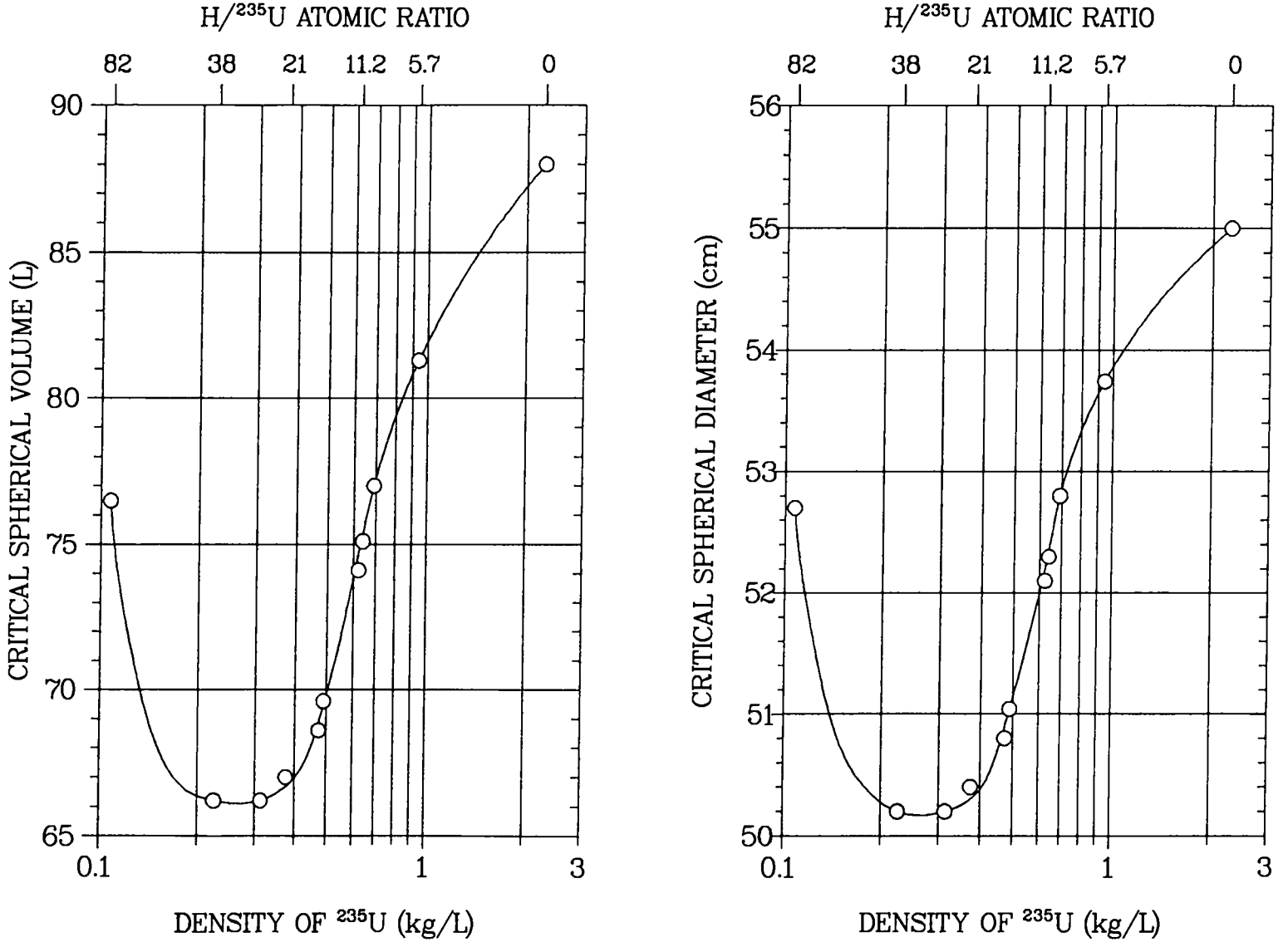


Fig. 2. Critical volume and diameter of U(93) as liquid UF_6-HF at 75° C as a function of ^{235}U density. The system was contained in a water-reflected 0.4-cm-thick Monel sphere.

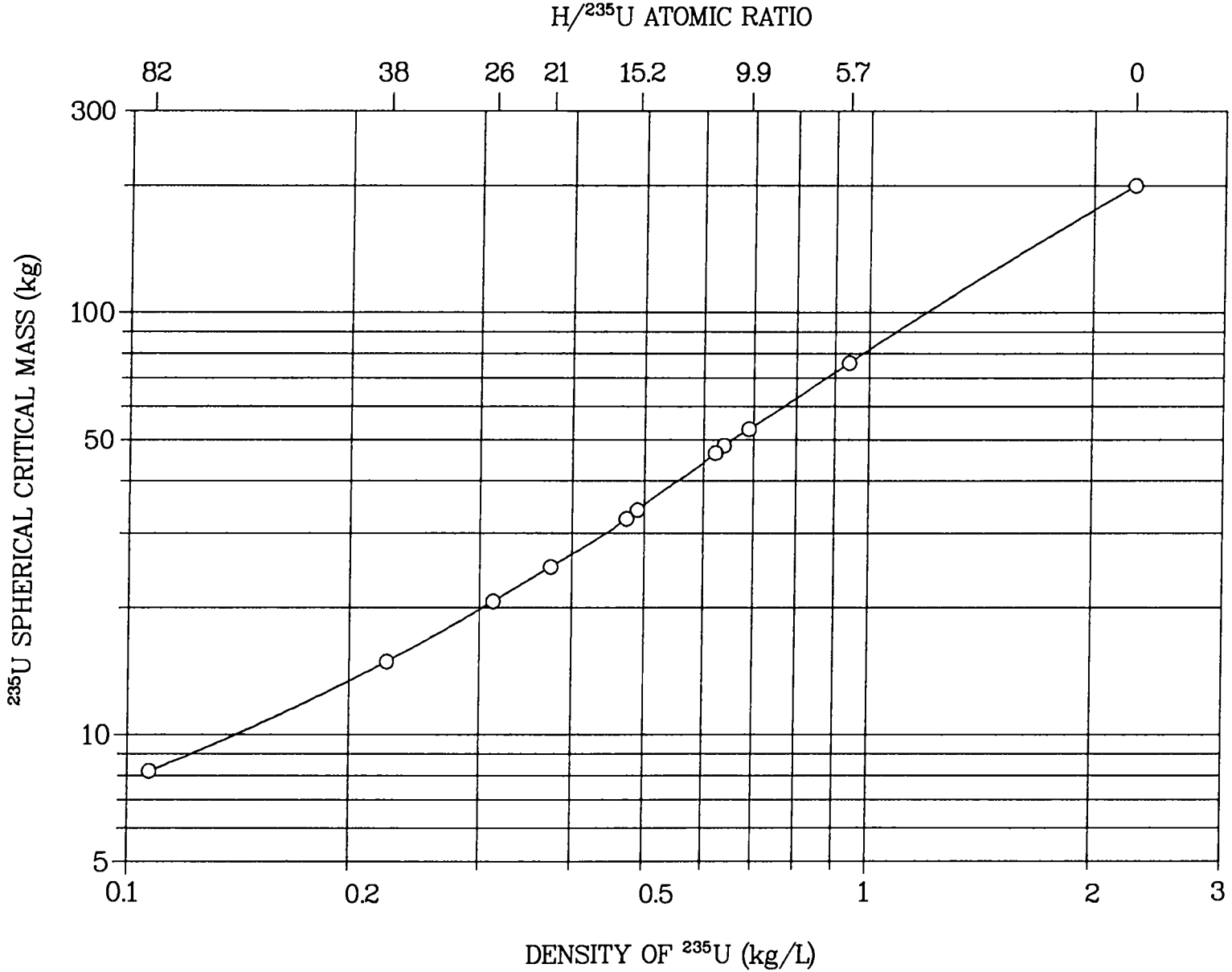


Fig. 3. Critical mass of U(93) as liquid UF_6-HF at 75°C as a function of ^{235}U density. The system was contained in a water-reflected 0.4-cm-thick Monel sphere.

As Fig. 2 indicates, the minimum critical volume occurs at H/U about 30. Further, UF_6-HF mixtures at any concentration cannot be made critical in water-reflected spheres 45-cm or less in diameter provided the temperature is at least 75°C.

TRANSITION TO FACILITIES FOR CRITICAL EXPERIMENTS

Returning to the very early experiments, it may be noted that they were conducted in improvised facilities. Two Los Alamos fatalities[25] underlined the need for improved protection of personnel, and permanent facilities designed specifically for the safe performance of critical experiments were in operation at Los Alamos in 1947 and at Oak Ridge in 1950 after experiments starting in 1946 at an interim facility. There followed at Oak Ridge extensive programs with uranium enriched in ^{235}U and ^{233}U, which contribute to the remainder of this report.

In 1951, critical experiments with plutonium solutions began in a Hanford facility that was temporary but had adequate personnel protection.[26] The permanent plutonium critical-mass facility at Hanford became operable in 1961.[27] These facilities provided much of the data for moderated plutonium systems that are included in this report.

Other laboratories that have provided results of critical experiments are at Aldermaston, England, operation beginning in 1952;[28] Livermore (from 1955 in its appropriate facility);[29] Dounreay, Scotland (from 1957);[28] Saclay, France (from 1960);[30] Valduc, France (from 1963);[30] and Rocky Flats (from 1965).[31] We have no information on USSR facilities at which criticality data have been generated.

RELATIONS FOR CONVERSION TO STANDARD CONDITIONS

Many of the data correlations that appear in this report required the conversion of experimental information to certain "standard" conditions. Two of the more significant types, shape and density conversions, are considered immediately. Other types, such as the correction for variations in ^{235}U enrichment, fit more naturally into later sections.

CYLINDER-SPHERE CONVERSIONS

Ratios of critical masses of cylinders (height h and diameter d) to those of spheres appear vs h/d in Fig. 4 for enriched uranium solutions* and in Fig. 5 for enriched uranium metal. The values for solutions and U(93) metal reflected by polyethylene and Plexiglas are derived from measurements at Oak Ridge.[32–36] Those for U(94) metal, unreflected and reflected by paraffin or water, are from Los Alamos.[37–39] Early critical data for plutonium solutions originated at Hanford[26] and for ^{233}U solutions at Oak Ridge.[40]

For extrapolation of experimental critical dimensions to those of broad slabs and long cylinders, the following method is useful. The dimensions of critical cylinders of different sizes and of a critical sphere, all of the same composition, are related to each other through the expression for the geometric buckling, B^2, provided appropriate values of the cylinder extrapolation distances are used. Effective values of cylinder extrapolation distances were obtained from the following relation using cylinder and sphere dimensions and sphere extrapolation distances of Table 5.

$$B^2 = \left(\frac{2.405}{r_c + \delta_c}\right)^2 + \left(\frac{\pi}{h + 2\delta_c}\right)^2 = \left(\frac{\pi}{r_s + \delta_s}\right)^2$$

where r_c = the radius of the cylinder
h = the height of the cylinder
r_s = the radius of the sphere
δ = the effective extrapolation distance appropriate to these dimensions.

TABLE 5. SPHERE EXTRAPOLATION DISTANCES FOR GENERAL APPLICATION OF FIG. 6 WITH WATER REFLECTION.

	Sphere Extrapolation Distance δ_s (cm)			
Atomic Ratio H/X[a]:	50	200	500	1000
$^{235}UO_2F_2$	5.8	5.4	5.2	5.2
$^{233}UO_2F_2$	5.6	5.2	5.0	5.0
$Pu(NO_3)_4$	6.3	5.8	5.4	5.2

[a] X≡^{235}U, ^{233}U or Pu.

* Aqueous solutions are implied throughout this document.

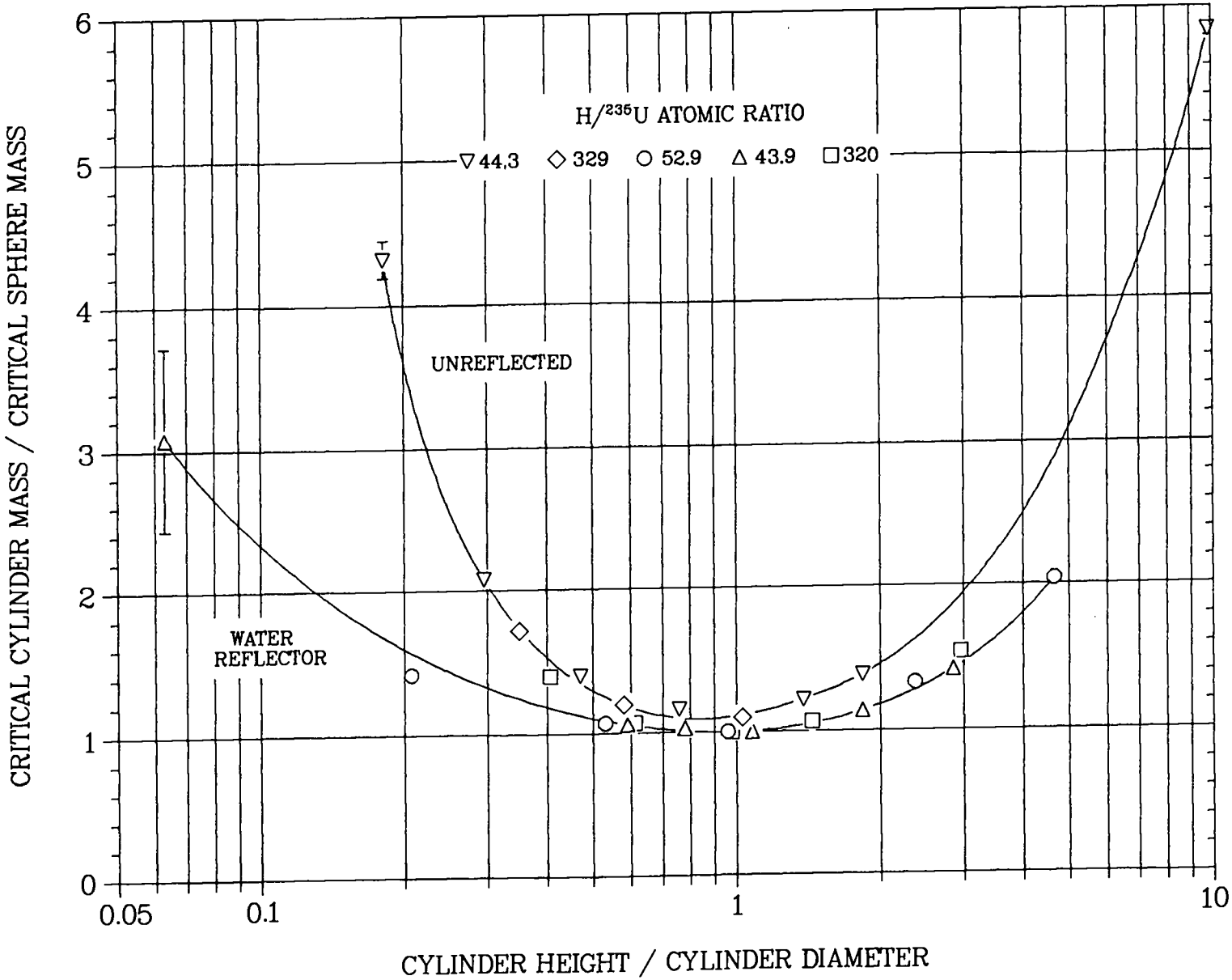


Fig. 4. The ratio of cylindrical to spherical critical masses of $U(93)O_2F_2$ solutions, unreflected and with water reflector, as a function of cylinder height to cylinder diameter ratio.

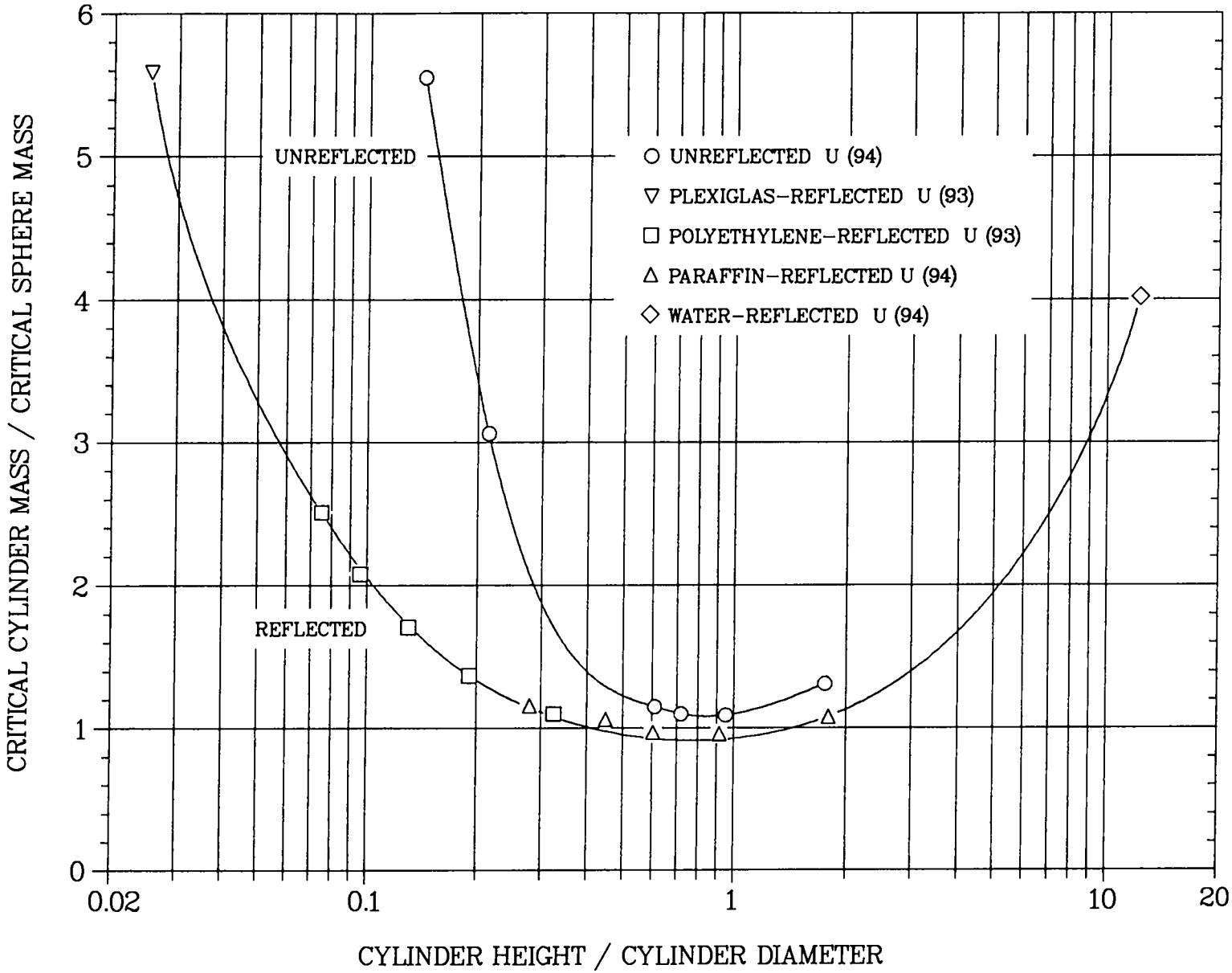


Fig. 5. The ratio of cylindrical to spherical critical masses, for U(>90) metal, unreflected and with hydrogenous reflector, as a function of cylinder height to cylinder diameter ratio.

The resulting ratios of cylinder to sphere extrapolation distances appear in Fig. 6, in which some dimensions were obtained experimentally and some were computed by the TWODANT[13] code with Hansen-Roach cross sections.[10] Figure 7 gives similar extrapolation distances for reflected and unreflected U(93.5) metal disks, where $\delta_s = 2.0$ cm is assumed for an unreflected sphere and reflector savings are consistent with Fig. 5.

In Figs. 6 and 7 the abscissa was chosen such that at zero the value of δ_c determines the thickness of an infinite slab [$= (\pi/B) - 2\delta_c$] and at unity the value determines the radius of an infinite cylinder [$= (2.405/B) - \delta_c$]. The calculated end points of Fig. 6 were obtained by means of the ONEDANT code[12] and Hansen-Roach cross sections.[10] In Table 5, the value of the sphere extrapolation distance for $^{235}UO_2F_2$ solution at $H/^{235}U = 50$ was obtained from Stratton's report LA-3612[14] by combining the extrapolation distance without reflector from his Eq. 2 and the reflector savings from his Table V. Results are $\delta_s =$ 5.8 cm with a water reflector and $\delta_s = 2.2$ cm with a 0.13-cm-thick stainless steel reflector. Other extrapolation distances in Table 5 (for more dilute ^{235}U solutions, ^{233}U solutions, and Pu solutions) were obtained from sphere, infinite cylinder and infinite slab dimensions calculated by ONEDANT.[10,12] The listed sphere extrapolation distances, then, are those required to bring both calculated end point extrapolation distances into coincidence with the end points of Fig. 6.

With 0.13 cm stainless steel reflection, $\delta = 2.2$ cm from Stratton's spheres applies universally to the transformation of solution cylinders. It has been confirmed empirically within $\pm$ 2% for $U(93.2)O_2F_2$, $^{233}UO_2F_2$, and $Pu(NO_3)_4$ + 1 $\underline{N}$ HNO_3 solutions over the experimentally available ranges of height-to-diameter ratios.

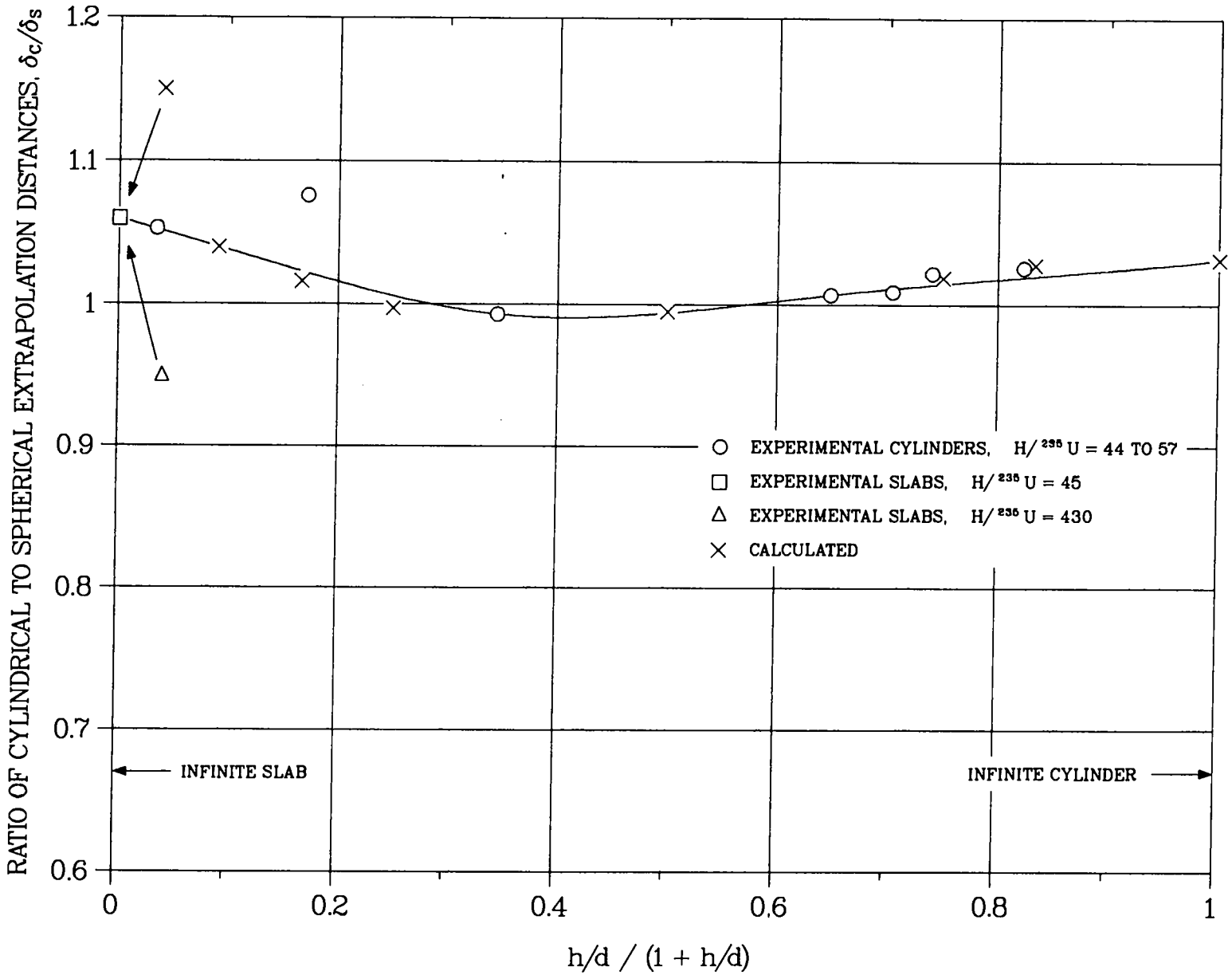


Fig. 6. Ratio of cylindrical extrapolation distance to that of sphere for water reflected $U(93)O_2F_2$ solutions. Cylinder height and diameter are h and d respectively.

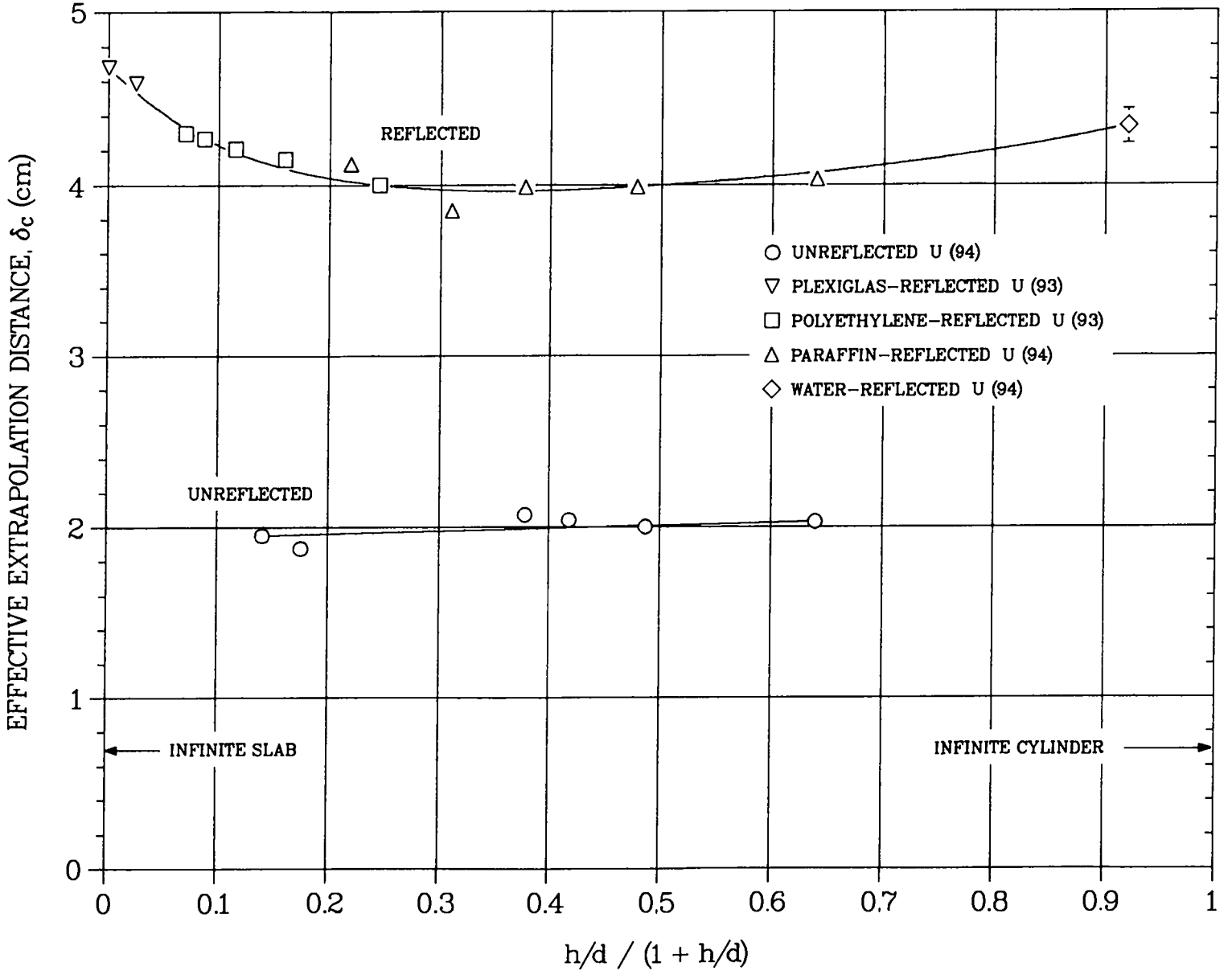


Fig. 7. Effective extrapolation distances of U(>90) metal cylinders, unreflected and with hydrogenous reflectors. Cylinder height and diameter are h and d respectively.

CORE-DENSITY CONVERSIONS

A change in the density of a fissile sphere by the ratio ρ/ρ_o leads to a changed critical mass, m_c, that may be expressed as

$$m_c \,/\, m_{co} \;=\; (\,\rho \,/\, \rho_o\,)^{-n}$$

where n is constant over a considerable range of density ratios. In fact, where density of both spherical core and reflector is changed by the same ratio and the ratio of reflector thickness to core radius is maintained, then $n = 2$ (the value for an unreflected sphere). Similarly, in the case of an infinite slab, the critical mass per unit area is necessarily independent of ρ (i.e., $n = 0$).

Where reflector characteristics remain constant, however, the value of n associated with the density change of a spherical core depends considerably upon the system. Combined Los Alamos, Livermore, and Rocky Flats data for U(93.5) metal and δ-phase plutonium cores[20,41,42] seem to follow a unique relation between the density exponent and the degree of reflection (see Fig. 8). The scatter associated with subcritical plutonium measurements would mask any small differences between the two fissile materials.

The experimental values of n (as determined by the UKAEA Atomic Weapons Research Establishment at Aldermaston, ORNL, and Los Alamos) for near-equilateral nonmetal cores are given in Table 6.[43–45]

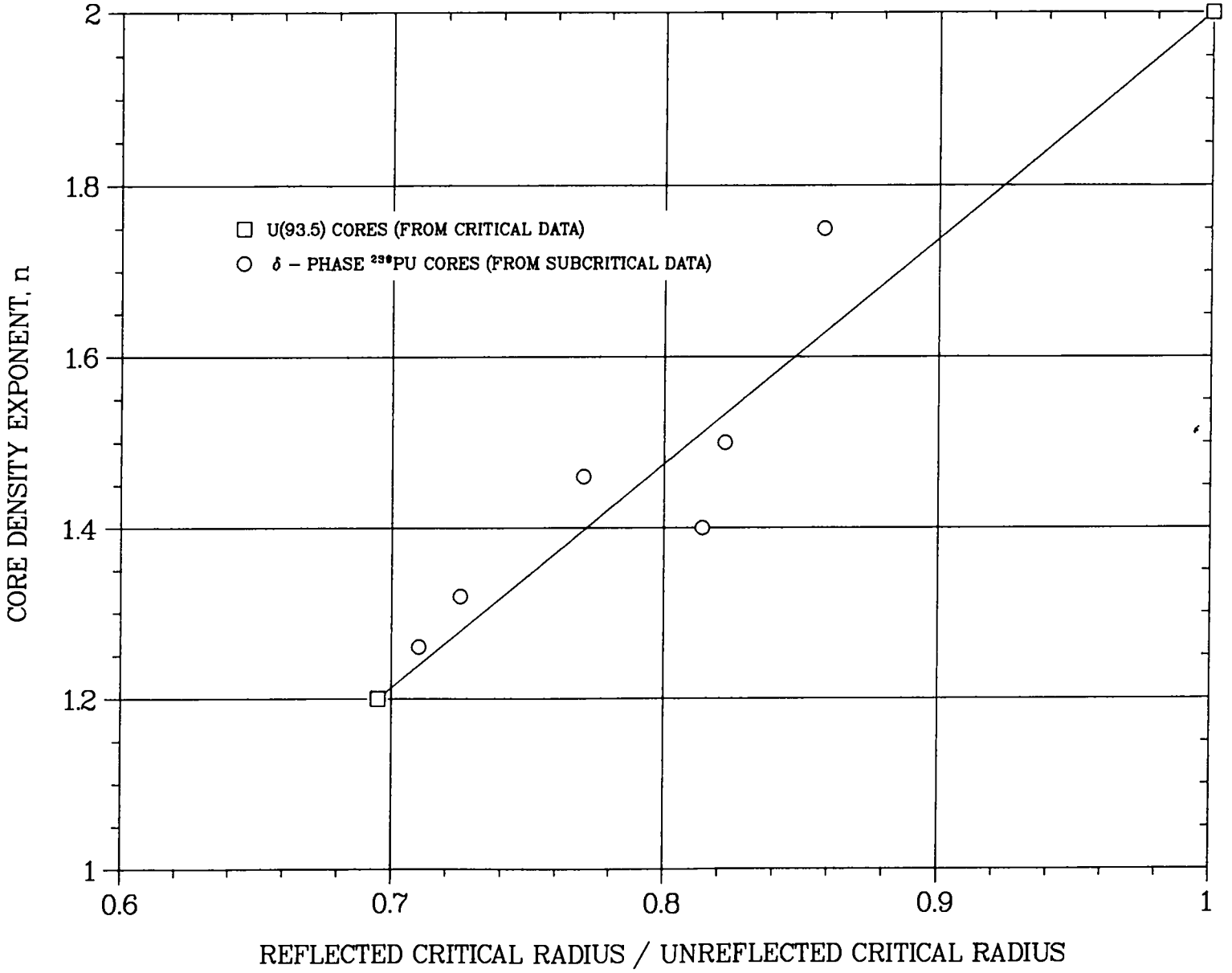


Fig. 8. Density exponents of unmoderated spherical cores in constant-density reflectors. Critical mass = constant (core density)$^{-n}$.

TABLE 6. EXPERIMENTAL VALUES OF THE NEGATIVE DENSITY EXPONENT, n, FOR NONMETAL CORES.

Core Composition	Core $H/^{235}U$	Reflector	n	Reference
$U(30)O_2$-paraffin	8.26	20.3-cm-thick Perspex[a]	1.46	43
$U(30)O_2$-paraffin	16.5	20.3-cm-thick Perspex	1.50	43
$U(30)O_2$-paraffin	16.5	20.3-cm-thick polyethylene	1.69	43
$U(30)O_2$-paraffin	82	20.3-cm-thick Perspex	1.56	43
$U(30)O_2$-paraffin	82	20.3-cm-thick polyethylene	1.67	43
$U(30)O_2$-paraffin	82	thick water	1.65	43
$U(93)O_2(NO_3)_2$	230	thick water	1.88[b]	44
$U(93)H_3C$	3.2	22.2-cm-thick U(0.7)	1.57	45

[a] Methacrylate plastic, called Plexiglas in the U.S.
[b] Possibly influenced by the manner in which voids were introduced.

The lack of experimental core-density exponents for solutions forces the use of computed values. Figure 9 shows such exponents for ^{235}U calculated by the DSN code[11] using Hansen-Roach cross sections.[10] Hanford calculations for ^{239}Pu used a similar code (DTK) but different cross sections (from GAMTEC-II).[46]

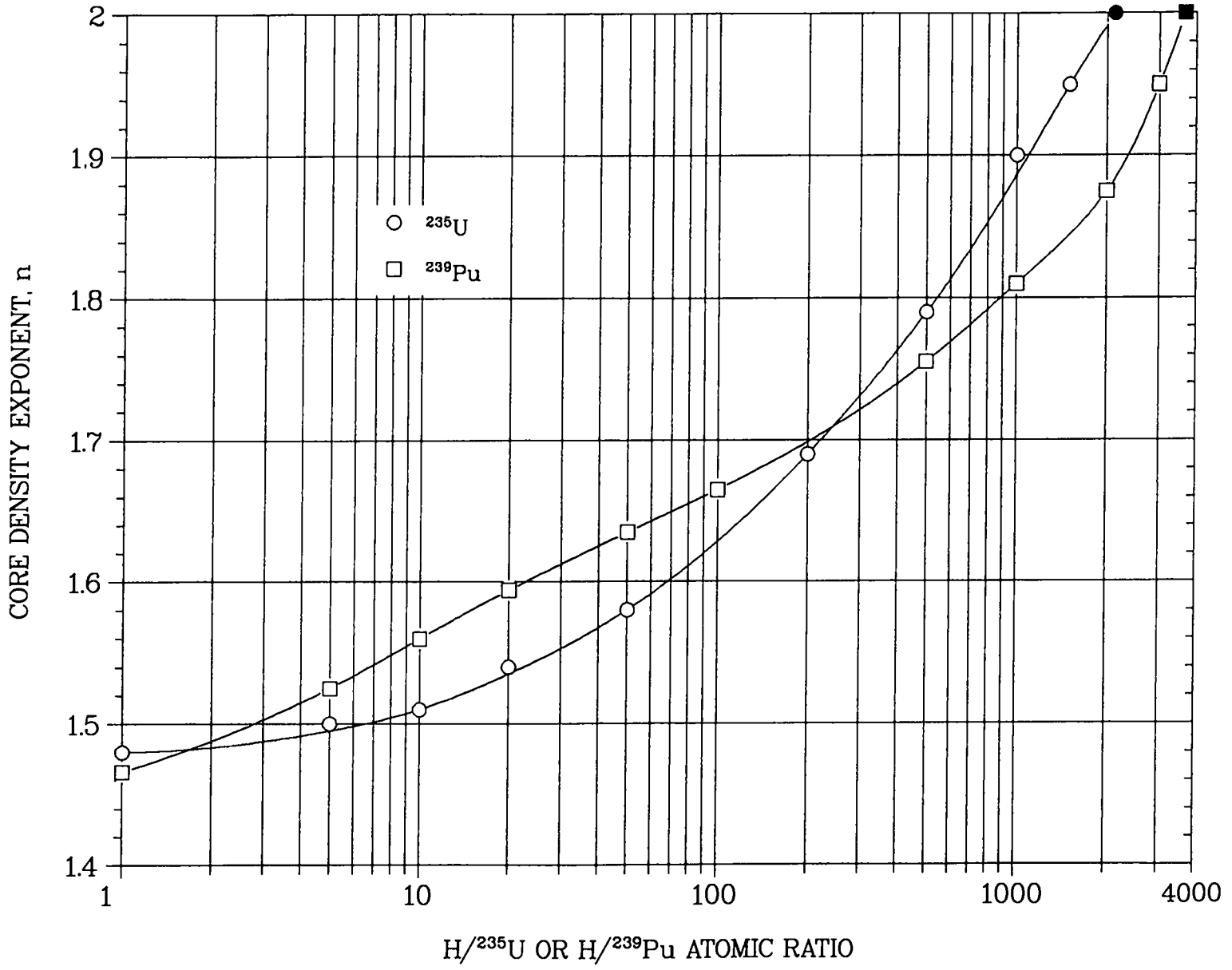


Fig. 9. Calculated core-density exponents for water-reflected spheres of homogeneous metal-water mixtures. The solid symbols represent limiting conditions.

HOMOGENEOUS HYDROGEN-MODERATED URANIUM

HIGHLY ENRICHED URANIUM

Figures 10 and 11 represent critical masses and critical volumes of homogeneous water-moderated spheres of U(93.2), both bare (except for the thin-wall container) and water reflected. For a water-reflected sphere of U(93.2) metal at a density of 18.75 g/cm^3, the critical mass is 22.8 kg ^{235}U and the critical volume is 1.30 L. Estimates of diameters of infinite critical cylinders of U(93.2)O_2F_2 solution appear in Fig. 12, and corresponding estimates of thicknesses of infinite critical slabs appear in Fig. 13. Values for water-reflected U(93.2)metal are 7.5 cm infinite cylinder diameter and 1.4 cm infinite slab thickness. It should be noted that the curve for bare infinite slabs is fictitious because a slab of infinite extent would intercept neutrons returned from material at any distance. Nevertheless, it may be useful for comparison with similarly fictitious calculations.

The branched curves of the figures show how concentrated UO_2F_2 solutions depart from ideal metal-water mixtures. As indicated by the computed curve for UO_2-water in Fig. 10, densities of the fissile isotope in metal-water mixtures are greater than actually found in practice; hence, critical values are lower limits that may be quite conservative. For convenience the assumed relations between the density of ^{235}U and the atomic ratio H/^{235}U for solutions and metal-water mixtures are given in Table 7.[47] Similar relations for ^{233}U and Pu (Ref. 26) are also included.

The appendix gives theoretical densities of unmoderated common uranium compounds.

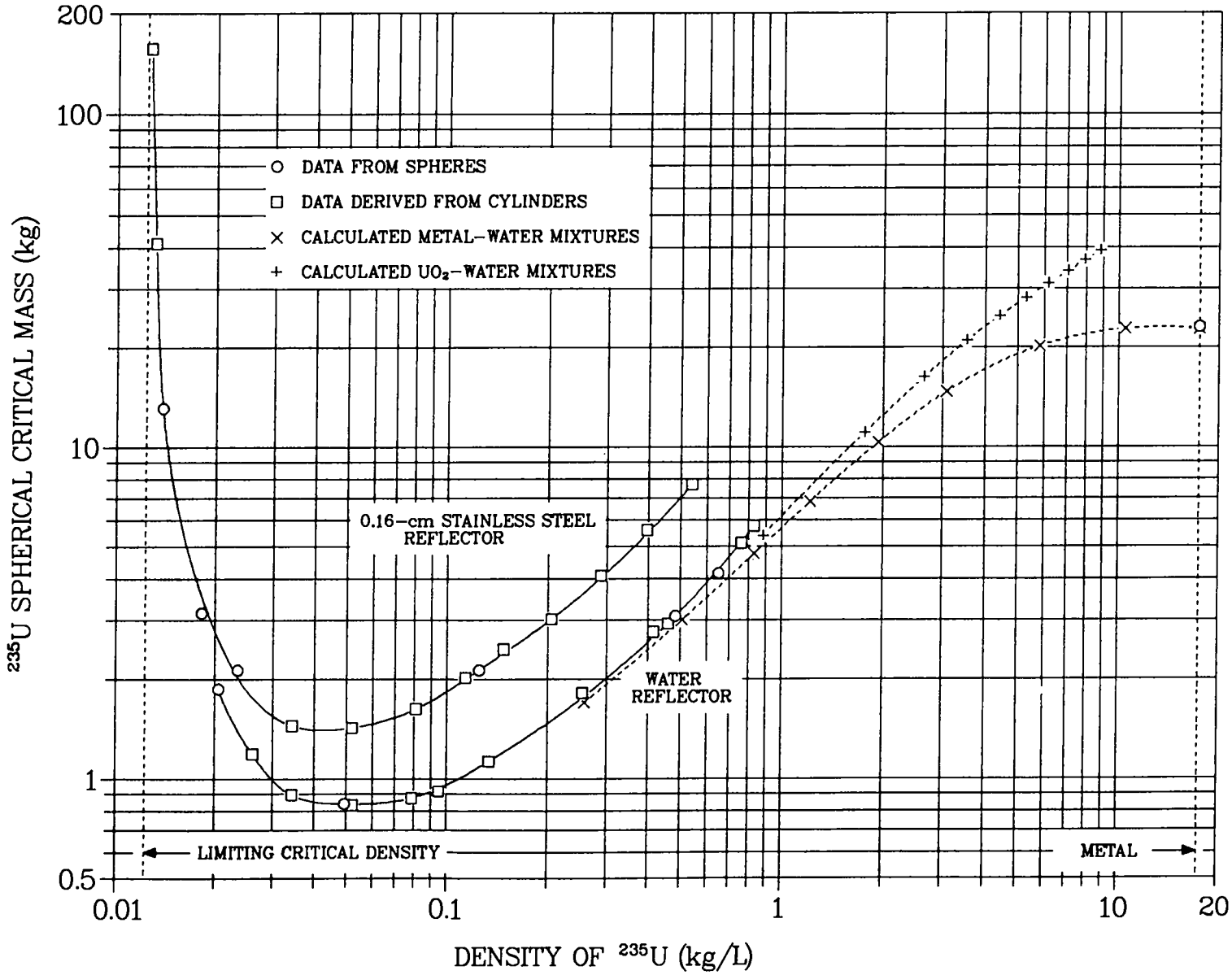


Fig. 10. Critical masses of homogeneous water-moderated U(93.2) spheres. Solution data appear unless indicated otherwise.

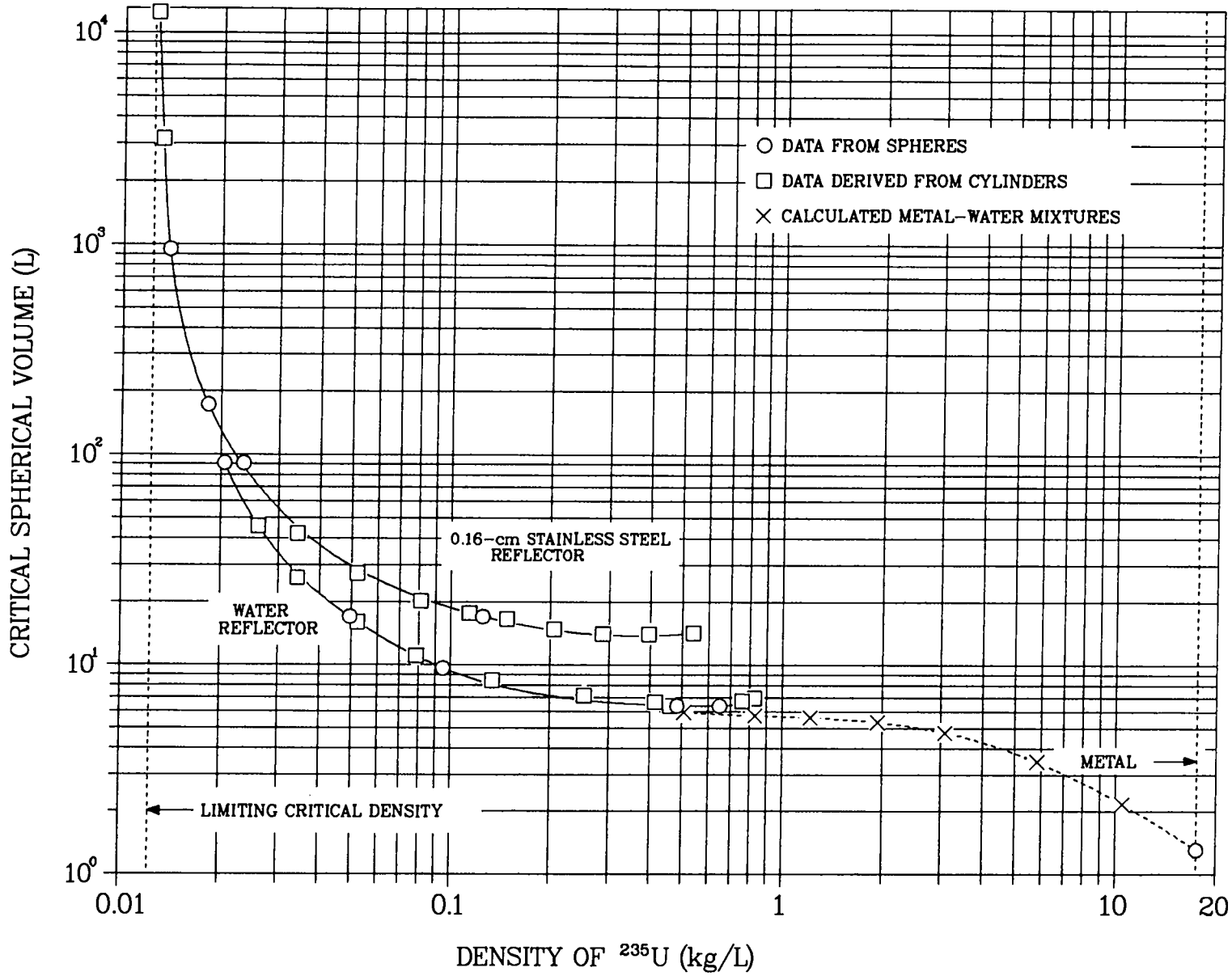


Fig. 11. Critical volumes of homogeneous water-moderated U(93.2) spheres. Solution data appear unless indicated otherwise.

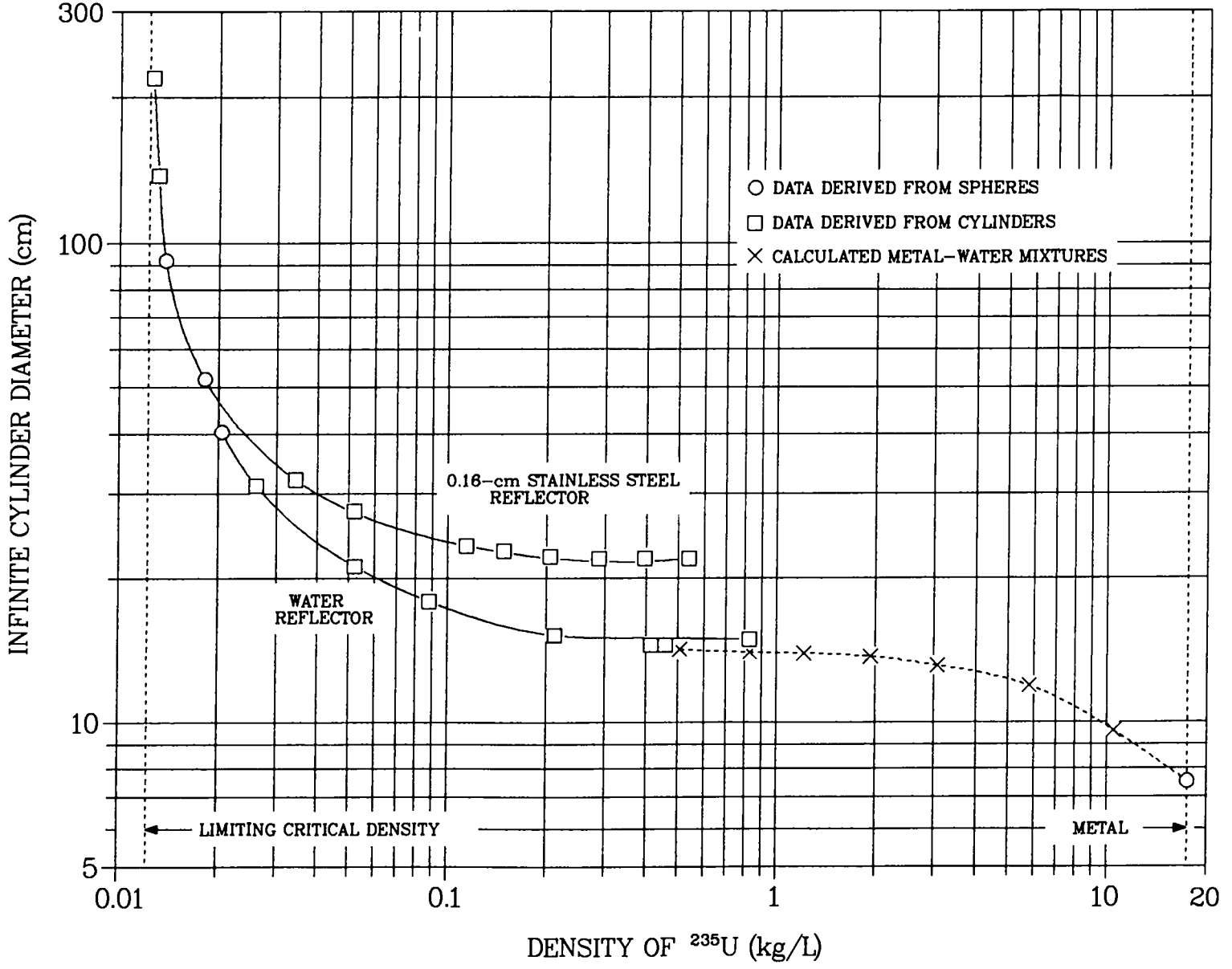


Fig. 12. Estimated critical diameters of infinitely long cylinders of homogeneous water-moderated U(93.2). Solution data appear unless indicated otherwise.

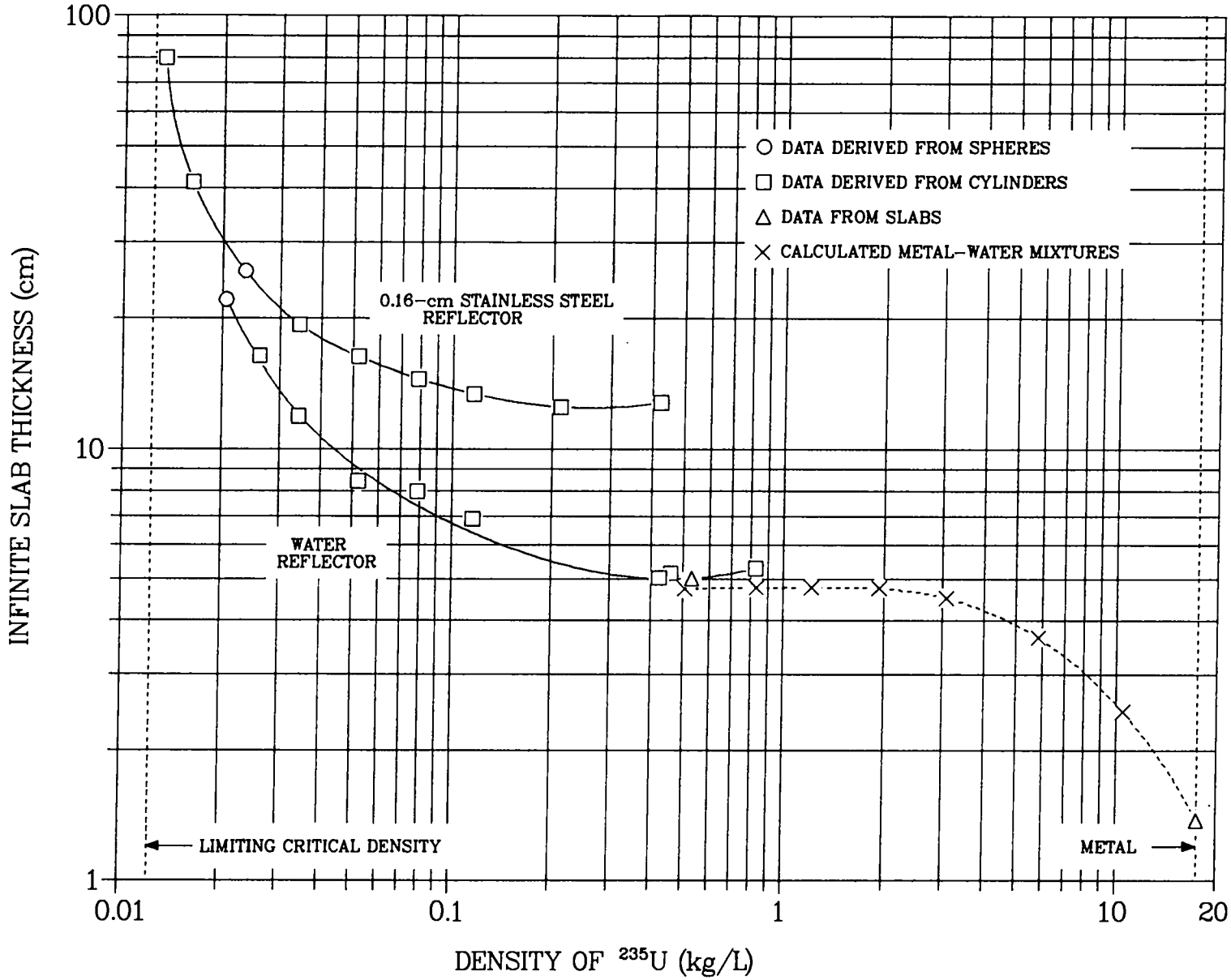


Fig. 13. Estimated critical thicknesses of slabs, infinite in other dimensions, of homogeneous water-moderated U(93.2). Solution data appear unless indicated otherwise. Unreflected infinite slabs are fictitious.

TABLE 7. DENSITY OF X VS. H/X ATOMIC RATIO
[X = ^{235}U as U(93.2), ^{233}U as ^{233}U(98.7 wt%), or Pu].

	^{235}U Density(g/cm^3)		^{233}U Density(g/cm^3)		Pu Density(g/cm^3)		
H/X	Metal-H_2O	Solution[a]	Metal-H_2O	Solution[a]	αPu-H_2O	δPu-H_2O	Solution[b]
0	17.53		18.28		19.6	15.65	
1	10.48		10.71		11.27	9.85	
2	7.48		7.57		7.91	7.18	
3	5.81		5.86		6.09	5.66	
5	4.02		4.03		4.18	3.96	
10	2.27		2.27		2.34	2.27	
20	1.21	1.06	1.21	1.07	1.24	1.22	
30	0.83	0.76	0.82	0.76	0.85	0.84	
50	0.51	0.48	0.50	0.48	0.52	0.51	(0.429)[c]
100	0.257	0.252	0.255	0.250	0.263	0.261	0.234
200	0.129	0.128	0.128	0.127	0.132		0.122
300	0.086	0.086	0.086	0.085	0.088		0.083
500		0.052		0.051			0.050
1000		0.0260		0.0258			0.0254
1500		0.0175		0.0179			0.0170
2000		0.0132		0.0134			0.028
3000							0.0085

[a] UO_2F_2 solution.
[b] $Pu(NO_3)_4$ solution with 1 N HNO_3, Pu contains 3% ^{240}Pu. Water densities from the relations on page 69, Ref. 4.
[c] Solution probably unstable.

Sources of experimental data and the nature of conversions to the conditions of Figs. 10 to 13 are as follows. The portions of those figures for the $H/^{235}U$ range greater than 20 are based on many critical-solution measurements of a variety of cylinders,[32,33] some spheres or cubes,[22,48-51] and a slab.[34]

The value of thickness of a critical infinite solution slab reported in Ref. 34 was obtained by extrapolating the reciprocal critical height of vertical finite slabs to zero. Alternatively, the infinite slab thickness may be obtained from the least subcritical finite slab by means of Table 5 and Fig. 6. The result at $H/^{235}U = 44.7$, 5.0 cm (including Plexiglas correction of 0.25 cm), compares with 4.5 cm reported in Ref. 34. There was a similar reexamination of the critical infinite slab thickness reported for Plexiglas-reflected U(93) metal.[35] The result, 1.38 cm, compares with the reported value of 1.52 cm (0.6 in.).

The most nearly equilateral critical cylinders are generally selected for conversion to spheres, elongated for infinite cylinders, and squat for infinite slabs. Conversions to the required shape make use of Fig. 6 and Table 5. Extrapolation of critical solutions concentration data to zero buckling gave 12.30 $\pm$ 0.10g of $^{235}U/L$ as the limiting critical concentration,[51] and 12.05 $\pm$ 0.03 resulted from measurements at the Hanford Physical Constants Testing Reactor (PCTR).[52]

Although they do not apply directly to the curves, critical data for slightly moderated solids are available for checking calculated points, for example, cores of effective composition $U(93.15)H_{2.97}\ C_{1.11}O_{.25}$ were reflected by natural uranium or iron.[45] More nearly appropriate is the critical mass of paraffin-reflected $U(95.3)F_6C$ mixed with polyethylene, of $H/^{235}U = 10$ (Ref. 22). The enriched-uranium-metal points were based on Los Alamos values[20,53,54] and ORNL slab data[55] (with DSN correction from Plexiglas to water reflector), supported by measurements at Lawrence Livermore National Laboratory (LLNL).[56,57] Shape conversions for the metal made use of extrapolation distances from Fig. 7. Core-density corrections for water-reflected spheres were made by using the computed relations of Fig. 9 and, for metal, the experimental points of Fig. 8. In regions of scanty or uncertain data, the curves of Figs. 10 to 13 are guided in form by results of DSN calculations.

Points of Fig. 10 for water-reflected $U(93)O_2$-H_2O (Table 8) mixtures are those reported by Magnuson of Oak Ridge.[58] They result from calculations validated by comparison with dense critical arrays of UO_2-$C_5O_2H_8$ units reflected by polyethylene and containing internal methyl methacrylate.

TABLE 8. CALCULATED CRITICAL PARAMETERS FOR WATER REFLECTED $U(93.3)O_2$-H_2O MIXTURES

Uranium Oxide Fraction	H_2O Fraction	$H/^{235}U$ Atom Ratio	^{235}U Density (g/cm^3)	Sphere Radius (cm)	Sphere ^{235}U Mass (kg)	Slab Thickness (cm)	Cylinder Radius (cm)
1.	0.0	0.0	8.817	10.19	39.08	3.376	6.22
0.9	0.1	0.328	7.936	10.33	36.64	3.502	6.32
0.8	0.2	0.738	7.054	10.47	33.91	3.638	6.43
0.7	0.3	1.266	6.172	10.64	31.14	3.780	6.55
0.6	0.4	1.969	5.290	10.83	28.15	3.942	6.69
0.5	0.5	2.954	4.409	11.04	24.85	4.114	6.84
0.4	0.6	4.431	3.527	11.23	20.92	4.292	6.97
0.3	0.7	6.892	2.645	11.36	16.24	4.486	7.08
0.2	0.8	11.82	1.763	11.44	11.06	4.640	7.15
0.1	0.9	26.58	0.882	11.34	5.39	4.842	7.18

Outside ORNL, there have been limited critical experiments with highly enriched uranium solutions. Results do not appear in Figs. 10-13 because they are less easily interpreted than the ORNL data. The ALECTO series in France was primarily a comparision of ^{235}U, ^{239}Pu, and ^{233}U critical solutions in similar geometries not necessarily as clean as possible.[59] $U(90)O_2(NO_3)_2$ solutions ranging from about 30-300 g $^{235}U/L$ were critical in 25- or 30-cm-diam cylinders with partial water or paraffin reflector and 30- or 42-cm-diam cylinders without reflector. Further French experiments directed toward excursions studies yielded critical masses of $U(93)O_2(NO_3)_2$ solution ranging from 22 to 380 g U/L, in 30- and 80-cm-diam cylinders.[60]

Critical experiments in the USSR used $UO_2(NO_3)_2$ solutions with ^{235}U enrichments of 90%, 10%, and 5%. The U(90) solutions were parallelepipeds, without reflector, and with partial water and water-steel reflectors.[61] The range of concentrations was similar to that of ALECTO.

Two critical unreflected spheres of $U(92.14)O_2F_2$ solution have been reported informally from the UK.* Results, for which there is no elaboration, appear in Table 9.

* John G. Walford and J. C. Smith, Dounreay Experimental Reactor Establishment, Dounreay, United Kingdom Atomic Energy Authority, 1963.

URANIUM OF VARIOUS ENRICHMENTS

Tables 9 and 10 give critical data for homogeneous hydrogen-moderated units of uranium enriched in ^{235}U to various degrees. In addition to UO_2F_2 solutions, there are solid uranium-bearing mixtures in which the hydrogenous material is polyethylene, paraffin, or Sterotex (glycerol tristearate, $[(C_{17}H_{35}COO)_3C_3H_5]$. In the tables there has been no attempt to convert to a common composition. In Table 9, critical values of one-dimensional forms (spheres, infinite cylinders and infinite slabs) are obtained from Oak Ridge reports of critical experiments. They are derived from quoted values of buckling, extrapolation distance and reflector savings. Results of other experiments at various enrichments appear in Table 10.

Reports of U(4.89), U(3.00) and U(2.00) systems give values of buckling, extrapolation distance, and reflector saving, from which spherical equivalents are derived (as well as equivalent infinite cylinders and slabs). Otherwise, listed spherical equivalents are as reported with the original data or as they appear in Ref. 14. The entries for U(95.3) and U(29.8) are from the only heterogeneous systems for which there is sufficient experimental information to permit correction to homogeneous compositions.

Critical masses and critical volumes of hydrogen-moderated spheres of U(93), U(30.3), U(4.89), U(3.00), and U(2.00) are displayed in Figs. 14 and 15. Values for U(3.00) and U(2.00) include those listed in Table 9 for $U(3.00)F_4$-paraffin and $U(2.00)F_4$-paraffin compacts. Dashed curves that extend beyond experimental ranges follow points computed by the MCNP Monte Carlo code[15] and the associated cross section set.* Critical data for uranyl fluoride-water mixtures at the four smaller enrichments are reproduced by this means within 0.01 k_{eff} and the method is used for conversion to U(2.00) and U(3.00) solutions.** It may be noted that the use of Hansen-Roach cross sections leads to 3% to 4% overestimates of $U(2.00)F_4$-paraffin critical sphere radii.[14]

Figures 16 and 17 give estimated infinite cylinder diameters and infinite slab thicknesses for U(93), U(30.3), U(4.89), U(3.00), and U(2.00). Values for the latter four ^{235}U enrichments were obtained from sphere radii by means of extrapolation distances consistent with the sphere radii, infinite cylinder diameters, and infinite slab thicknesses tabulated in Ref. 14.

* Robert C. Little, Los Alamos National Laboratory, Los Alamos, NM 87545, 1986.

** N. L. Pruvost, Los Alamos National Laboratory, Los Alamos, NM 87545 (1986).

TABLE 9. CRITIAL SPECIFICATIONS OF ONE-DIMENSIONAL FORMS OF URANIUM AT SEVERAL ENRICHMENTS

$H/^{235}U$ Atomic Ratio	^{235}U Density (g/cm^3)	Sphere Volume (L)	Infinite Cylinder Diameter (cm)	Infinite Slab Thickness (cm)
		$U(4.89)_3O_8$ - Sterotex, Unreflected[62]		
102	0.094	271	60.2	38.9
124	0.089	208	55.2	35.6
147	0.083	194	53.9	34.7
172	0.070	176	52.2	33.7
199	0.065	164	51.0	32.9
245	0.056	152	49.7	32.0
320	0.048	136	48.0	30.9
396	0.040	135	47.9	30.9
449	0.037	140	48.7	31.3
503	0.034	152	49.8	32.2
757	0.022	273	60.8	39.3
		$U(4.89)_3O_8$ - Sterotex, Water Reflected[62]		
102	0.094	152	46.3	23.4
124	0.089	112	41.5	20.6
147	0.083	105	40.5	20.3
172	0.070	95	39.2	19.6
199	0.065	91	38.3	19.1
245	0.056	83	37.7	19.0
320	0.048	77	36.9	18.9
396	0.040	80	37.5	19.7
449	0.037	85	38.7	20.6
503	0.034	95	40.3	22.0
757	0.022	195	52.2	30.0
		$U(4.89)O_2F_2$ Solution, Aluminum Container, Unreflected[62]		
524	0.0425	69	38.6	24.7
643	0.0356	80	40.6	26.0
735	0.0318	94	42.8	27.5

TABLE 9. (cont.)

$H/^{235}U$ Atomic Ratio	^{235}U Density (g/cm^3)	Sphere Volume (L)	Infinite Cylinder Diameter (cm)	Infinite Slab Thickness (cm)
		$U(4.89)O_2F_2$ Solution, Aluminum Container, Water Reflected[62]		
524	0.0425	45	31.8	17.9
643	0.0356	53	33.8	19.3
735	0.0318	65	36.3	20.9
1000	0.0240	132	46.7	280
		$U(3.00)F_4$ - Paraffin, Unreflected[63]		
133	0.093	208	55.3	34.6
277	0.066	100	43.0	26.5
		$U(3.00)F_4$ - Paraffin, Plexiglas-Paraffin or Plexiglas-Polyethylene Reflected[63]		
133	0.093	124	43.6	22.9
277	0.066	60	34.1	17.6
		$U(2.00)F_4$ - Paraffin, Unreflected[63]		
195	0.0627	379	67.6	42.4
294	0.0528	239	57.8	36.1
406	0.0444	202	54.6	34.0
496	0.0394	201	54.7	34.2
614	0.0345	224	56.6	35.5
972	0.0245	513	74.8	47.1
		$U(2.00)F_4$ - Paraffin, Plexiglas-Paraffin or Plexiglas-Polyethylene Reflected[63]		
195	0.0627	257	56.6	31.5
294	0.0528	161	48.3	26.5
406	0.0444	139	46.0	25.4
496	0.0394	142	46.6	26.2
614	0.0345	163	49.0	27.8
972	0.0245	413	67.9	40.2

TABLE 10. CRITICAL MASSES AND VOLUMES OF HOMOGENEOUS HYDROGEN-MODERATED SPHERES, HEMISPHERES, AND CUBES OF URANIUM AT SEVERAL ENRICHMENTS

$H/^{235}U$ Atomic Ratio	^{235}U Density (g/cm^3)	Reflected[a] Volume (L)	Reflected[a] Mass (kg ^{235}U)	Unreflected Volume (L)	Unreflected Mass (kg ^{235}U)
		$U(95.3)F_4CF_2$ - Polyethylene Cube[22]			
		Paraffin Reflected			
10	1.48	10.2	15.1	—	—
		$U(44.6)O_2F_2$, Water Solution, Spheres[b]			
365	0.0705	—	—	22.27	1.570
635	0.0407	—	—	34.91	1.421
		$U(44.6)O_2F_2$, Water Solution, Sphere Dimensions Transformed from Cylinders[b]			
258	0.0974	11.5	1.12	17.6	1.71
493	0.0518	19.5	1.01		
678	0.0379	29.5	1.12		
		$U(37.5)F_4$-CF_2 Cube[62]			
0.1	1.18	156	184	~408	~482
		$U(30.45)O_2F_2$, Water Solution, Spheres[b]			
76	0.288	—	—	22.27	6.40
218	0.1130	—	—	22.27	2.517
351	0.0716	14.85	1.063	—	—
534	0.0478	—	—	34.91	1.667
573	0.0445	22.27	0.991	—	—
783	0.0328	34.91	1.143	—	—
1037	0.0248	—	—	91.34	2.265
1193	0.0216	91.34	1.973	—	—
277	0.0900	17.44	1.571	—	—
488	0.0520	—	—	45.67	2.375

TABLE 10. (cont.)

$H/^{235}U$ Atomic Ratio	^{235}U Density (g/cm^3)	Reflected[a] Volume (L)	Reflected[a] Mass (kg ^{235}U)	Unreflected Volume (L)	Unreflected Mass (kg ^{235}U)
		$U(30.45)O_2F_2$, Water Solution, Hemispheres,[a]			
728	0.0352	45.67	1.608	—	—
		$U(30.3)O_2F_2$, Water Solution, Sphere Dimensions Transformed from Cylinders[64]			
76.7	0.288	11.3	3.26	19.5	5.62
106	0.220	11.6	2.54	20.0	4.38
167	0.146	11.6	1.70	20.0	2.93
257	0.0978	13.0	1.28	22.1	2.16
378	0.0675	16.1	1.08	26.3	1.77
439	0.0584	17.1	1.00	27.7	1.62
657	0.0394	27.8	1.10	42.1	1.66
815	0.0317	38.1	1.24	55.5	1.76
		$U(30.14)O_2$-CH_2 Compacts, Cube Dimensions Transformed from Parallelepipeds[43]			
		Perspex Reflected			
8.14	1.570	13.9	21.8	31±2	49±3
8.14	1.190[c]	27.6	32.9	—	—
16.3	1.130	9.37	10.59	26.5	30.0
16.3	0.845[c]	19.37	16.40	—	—
39.2	0.668	6.66	4.45	18.40	12.29
81.3	0.332	5.83	1.94	15.34	5.09
81.3	0.248	12.1	3.00[d]	38.3[d]	9.51[d]
81.4	0.244[e]	9.58	2.38[d]	25.6[d]	6.24[d]
		Polyethylene Reflected			
8.14	1.190	32.4	38.6	—	—
16.3	1.130	10.21	11.54	26.5	30.0
16.3	0.845	22.3	18.9	—	—
39.2	0.668	7.21	4.82	18.40	12.29
81.3	0.332	6.32	2.10	15.34	5.09
81.3	0.248[b]	13.7	3.39	38.3[c]	9.51[c]
81.4	0.244[d]	10.58	2.58	25.6[c]	6.24[c]

TABLE 10. (cont.)

$H/^{235}U$ Atomic Ratio	^{235}U Density (g/cm^3)	Reflected[a] Volume (L)	Reflected[a] Mass (kg ^{235}U)	Unreflected Volume (L)	Unreflected Mass (kg ^{235}U)
		$U(29.83)F_4CF_2$-Polyethylene Cube,[23] Paraffin Reflected			
32	0.542	2.66	7.45		
		$U(18.8)F_4$-CF_2 Cube, heterogeneous[62]			
0.14	0.591	733	433		
		$U(14.7)O_2SO_4$, Water Solution, Sphere[65]			
	~0.081	14.8	~1.2		
		$U(4.98)O_2F_2$, Water Solution, Sphere[66,67,f]			
490	45.3	44.4	2.01	68.5	3.11
		$U(1.42)F_4$-Paraffin Compacts, Transformed to Sphere[14,b]			
		Water Reflected			
418	0.0353	697	24.6	847	29.9
562	0.0305	728	22.2	873	26.6
		Polyethylene Reflected			
418	0.0353	665	23.5	—	—
562	0.0305	717	21.9	—	—

[a] Water reflected unless indicated otherwise.
[b] John G. Walford and J.C. Smith, Dounreay Experimental Reactor Establishment, Dounreay, United Kingdom Atomic Energy Authority, 1963.
[c] J. R. Dominey and A. F. Thomas, Atomic Weapons Research Establishment, Aldermaston, United Kingdom Atomic Energy Authority, 1962.
[d] Estimated from replacement measurements on one face.
[e] Extra graphite was added to change the effective composition of the wax to CH.
[f] See Fig. 18.

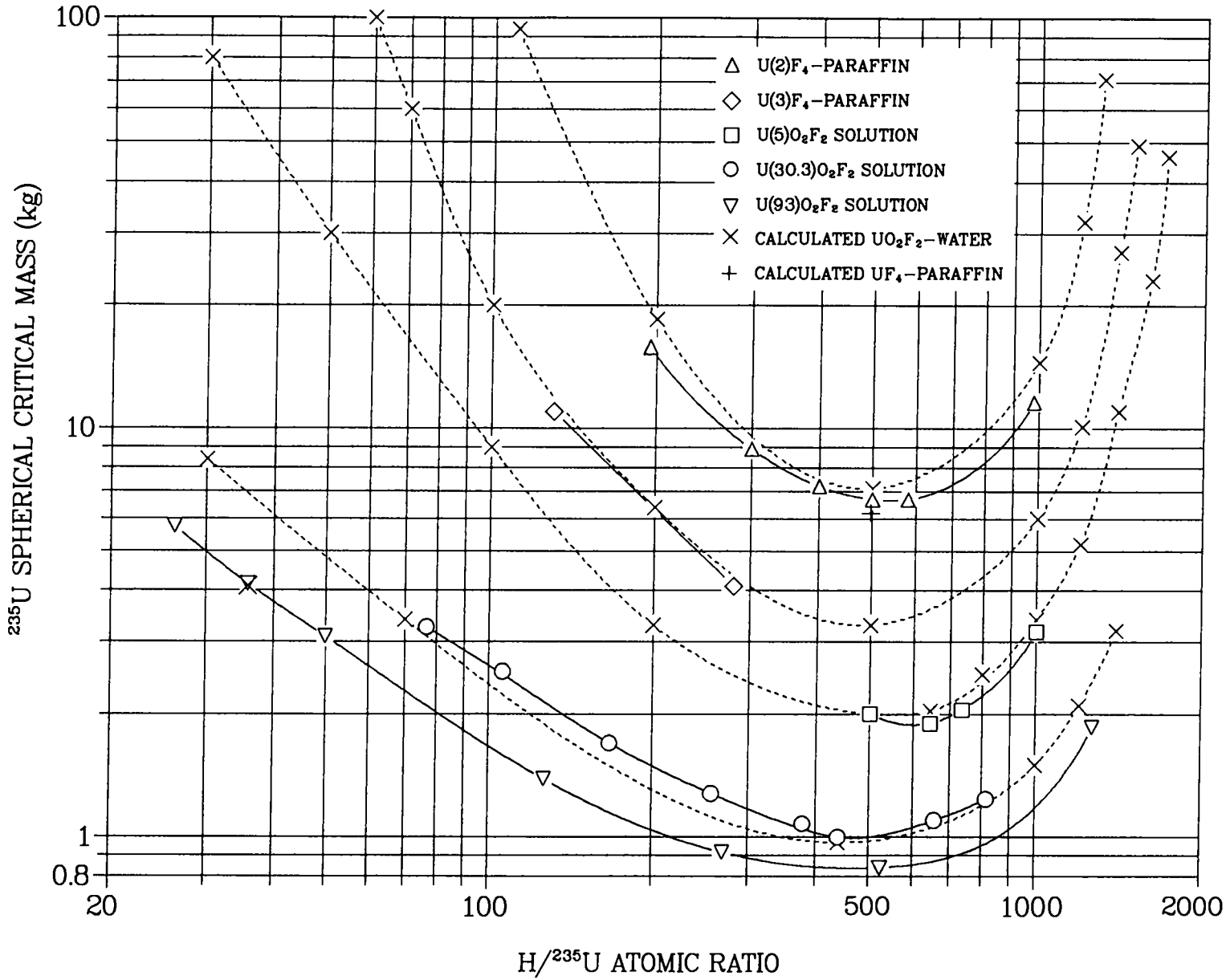


Fig. 14. Critical masses of water-reflected spheres of hydrogen-moderated U(93), U(30.3), U(5.00), U(3.00) and U(2.00).

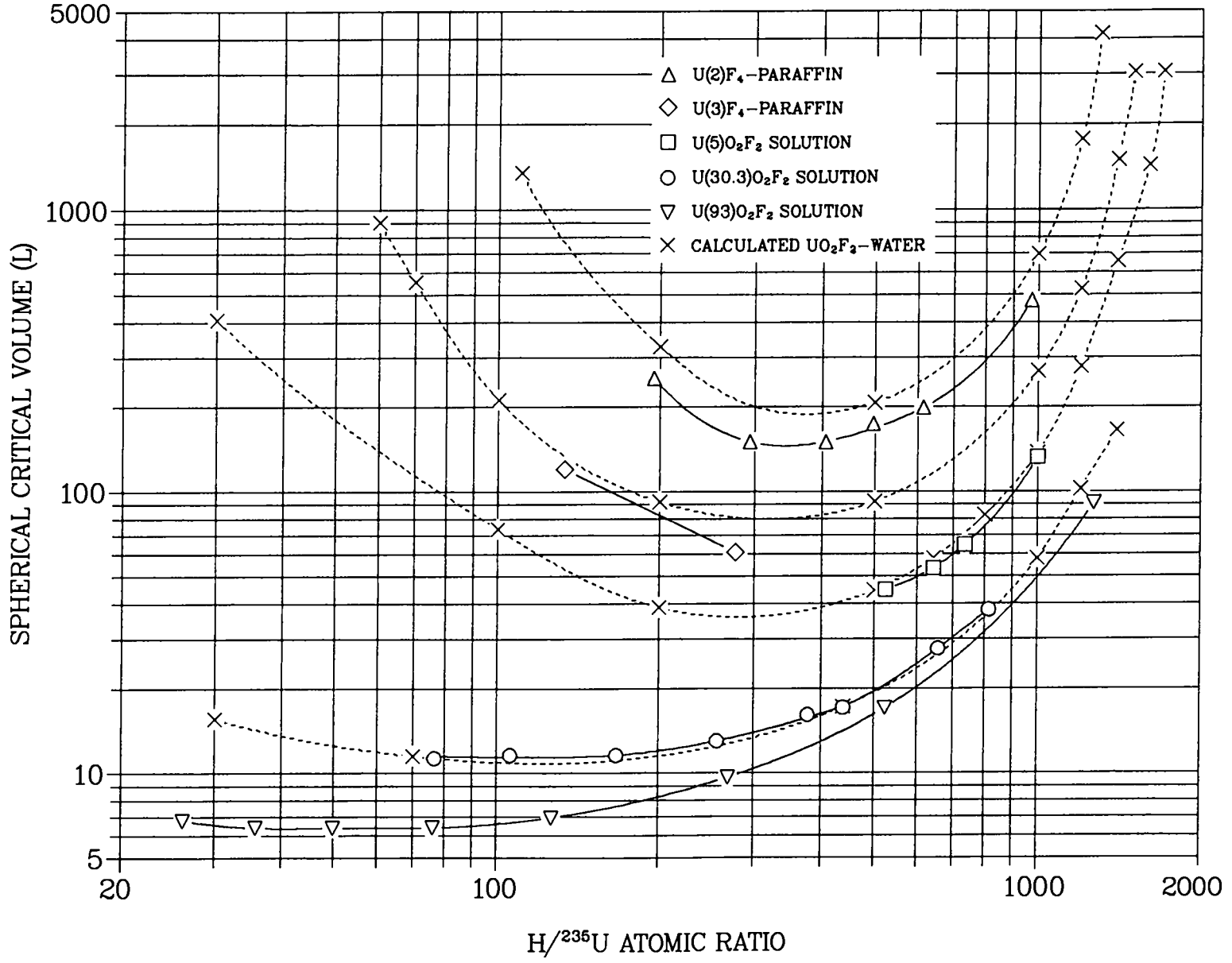


Fig. 15. Critical volumes of water-reflected spheres of hydrogen-moderated U(93), U(30.3) U(5.00), U(3.00) and U(2.00).

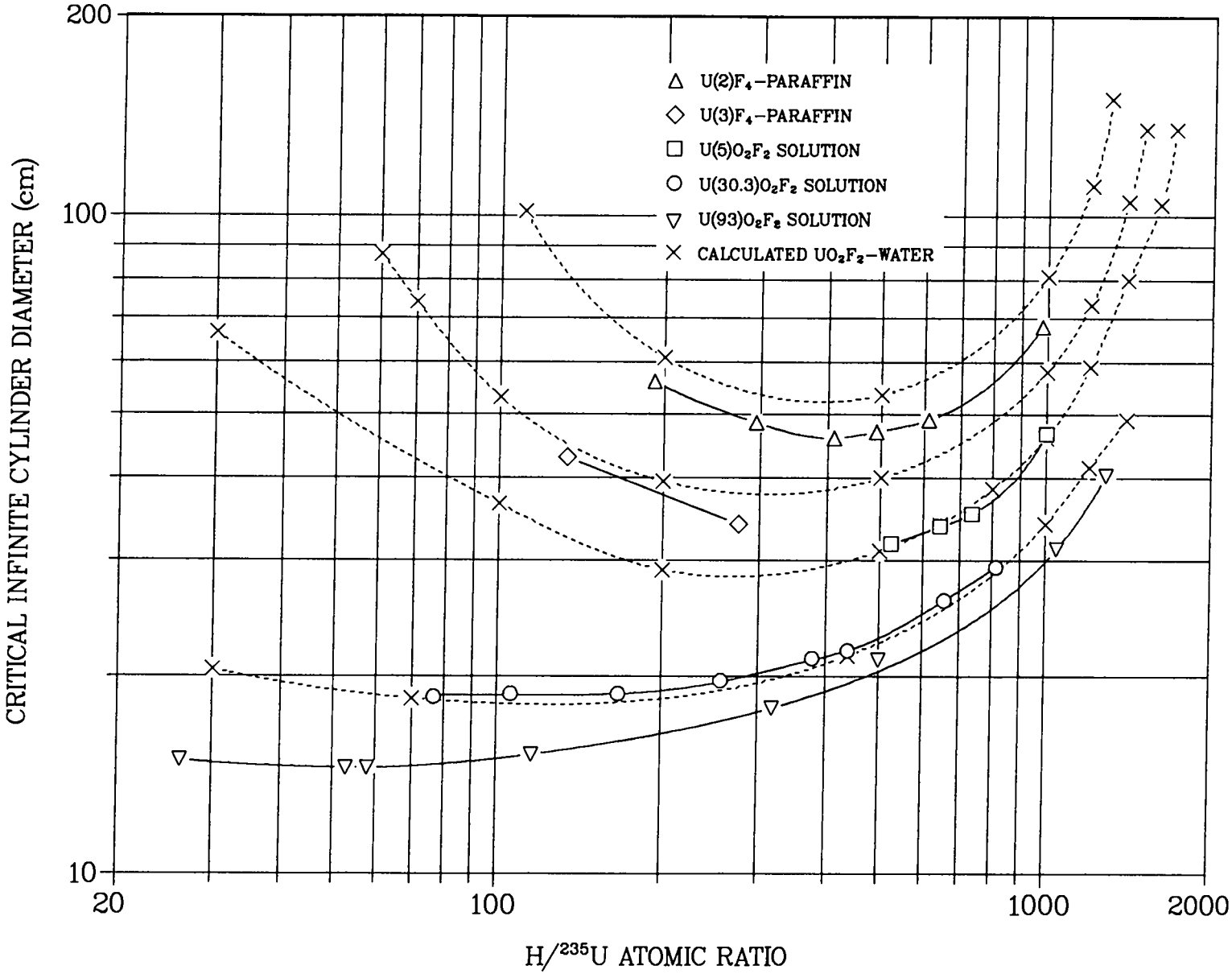


Fig. 16. Estimated critical diameters of infinitely long water-reflected cylinders of hydrogen-moderated U(93), U(30.3), U(5.00), U(3.00) and U(2.00).

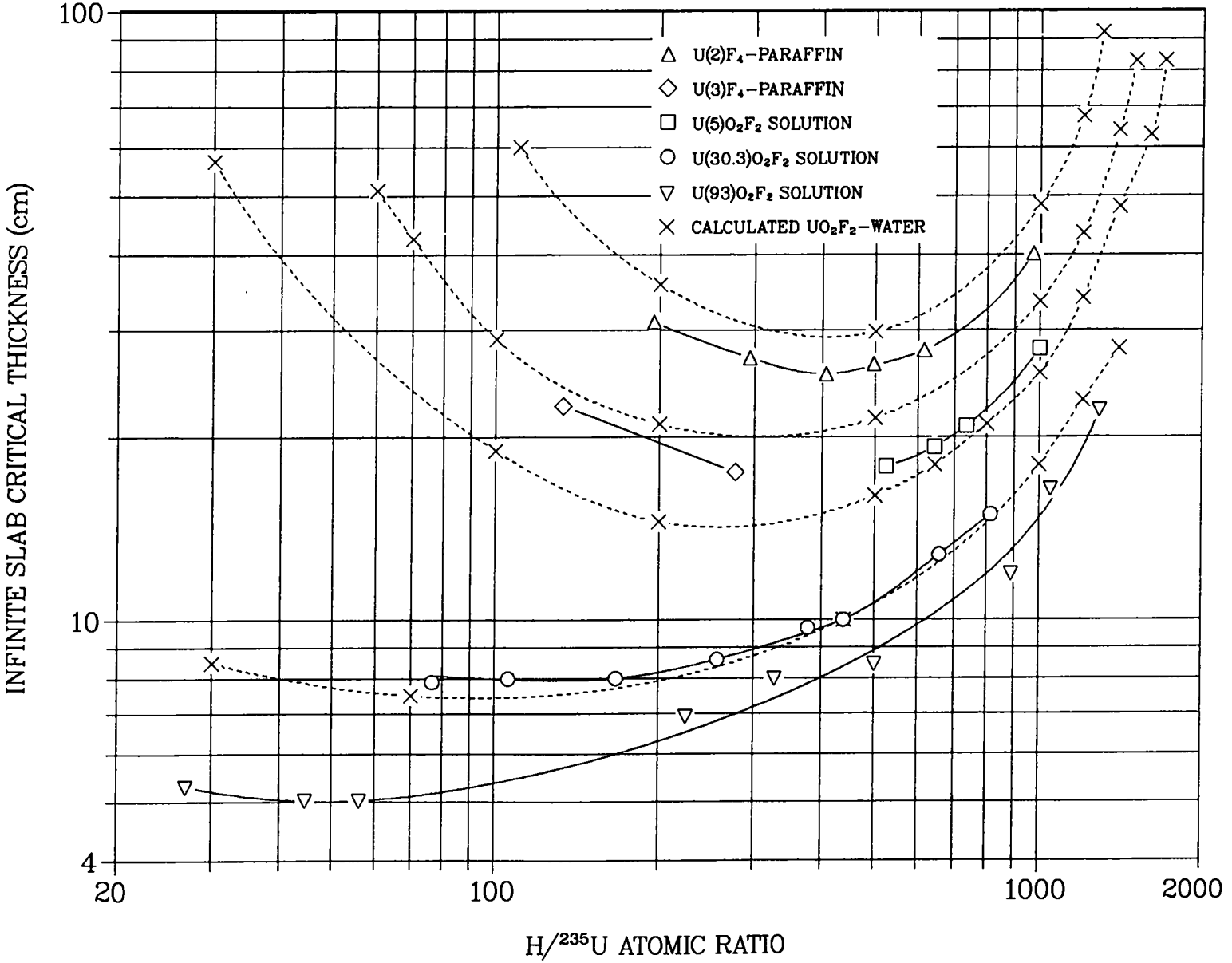


Fig. 17. Estimated critical thicknesses of water-reflected slabs, infinite in other dimensions, of hydrogen-moderated U(93), U(30.3), U(5.00), U(3.00) and U(2.00).

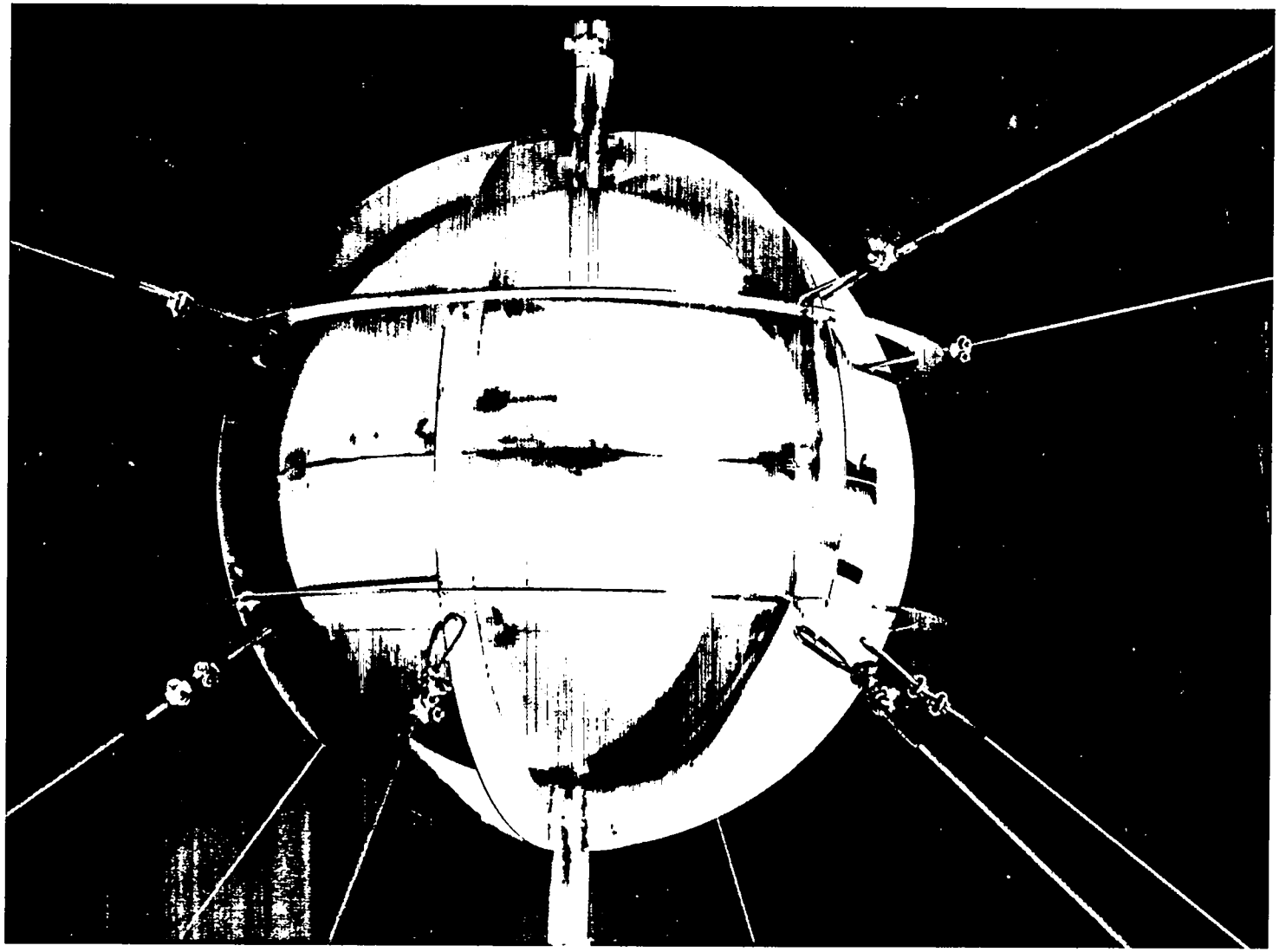


Fig. 18. Spherical container, suspended by stainless-steel cables, for establishing unreflected critical specifications of $U(5.00)O_2F_2$ solution.[66] *The stainless-steel-wall thickness was 0.05 cm, and the inside diameter was 50.77 ± 0.03 cm.*

Instead of being listed in Table 10, data from Los Alamos for unmoderated uranium metal at various enrichments appear in Fig. 19.[68-71] Because the curves for unreflected uranium and uranium with a natural uranium reflector appear to be parallel, it is assumed that the curve for water reflection would also be parallel as indicated in Fig. 19. Exponential experiments, which supplement critical experiments, indicate that unmoderated uranium cannot become critical if the ^{235}U content is below 5 or 6 wt%. A cooperative European reactor physics program has narrowed this limiting critical enrichment to 5.56 $\pm$ 0.02 at% ^{235}U, i.e. U(5.49).[72]

This ^{235}U enrichment limit applies to single undiluted uranium units. As shown by the following Oak Ridge critical data, it does not apply to clusters of metal units in water. For example, ten 442-kg U(1.95) metal annuli were critical as a triangular lattice with 2.54-cm optimum separation of annuli.[73,74] The dimensions of these units were 18.3-cm-o.d., 6.60-cm-i.d. and 101.6-cm-long. The critical mass, 86 kg ^{235}U compares with 54 kg ^{235}U in a similar 9-unit critical array of U(1.95) annuli with the outside diameter reduced to 15.7 cm and other dimensions the same.

Optimum critical lattices of massive U(3.85) annuli, and rods consisting of annuli with water-filled interiors, again were triangular with 2.5-cm separation of units. As examples, the interpolated critical mass of 332-kg annuli, 18.3-cm-o.d., 6.6-cm-i.d. and 76-cm-long, was 57-kg ^{235}U, and that of 380-kg rods of the same outside diameter and length was 90 kg ^{235}U.[73,74]

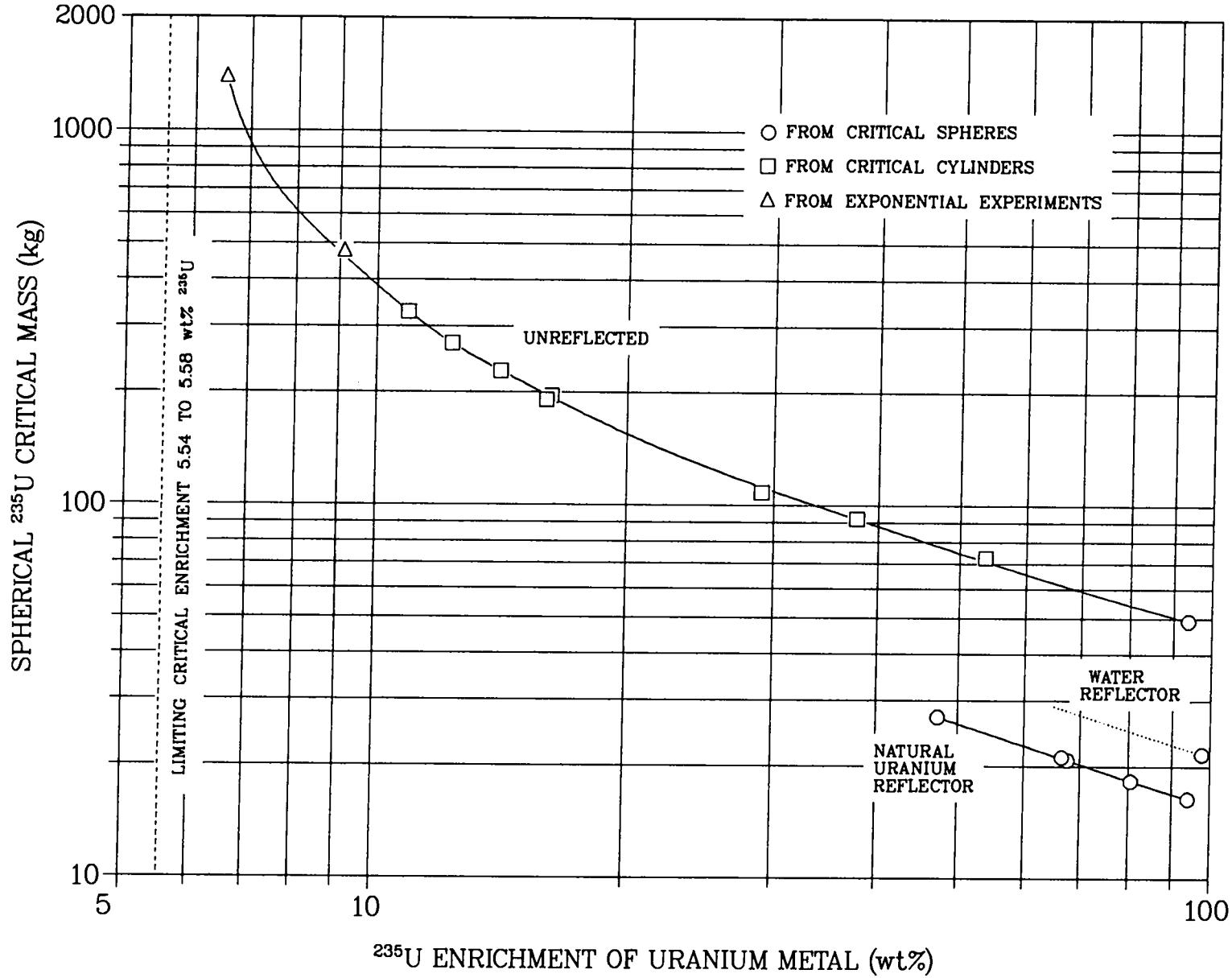


Fig. 19. Critical mass vs ^{235}U enrichment of uranium metal. The dashed line represents the enrichment below which a piece of uranium metal cannot become critical, as determined by an international reactor physics program.

HETEROGENEOUS WATER-MODERATED URANIUM AT VARIOUS ENRICHMENTS

NEAR-HOMOGENEOUS HYDROGEN-MODERATED URANIUM ENRICHED IN ^{235}U

Parallelepipeds and a pseudocylinder of U(93.16) metal foil (0.005- to 0.030-cm-thick) interleaved with various combinations of 0.16-cm-thick Plexiglas plates and 0.46-cm- or 0.71-cm-thick graphite plates were effectively homogeneous.[75] An aluminum matrix at a mean density of 0.165 g/cm^3 was distributed throughout the core and comprised the reflector. Critical conditions appear in Table 11. Volumes of equivalent spheres are quoted in the reference.

WATER-MODERATED LATTICES OF SLIGHTLY ENRICHED URANIUM

Early measurements on lattices of slightly enriched uranium metal or oxide rods in water were summarized by Kouts et al. at the Geneva Conferences of 1955 and 1957.[76,77] Experiments with exponential columns established values of buckling and extrapolation distance for a number of lattice spacings at each rod diameter. For each diameter of U(1.027), U(1.143), and U(1.299) metal rods, a spacing leading to the largest value of buckling, thus smallest value of critical volume, was spanned. Such volumes for equilateral cylindrical lattices are listed in Table 12. (Equilateral cylindrical geometry is chosen as more appropriate for arrays of rods than spherical geometry.) There are less extensive data for lattices of aluminum-clad U(1.3)O_2 rods,[77] U(3.95)O_2 rods clad with Inconel[78] and U(4.02)O_2 rods clad with stainless steel.[77]

TABLE 11. ASSEMBLIES OF U(93) WITH HYDROGEN AND CARBON MODERATORS[a]

^{235}U Density (g/cm^3)	Atomic Ratio		Dimensions (cm)	Critical Volume (L)	
	$H/^{235}U$	$C/^{235}U$		Observed	Sphere
2.303	6.0	3.76	59.7 x 30.5 x 23.5	42.8	25.8
2.096	6.0	3.74	59.7 x 30.5 x 25.7	50.8	33.3
0.917	6.0	24.3	9.7 x 45.7 x 45.7	125	100
0.521	6.0	48.5	59.7 x 61.0 x 69.9	254	208
1.317	12.1	7.6	38.1 x 30.5 x 28.9	33.6	27.6
0.258	12.3	98.7	59.7 x 72.4 x 72.9	315	255
0.258	12.4	98.2	61.3 x 62.9 x 62.6	320	253
0.258	12.4	98.2	61.3 long x 69.1 diam	305	263
0.480	35.1	21.9	38.1 x 340.5 x 30.5	35.4	29.3
0.336	35.2	48.2	38.1 x 38.1 x 42.4	61.5	51.5
0.223	35.5	99.4	38.1 x 53.3 x 53.3	108	85

[a] A complete computational survey of critical masses of U(93.5)-water-graphite spheres resulted in Fig. 7 of Ref. 14.

TABLE 12. LATTICES OF SLIGHTLY ENRICHED URANIUM METAL RODS IN WATER

Enrichment in ^{235}U	Rod Diameter (cm)	Average ^{235}U Density (g/cm^3)	Critical Volume of Equilateral Cylinder at Optimum Lattice Spacing (L)
U(1.027)	0.98	0.055	524
	1.52	0.06	430
	1.90	0.065	393
U(1.143)	0.98	0.055	274
	1.52	0.065	238
U(1.299)	0.98	0.06	175
	1.52	0.075	155
U(2.0)	1.52	0.095	58.2
	2.35	0.12	56.6
U(3.063)	0.445	0.09	32.0
	0.762	0.105	29.8
	1.52	0.15	30.1
	2.35	0.175	35.1

Subsequent exponential experiments at Hanford provided the further optimum critical volumes for U(2.0) and U(3.063) that appear in Table 12.[79] Savannah River personnel extended the U(3) data to 5.1-cm- and 7.6-cm-diam rods.[80] It is illuminating to plot the U(3.063) critical volumes of Table 12 and corresponding critical masses against rod diameter, as in Fig. 20. The minimum critical volume of 29.5 L occurs with a rod diameter of about 1.1 cm. The critical mass minimum is far below the smallest rod diameter shown; in fact, computations indicate that it should occur at a diameter of about 0.35 cm. The corresponding volume would be extremely large.

Oak Ridge reported critical lattices in water* of U(4.95)**metal rods at a range of diameters.[67,78] Spherical critical masses were calculated from experimental values of buckling and corresponding extrapolation distance. A minimum spherical critical mass of 1.6 kg ^{235}U at a rod diameter of about 0.17-cm compares with a minimum spherical critical mass of 2.0 kg ^{235}U for U(5.00) solution with water reflector.

Experiments at Valduc established critical heights of water in lattices of U(4.75)O_2 rods at various pitches.[81] Each rod contained 0.79-cm-diam by 90-cm-long UO_2 at a density of 10.4 g/cm^3 and was clad with 0.06-cm thick aluminum. Extrapolation to fully immersed and reflected conditions led to a critical square lattice of 217 rods at an optimum pitch of 2.1 cm.

* Boron concentration of 0.140 g/L increased critical masses about 25%.

** Corrected by the author from U(4.89) reported in Ref. 78.

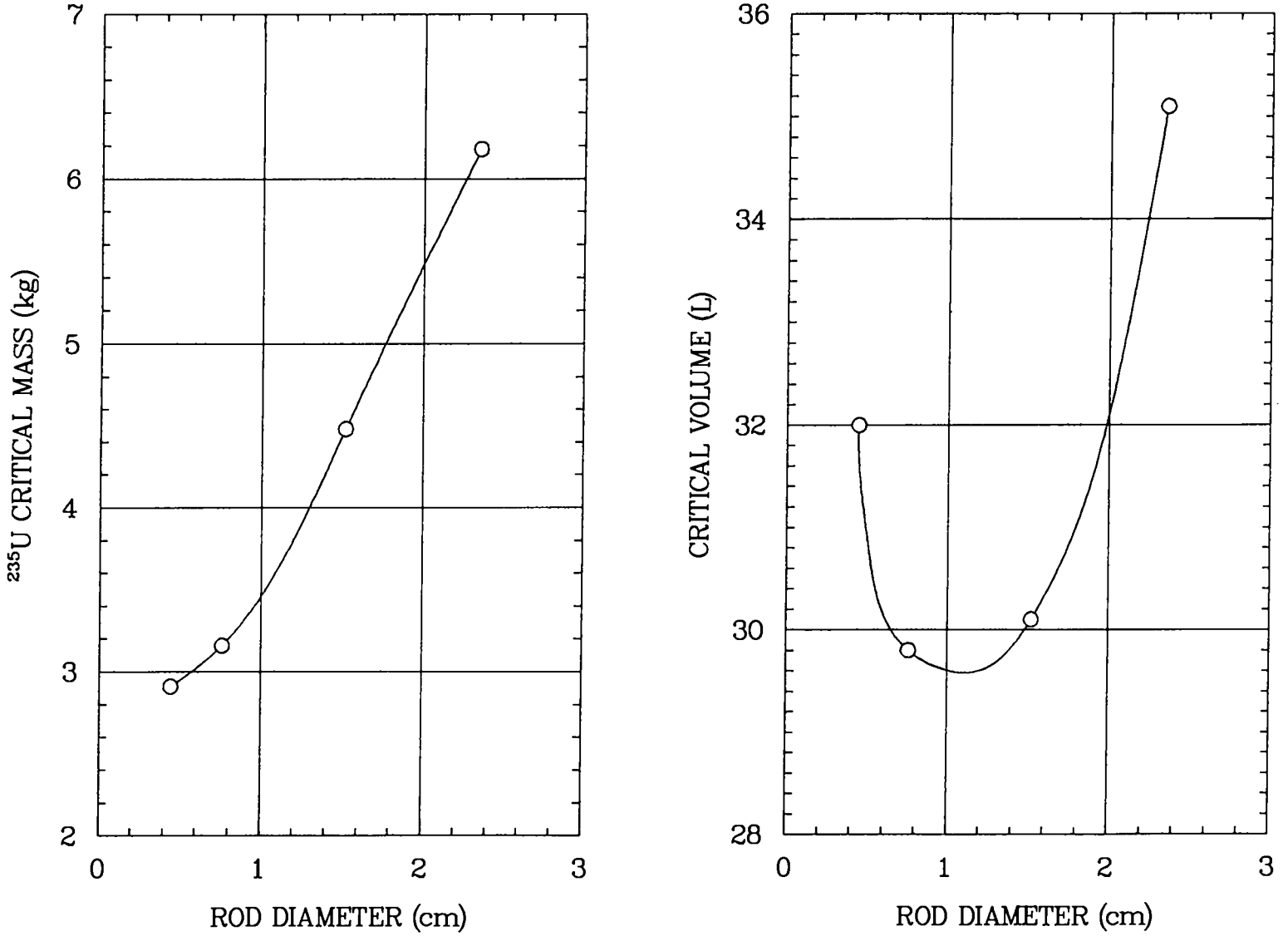


Fig. 20. Critical mass and volume for lattices of U(3.063) metal rods in water. At each rod diameter, the critical volume is that of an equilateral cylinder at optimum lattice spacing, and the critical mass corresponds.

WATER-SPACED UNITS OF ENRICHED URANIUM

Highly Enriched Uranium

Unlike slightly enriched uranium lattices, water lattices of uranium enriched to more than 10% ^{235}U have critical masses that are greater than the corresponding homogeneous systems. This is illustrated by results in Fig. 21 of Los Alamos measurements on lattices of U(93.5) metal as 2.54-cm cubes, 1.27-cm cubes and 0.318-cm-diam rods.[82] Surface spacing for minima in critical mass varies from 1.8 cm for the 2.54-cm cubes to 1.5 cm for the 0.318-cm rods.

A limited number of Oak Ridge experiments with MTR-type fuel elements provide guidance for pool storage and recovery operations.[83,84] These elements were roughly 7.6-cm-square in cross section and the fissile material, U(~93), was contained in a number of closely spaced clad plates with the active fuel length about 60 cm. The ^{235}U content was 140, 168 or 306 g per element. Measurements established, for each ^{235}U loading, the element separation in water required for optimum moderation and effects of greater separation on the critical number of elements.

As an example, 47 of the 306 g elements spaced about 4 cm apart were critical latticed in water. Further, 4 g $^{235}U/L$ as $U(93)O_2(NO)_2$ dissolved in the reflector-moderator reduced the critical mass of a compact lattice by about one-third, and 1.12 g B/L dissolved in that solution increased the critical mass by a factor of five. Influences of cadmium interspersed in various flooded storage patterns also were established.

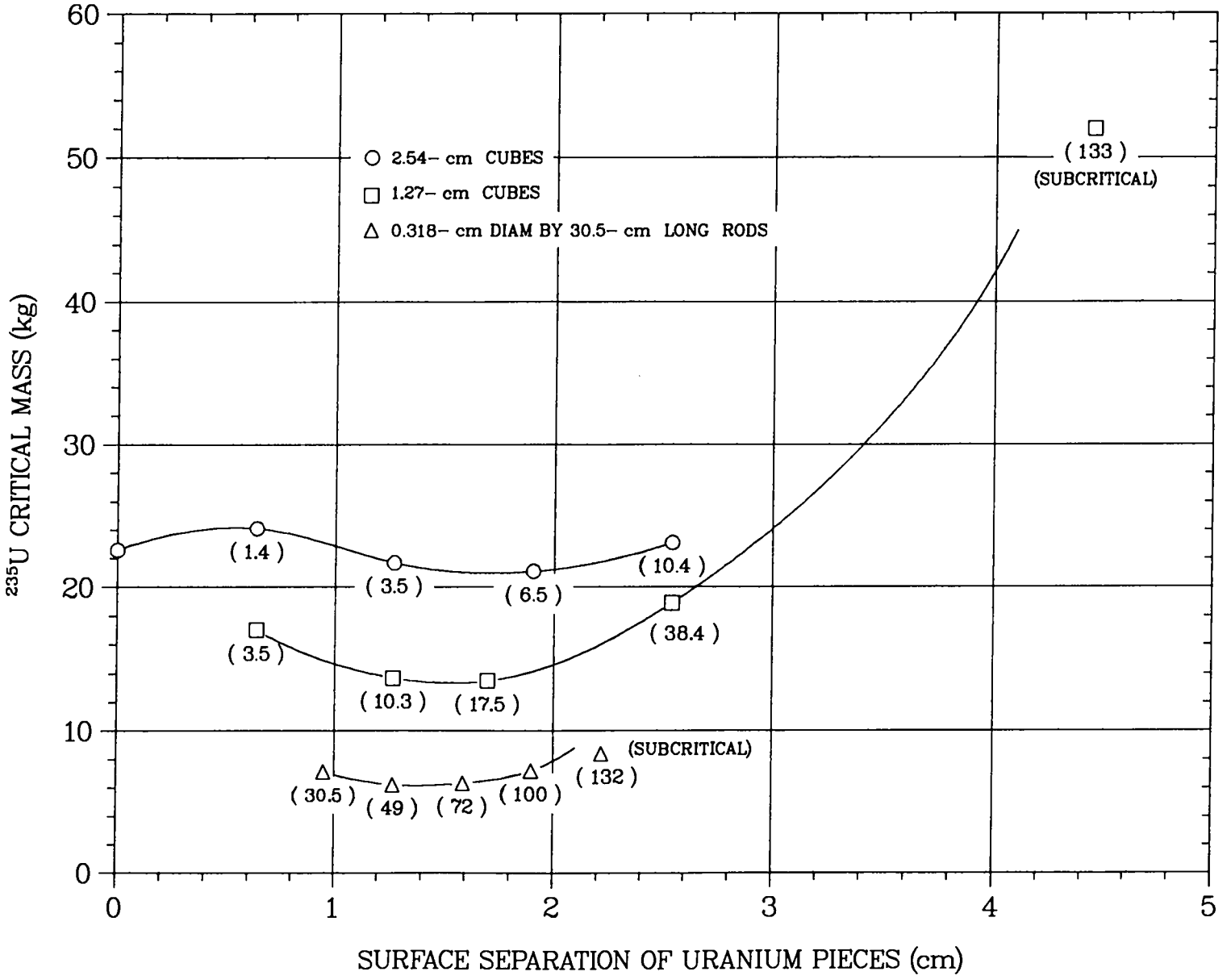


Fig. 21. Critical masses of water-immersed U(94) metal lattices as functions of spacing between units. Mean $H/^{235}U$ ratios are in parentheses.

Uranium of Various Enrichments

Over the range of ^{235}U enrichments, minimum critical masses of heterogeneous uranium in water compare with homogeneous values as shown in Fig. 22. Below about 10% ^{235}U enrichment, heterogeneous critical masses are smaller than corresponding homogeneous values. Heterogeneous minimum critical volumes (Fig. 23), diameters of infinite cylinders (Fig. 24), and thicknesses of finite slabs (Fig. 25), however, are less than corresponding homogeneous values throughout the entire enrichment range.

These figures, originally from Callihan, Ref. 9, have been modified to include the subsequent Oak Ridge experiments, in particular, the comparison at U($\sim$ 5) noted earlier. As a reminder, the minimum critical mass of U(4.95) metal rods latticed in water was 1.6 kg ^{235}U,[67,78] and that of water-reflected U(5.00) solution was 2.0 kg ^{235}U. Further, homogeneous values at 2.00 and 3.00 wt% enrichment are derived from ORNL experiments with U(3.00)F_4-paraffin and U(2.00)F_4-paraffin compacts.[63]

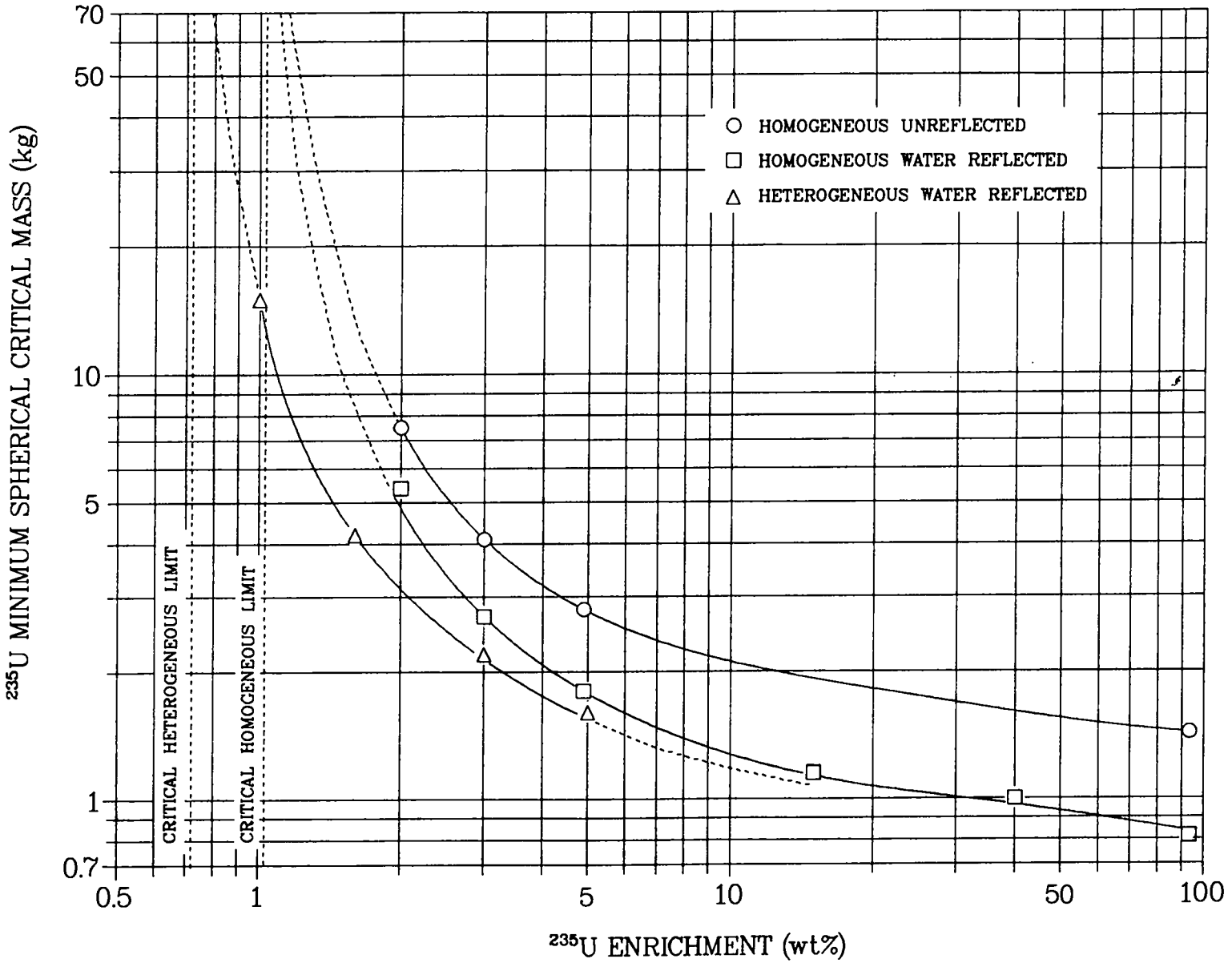


Fig. 22. Minimum spherical critical masses as functions of ^{235}U enrichment in homogeneous and heterogeneous hydrogen-moderated systems.

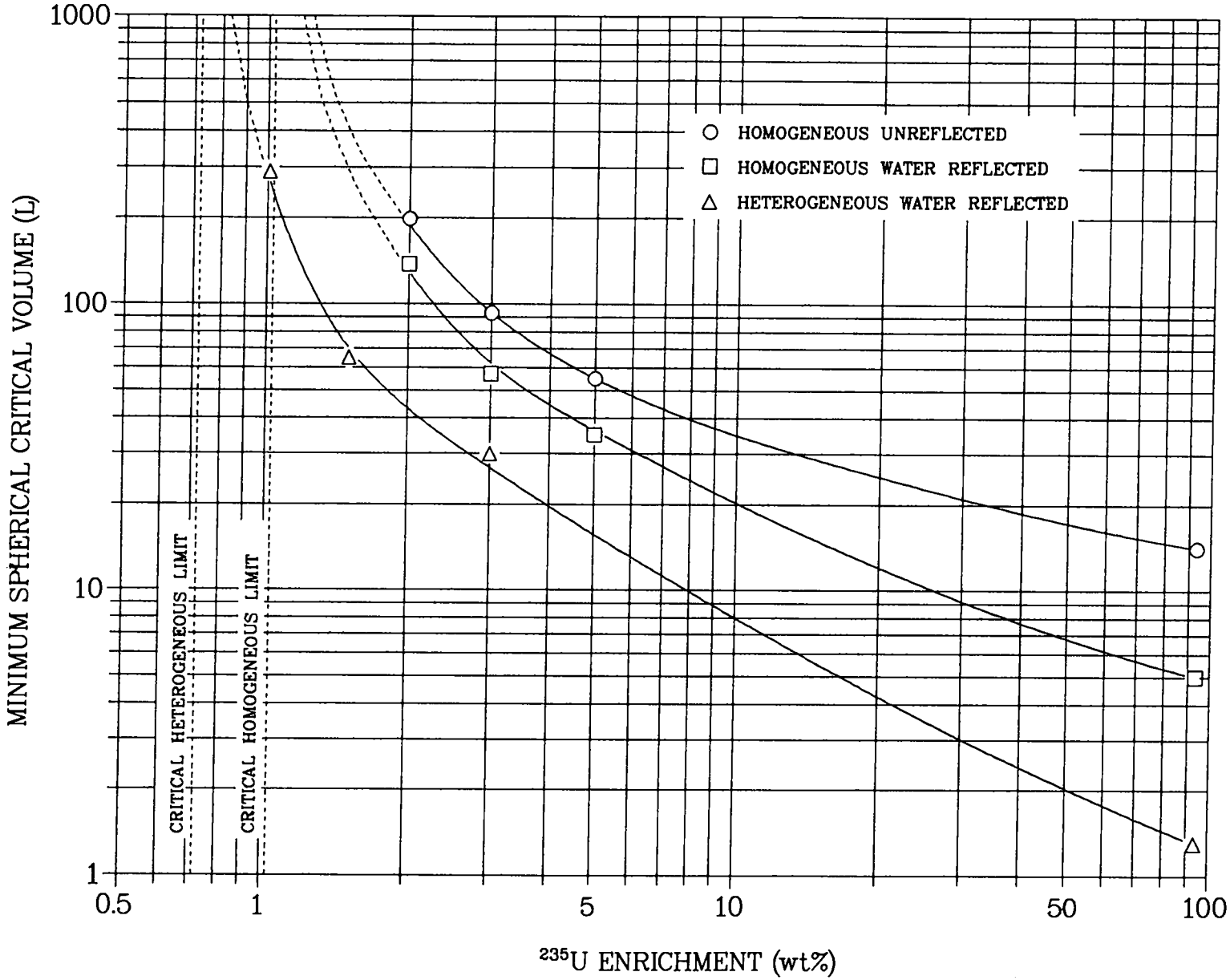


Fig. 23. Minimum spherical critical volumes as functions of ^{235}U enrichment in homogeneous and heterogeneous hydrogen-moderated systems.

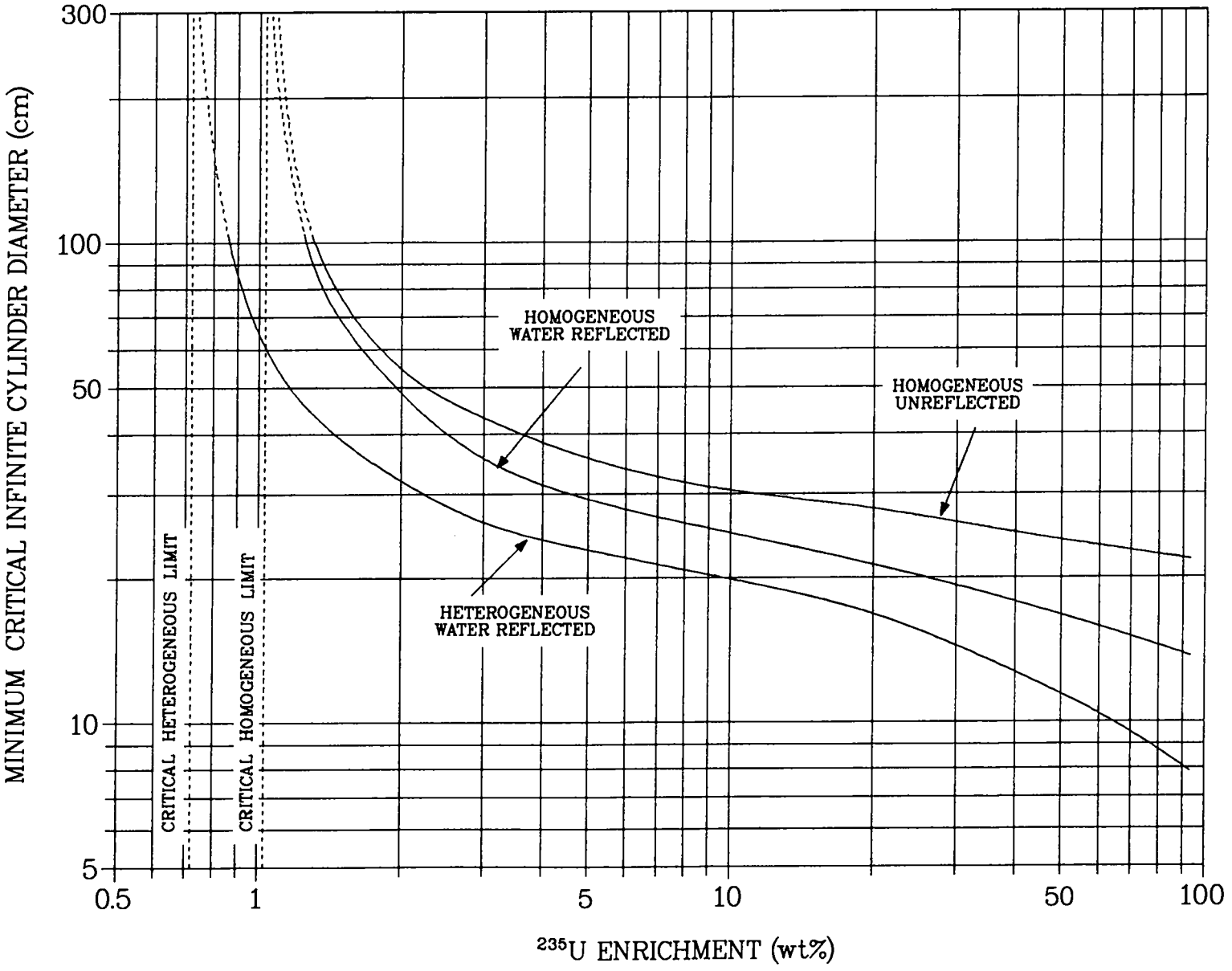


Fig. 24. Minimum critical infinite cylinder diameters as functions of ^{235}U enrichment in homogeneous and heterogeneous hydrogen-moderated systems.

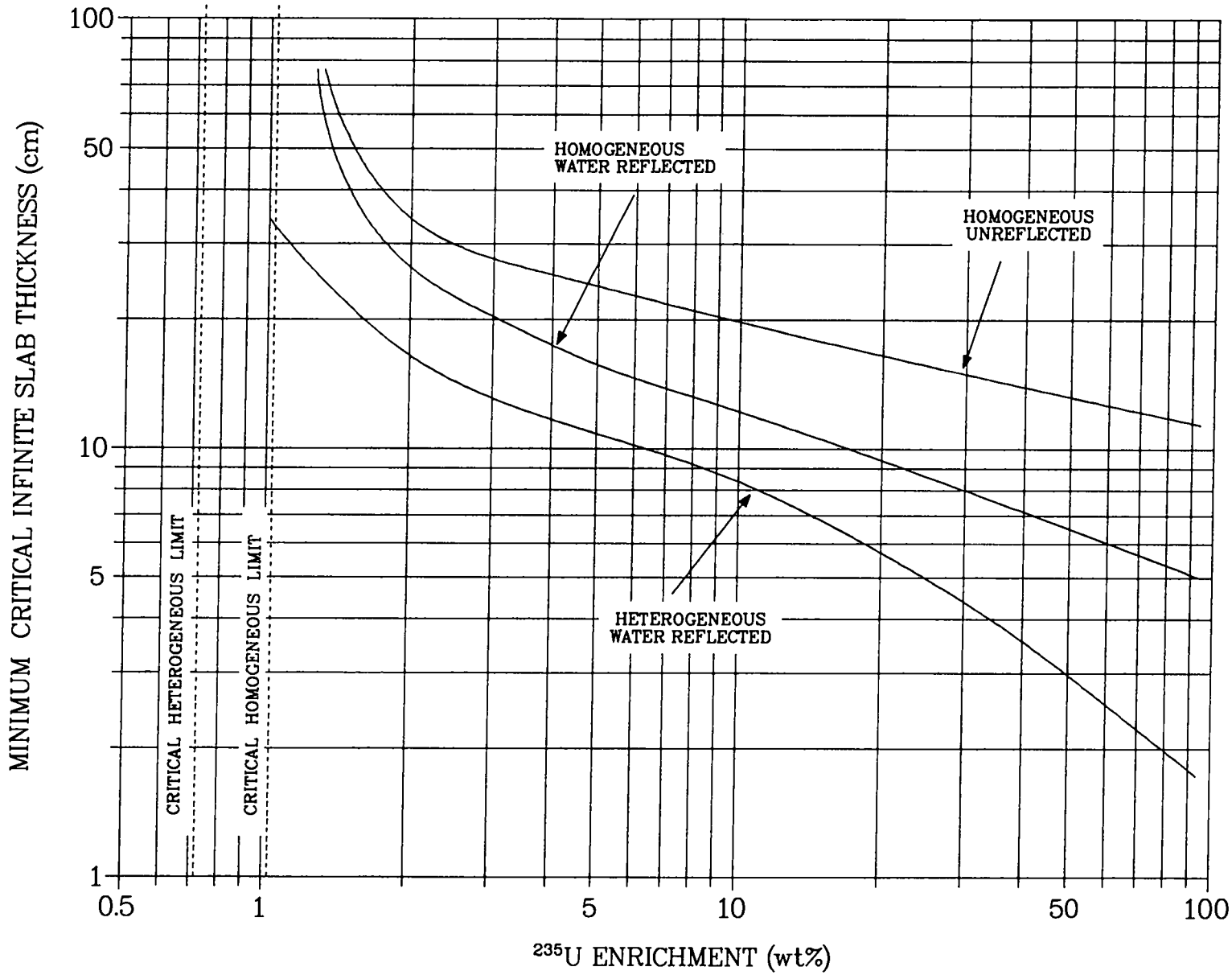


Fig. 25. Minimum critical infinite slab thicknesses as functions of ^{235}U enrichment in homogeneous and heterogeneous hydrogen-moderated systems. Unreflected infinite slabs are fictitious.

METAL-SOLUTION MIXTURES

All experimental information about another practical type of inhomogeneous system, combinations of massive fissile metal and fissile solution, is derived from subcritical measurements at Rocky Flats. One set of measurements applies to spaced 0.15-cm-thick disks of U(93) metal in solutions containing 102 and 308 g U(93)/L.[85] Other measurements are of more general interest because they may apply to U(93) billets in cleaning solution.[86,87] They relate concentrations of ^{235}U in solution to the critical thickness of a U(93) metal slab within the solution. Figure 26 gives results for 12.7- by 20.3-cm slabs in a 24-cm-diam, 41-cm-high solution, and Fig. 27 does the same for 25.4- by 41-cm slabs in a 76-cm-diam, 71-cm-high solution.

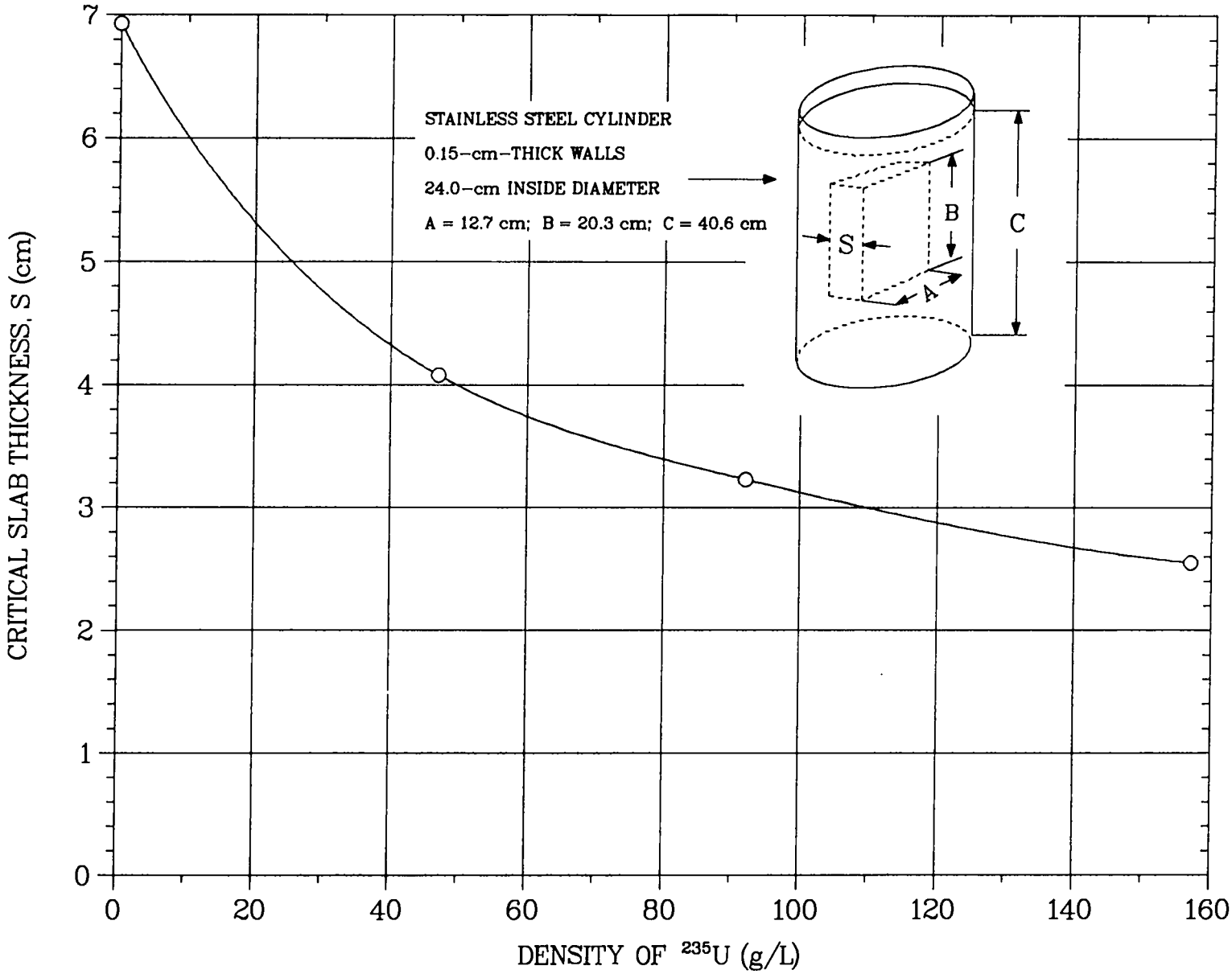


Fig. 26. Critical thicknesses of 12.7- by 20.3-cm U(93) metal slabs immersed in 24-cm-diam $U(93)O_2(NO_3)_2$ solutions at various ^{235}U densities.

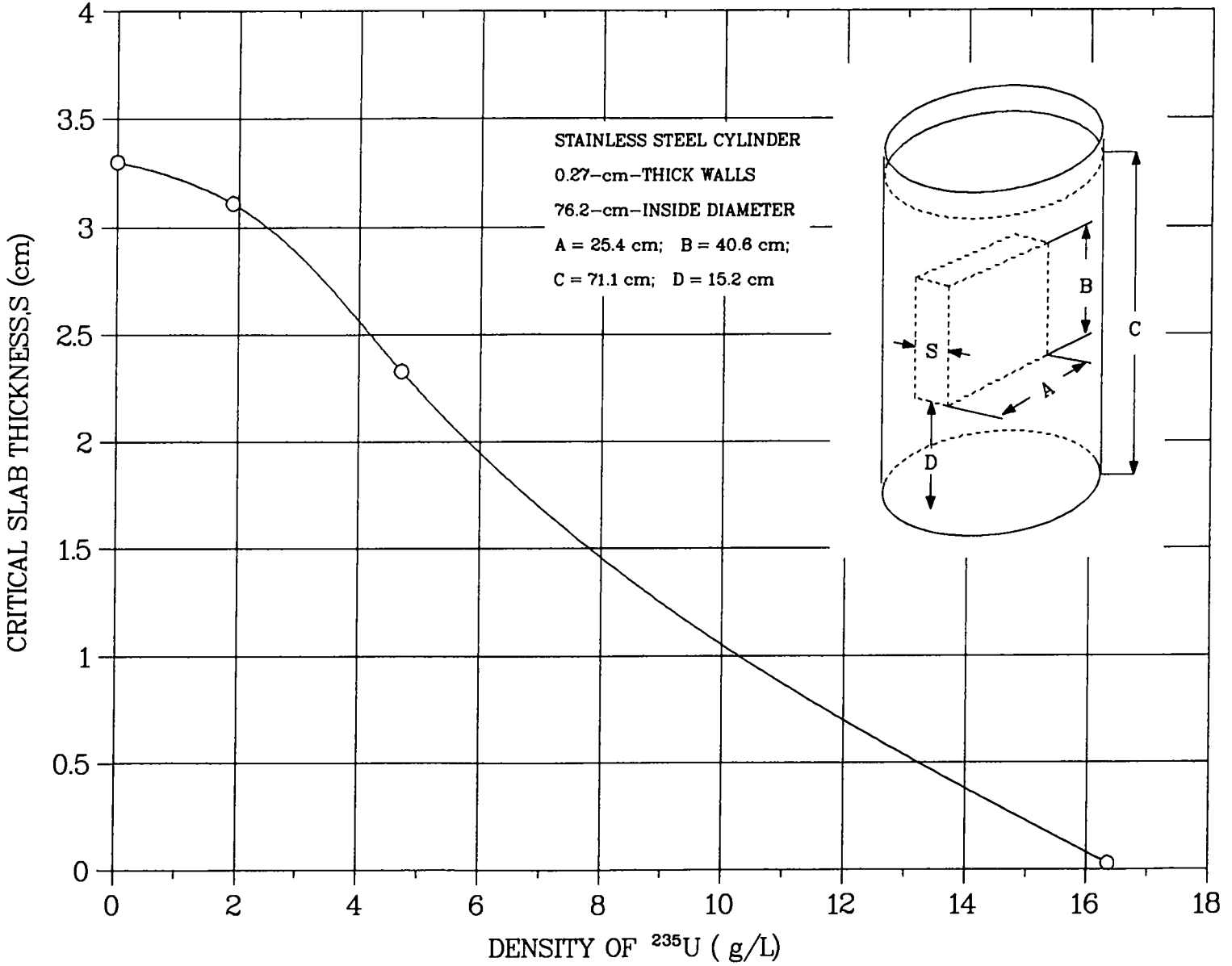


Fig.27. Critical thicknesses of 25.4- by 40.6-cm U(93)metal slabs immersed in 76-cm-diam $U(93)O_2(NO_3)_2$ solutions at various ^{235}U densities.

HOMOGENEOUS HYDROGEN-MODERATED PLUTONIUM

PLUTONIUM SOLUTIONS, LOW ^{240}Pu

Plutonium solutions, nominally $Pu(NO_3)_4$, are not as stable as uranyl fluoride and uranyl nitrate solutions.* The addition of nitric acid, one-half normal or greater, is necessary to prevent the formation of polymer over most of the plutonium concentration range. At 300 g Pu/L and greater, polymer can appear even in the presence of nitric acid. For comparison, uranyl fluoride solutions without excess acid may attain almost 1 kg U/L without precipitation.

Early Hanford critical experiments with plutonium solutions, both spherical and cylindrical, are reported in Ref. 26. Results for water-reflected solutions are summarized in Fig. 28 where there have been empirical adjustments for ^{240}Pu and nitrate contents. The relations between H/Pu and Pu density of Table 7 are supplemented by Fig. 29, which shows how that relation changes with HNO_3 molarity.[88] Results are supported by Harwell measurements on partially reflected solution.[89]

An extensive program at Saclay led, among other results, to the data summarized in Table 13.[59,90–92] Further, supplementary Hanford critical data for solution spheres with various reflectors are reported in Ref. 93 (see Fig. 30). Table 14 contains the items of Table 1 of that report for which corrections for neck and support of spheres are available. Data for reflectors of 1.27-cm-thick paraffin, 10.2-cm-thick concrete, and concrete with gaps are not included because such corrections are lacking. Reference 93 also includes critical masses of hypothetical α-phase ^{239}Pu-water mixtures over a range of plutonium densities, from calculations that give near agreement with experimental data.

Critical masses and volumes of water-moderated plutonium appear in Figs. 31 and 32, diameters of infinite cylinders in Fig. 33, and thicknesses of infinite slabs in Fig. 34.[26,94,95] Curves derived directly from experimental data apply to water-reflected $Pu(NO_3)_4$ solution with 1 N HNO_3 and 3.1% ^{240}Pu content of the plutonium. To show extreme hypothetical departures from solution values, computed curves for α-phase plutonium-water mixtures from Ref. 93 and Tables 15 and 16 are included in these figures.

The experimental value of infinite slab thickness that appears in Fig. 34 is derived from Ref. 94. Adjustments to 1 N HNO_3 and 3.1% ^{240}Pu were empirical.

The limiting critical concentration in Figs. 31-34, 7.6 g ^{239}Pu/L, was deduced at Hanford from specifications of a critical plutonium solution in a 4-ft-diam sphere.[96] This compares with 8.0 g ^{239}Pu/L as originally obtained from a Hanford Physical Constants Test Reactor (PCTR) measurement[52] and as reduced to 7.2 g ^{239}Pu/L in a reexamination of the data.[96]

* Eldon Christensen, Los Alamos National Laboratory, Los Alamos, NM (1985).

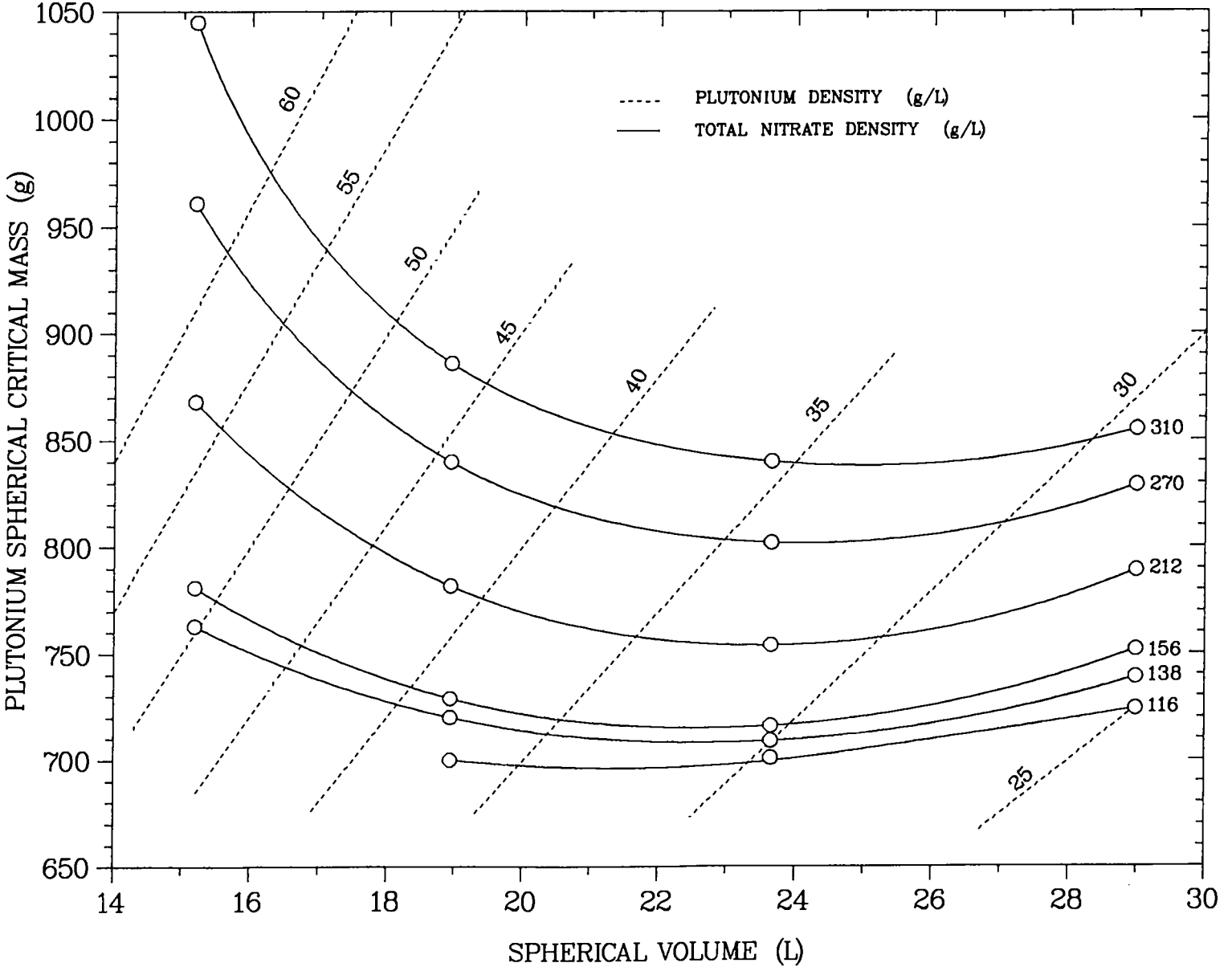


Fig. 28. Spherical critical masses and volumes of water-reflected $Pu(NO_3)_4$ solutions with various nitrate densities.

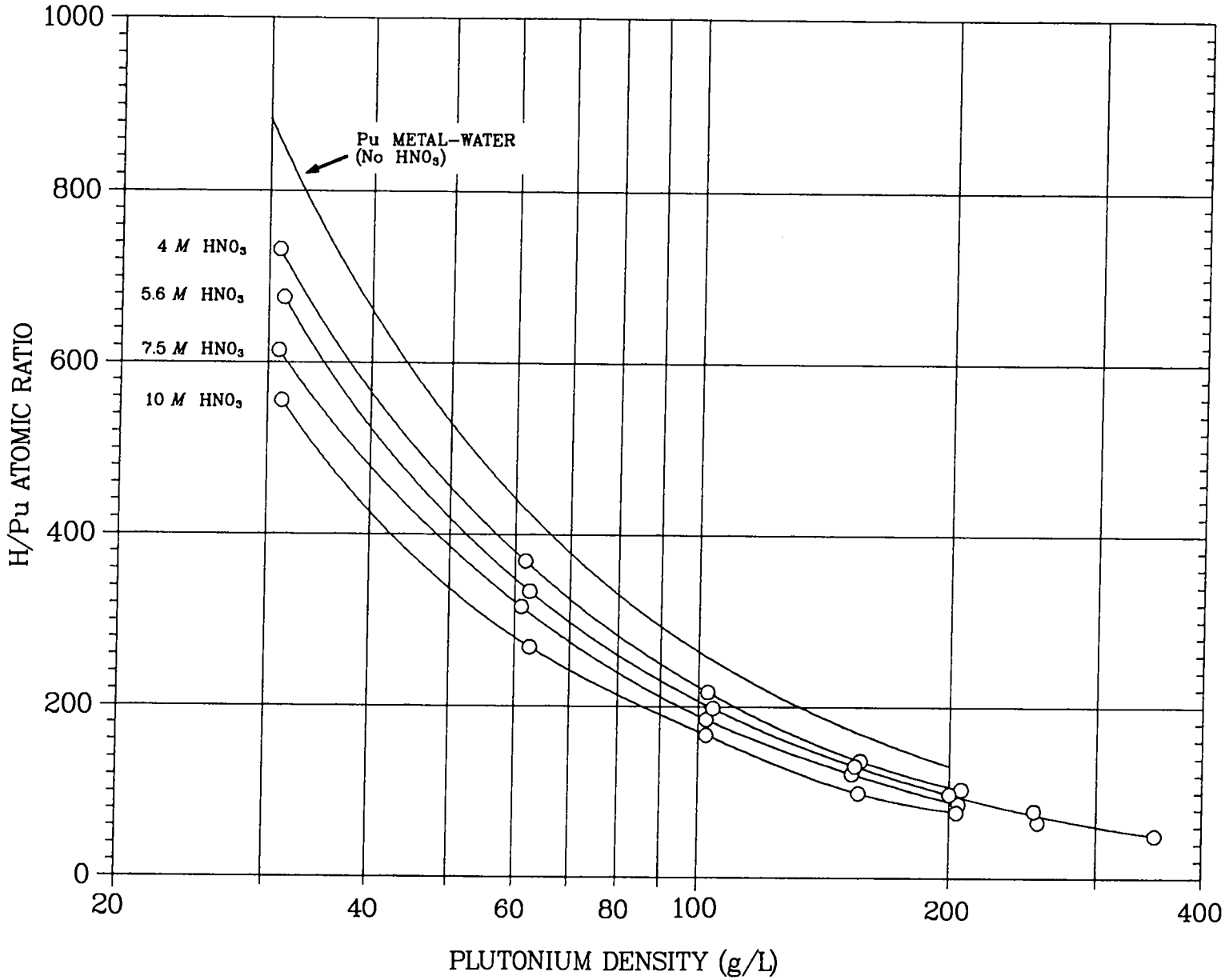


Fig. 29. Atomic ratios H/Pu as functions of plutonium density for $Pu(NO_3)_4$ solutions containing 4 M, 5.6 M, 7.5 M and 10 M HNO_3, and for idealized plutonium-water mixtures.

TABLE 13. CRITICAL DIMENSIONS OF CYLINDERS OF SOLUTIONS OF $Pu(NO_3)_4$, FROM FRANCE

Solution Composition				Critical Dimensions		
H/Pu	g Pu/L	Density (g/cm^3)	Total NO_3 (g/L)	Height (±0.05 cm)	Volume (L)	Mass (kg Pu)
			Cylinder o.d. 30.0 cm, Water Reflector			
560	44.7	1.13	174	31.2	21.3	0.95 ± 0.012
610	41.0	1.13	170	33.1	22.6	0.93 ± 0.012
660	38.0	1.12	162	35.4	24.1	0.92 ± 0.012
710	35.2	1.12	161	37.9	25.9	0.91 ± 0.012
780	32.6	1.11	157	41.1	28.1	0.92 ± 0.012
820	30.7	1.11	157	44.8	30.6	0.94 ± 0.012
880	28.9	1.11	153	49.6	33.9	0.98 ± 0.012
			Cylinder o.d. 30.0 cm, Concrete Reflector			
540	47.0	1.13	170	30.7	20.9	0.98 ± 0.015
670	37.5	1.12	166	35.0	23.9	0.90 ± 0.013
770	32.8	1.11	155	39.0	26.7	0.88 ± 0.012
900	28.2	1.10	142	45.4	31.1	0.88 ± 0.012
940	27.0	1.11	155	49.0	33.5	0.90 ± 0.012
			Cylinder o.d. 30.0 cm, Beech Wood Reflector			
580	43.6	1.13	162	33.7	23.0	1.00 ± 0.014
640	39.4	1.12	158	36.4	24.9	0.98 ± 0.014
710	35.4	1.11	153	40.1	27.3	0.97 ± 0.013
740	34.0	1.11	152	41.1	28.1	0.96 ± 0.013
810	31.2	1.10	145	46.6	31.9	0.99 ± 0.013
880	28.7	1.10	151	51.9	35.6	1.02 ± 0.013
			Cylinder o.d. 33.0 cm, Water Reflector			
540	45.5	1.13	177	26.3	20.5	0.93 ± 0.009
650	38.2	1.11	176	29.0	22.7	0.87 ± 0.008
750	33.3	1.12	168	32.7	25.7	0.86 ± 0.008
830	30.1	1.11	161	35.6	28.1	0.85 ± 0.007
890	28.0	1.10	152	38.5	30.4	0.85 ± 0.007
980	25.7	1.11	146	43.7	34.7	0.89 ± 0.007
1070	23.4	1.09	145	51.6	41.7	0.96 ± 0.007

Note: ^{240}Pu content ~1.5%; reflector thickness 40 cm on lateral surface only, stainless steel wall thickness 0.3 cm.

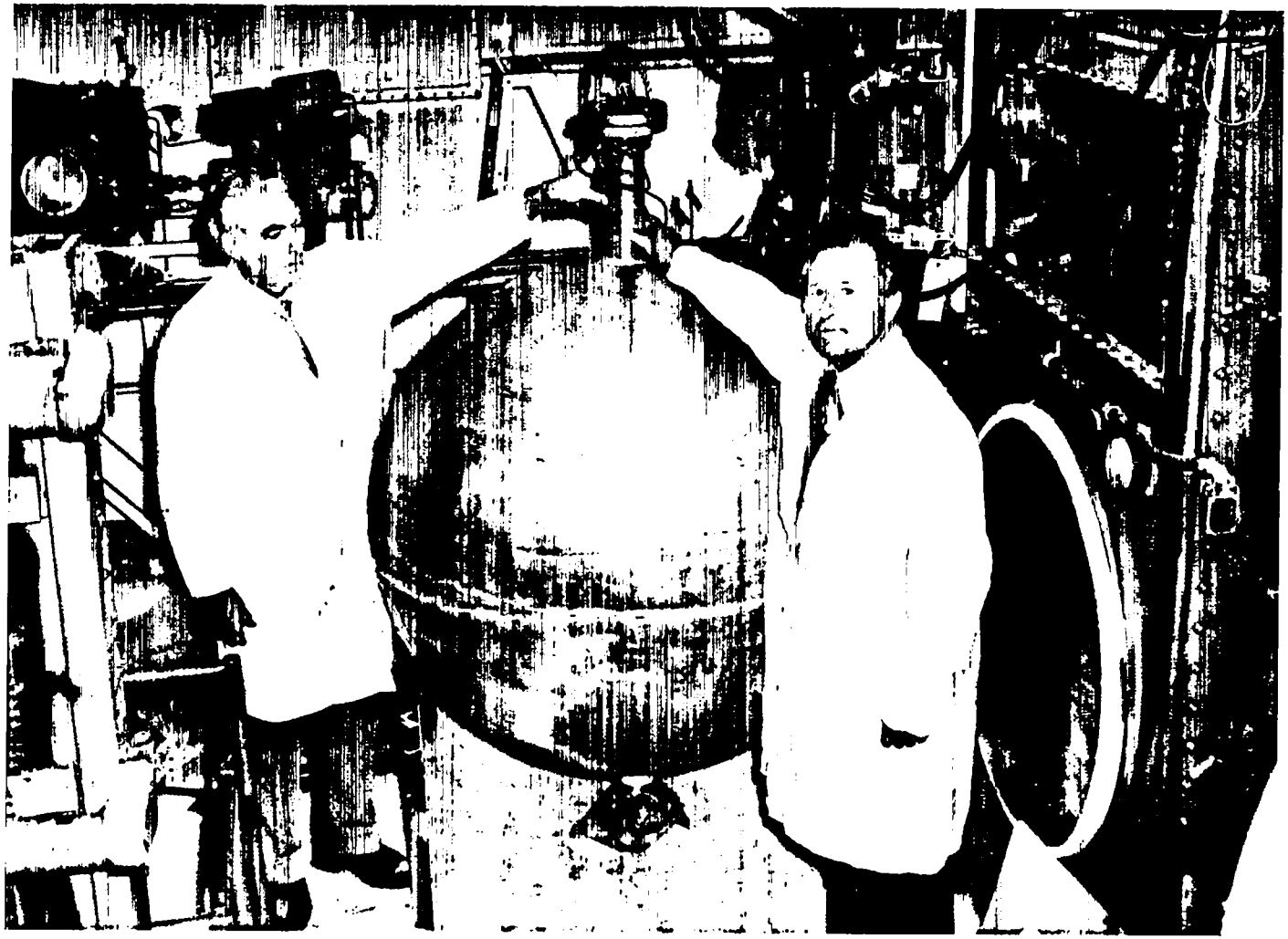


Fig. 30. Aluminum spherical container, of 949 L capacity, used at Hanford for measurements establishing the minimum critical concentration of plutonium in solution.[96] *The same sphere had been used at Oak Ridge for similar measurements with* ^{233}U *and* ^{235}U *solutions.*[51]

TABLE 14. CRITICALITY OF $Pu(NO_3)_4$ SOLUTIONS (4.6% ^{240}Pu)

Reflector	H/Pu	Density (g Pu/L)	HNO_3 Normality	Total NO_3 (g/L)	Critical Volume (L)[a]	Critical Mass (kg Pu)[a]
29.3-cm-diam Stainless Steel Sphere, 0.124-cm-thick Wall						
Water	354	73	0.2	86	12.9	0.94
Water	344	74	0.4	105	12.9	0.96
Water	243	100	1.9	230	12.9	1.29
Water	192	126	2.2	262	12.9	1.63
Water	87	269	1.1	346	12.9	3.47
Water	81	295[b]	0.8	303	12.9	3.81
Water	54	435[b]	0.8	372	12.9	5.61
28.6-cm Concrete	341	75	0.44	109	12.9	0.97
28.6-cm Concrete	100	236	1.16	318	12.9	3.04
35.6-cm-diam Stainless Steel Sphere, 0.112-cm-thick Wall						
2.54-cm Paraffin	443	54	4.3	323	23.4	1.25
2.54-cm Paraffin	256	85	6.4	482	23.4	1.99
2.54-cm Paraffin	223	98	6.1	473	23.4	2.28
25.4-cm Concrete	848	29.6	1.07	118	23.1	0.68
25.4-cm Concrete	651	36.6	4.39	310	23.1	0.85
25.4-cm Concrete	518	43.4	6.37	445	23.1	1.00
Water	754	33.2	2.08	164	23.1	0.77
Water	618	38.6	4.07	292	23.1	0.89
Water	466	47.5	6.66	462	23.1	1.10
Water + 0.183cm Stainless Steel	452	49.5	6.01	424	23.1	1.14
38.6-cm-diam Stainless Steel Sphere, 0.122-cm-thick Wall						
None	668	39.0	0.4	64	30.8	1.20
None	125	172	4.9	486	30.8	5.31
0.66-cm Stainless Steel	758	34.3	0.5	66	30.8	1.06
Water	1068	24.4	0.5	58	30.1	0.73
Water	553	38.7	7.7	517	30.1	1.16
Water + 0.203-cm Stainless Steel	1032	25.2	0.5	60	30.1	0.76

[a] Corrected for neck and support.

[b] Contains Pu(VI) and plutonium polymer, increasing the uncertainty to ± 5%

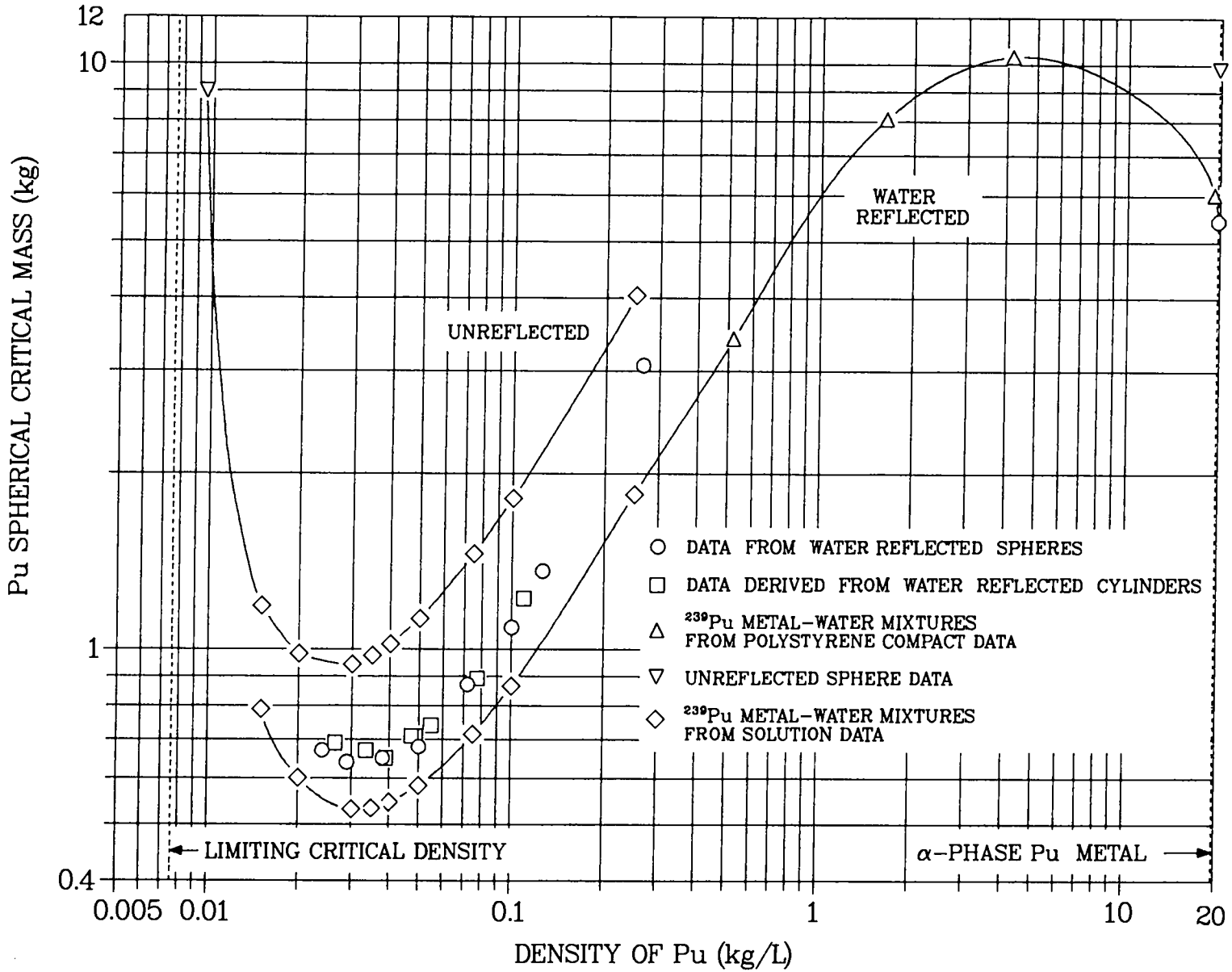


Fig. 31. Critical masses of homogeneous water-moderated plutonium spheres. The points suggesting an intermediate curve apply to water-reflected Pu $(NO_3)_4$ solution with 1 $\underline{N}$ HNO_3 and 3.1% ^{240}Pu content of the plutonium.

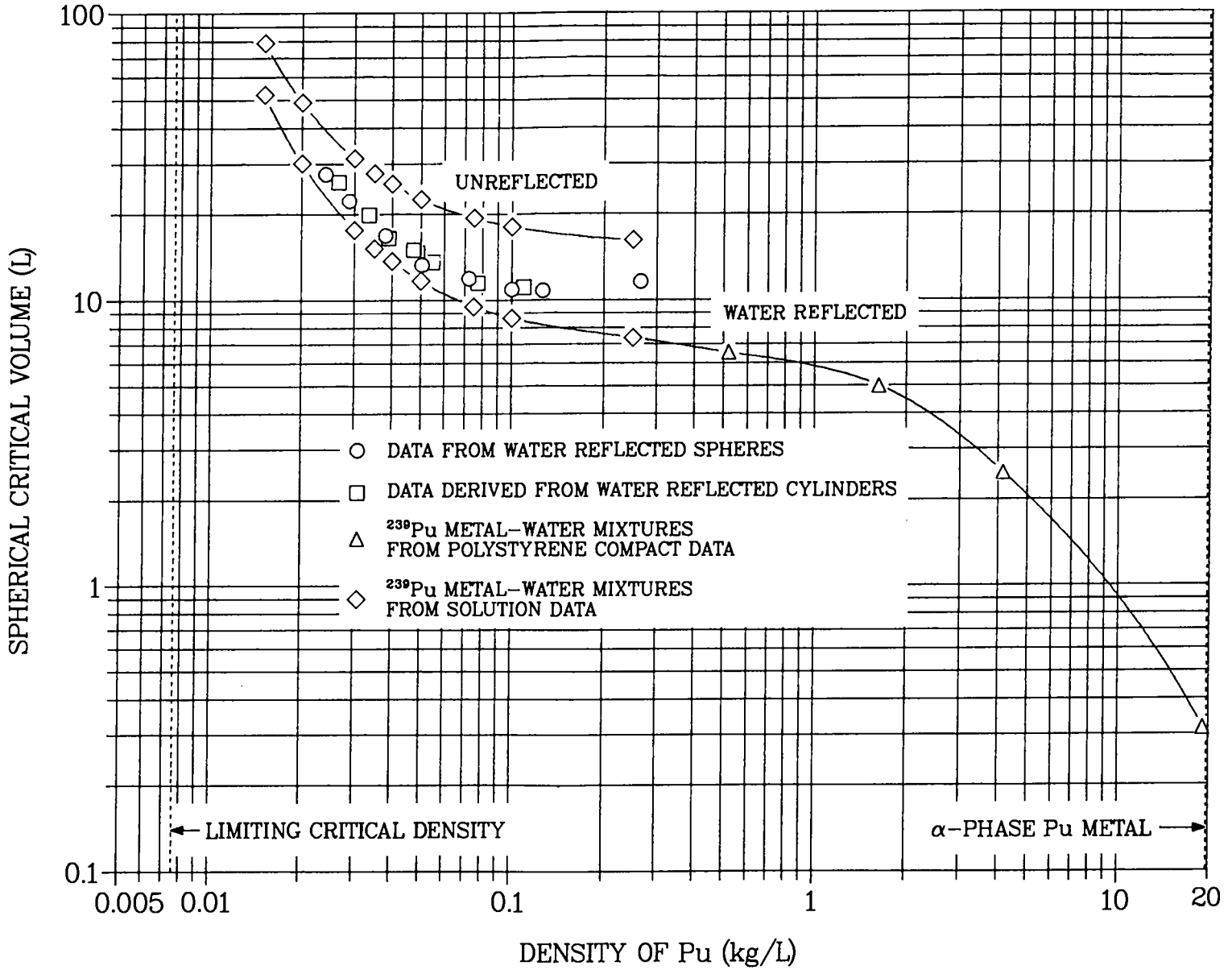


Fig. 32. Critical volumes of homogeneous water-moderated plutonium spheres. The points suggesting an intermediate curve apply to water-reflected Pu $(NO_3)_4$ solution with 1 N HNO_3 and 3.1% ^{240}Pu content of the plutonium.

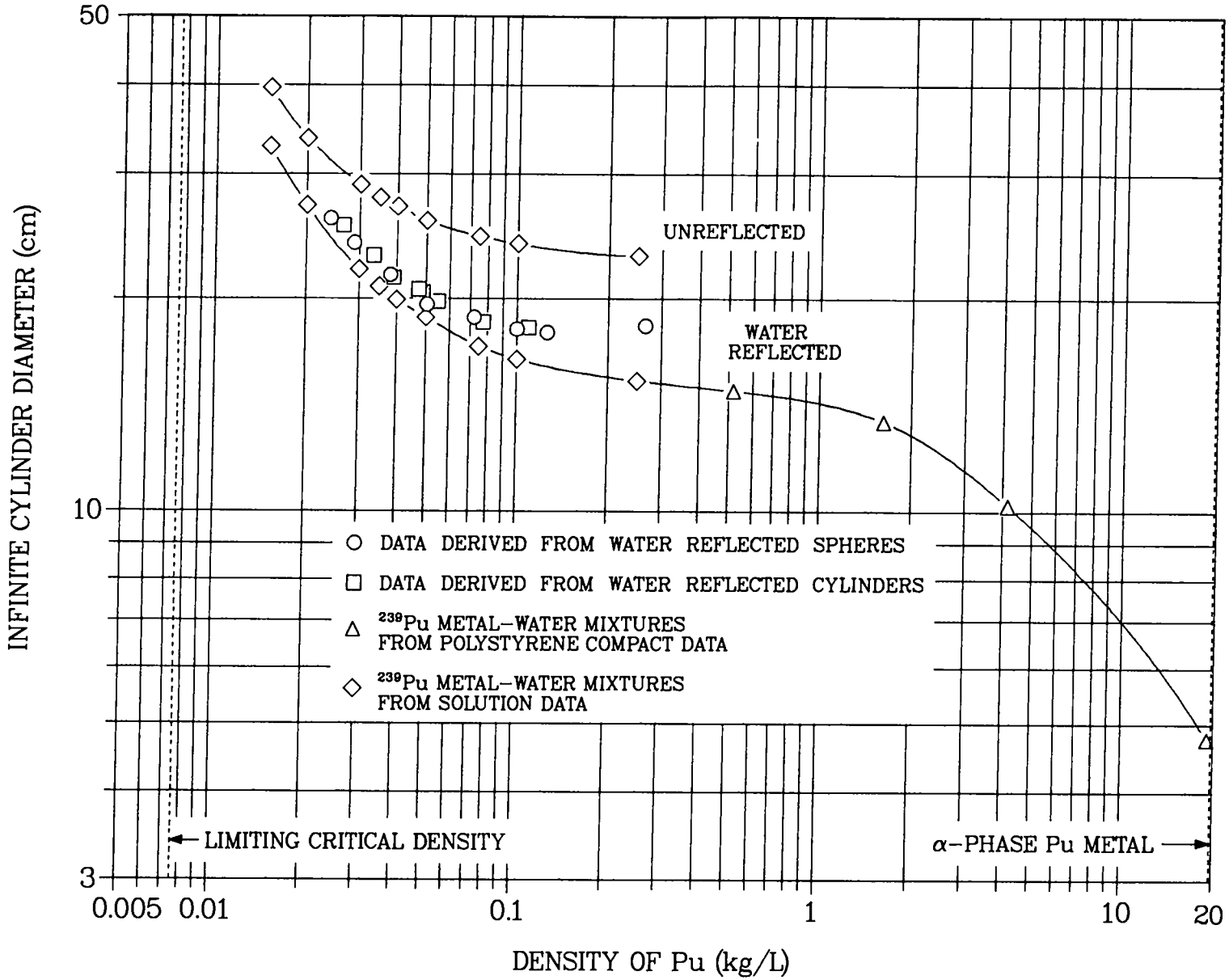


Fig. 33. Estimated critical diameters of infinitely long cylinders of homogeneous water-moderated plutonium. The points suggesting an intermediate curve apply to water-reflected Pu $(NO_3)_4$ solution with 1 N HNO_3 and 3.1% ^{240}Pu content of the plutonium.

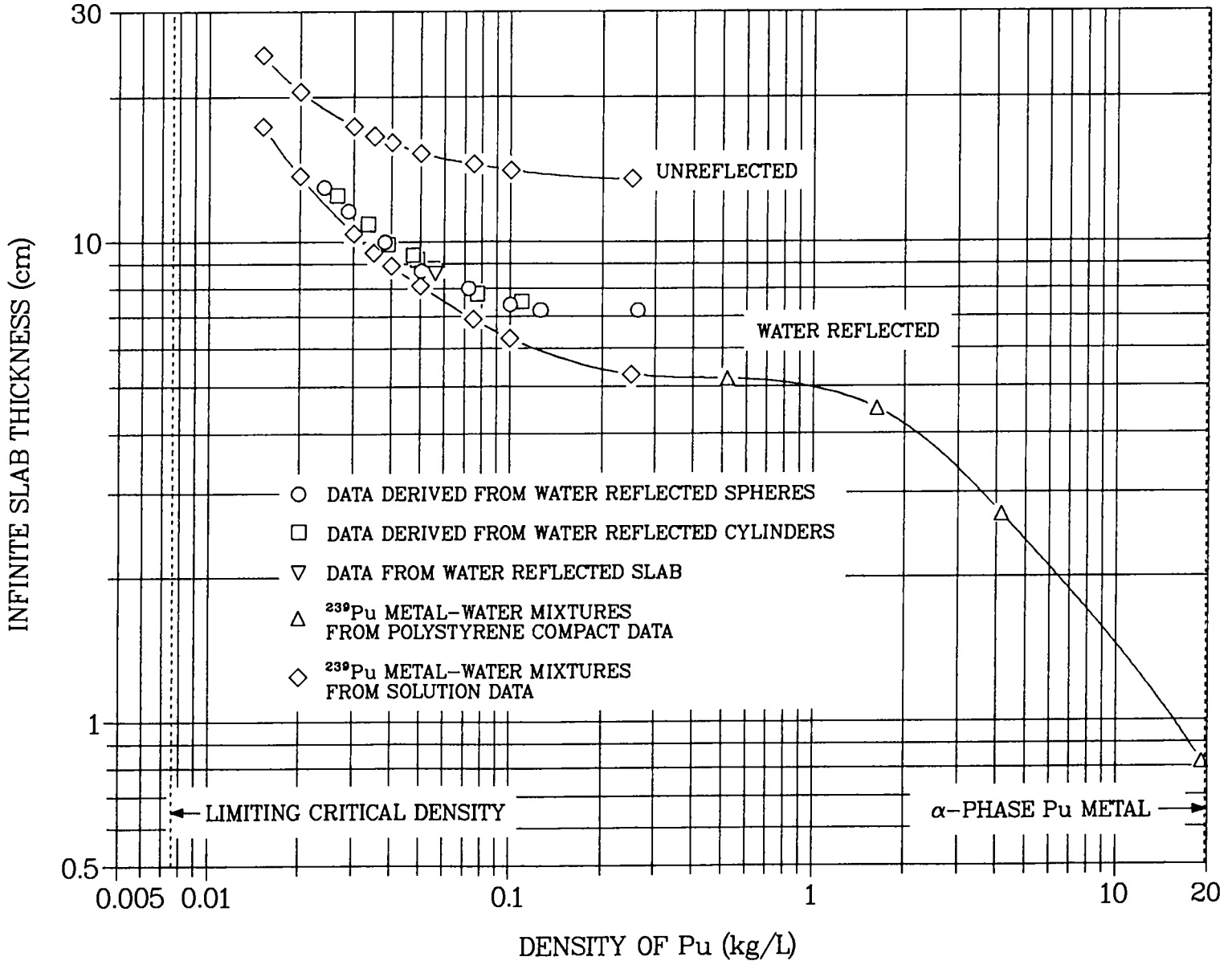


Fig. 34. Estimated critical thicknesses of slabs, infinite in other dimensions, of homogeneous water-moderated plutonium. Unreflected infinite slabs are fictitious. The points suggesting an intermediate curve apply to water-reflected Pu $(NO_3)_4$ solution with 1 N HNO_3 and 3.1% ^{240}Pu content of the plutonium.

TABLE 15. CRITICAL DIMENSIONS OF REFLECTED SPHERES, INFINITE CYLINDERS AND INFINITE SLABS DEDUCED FROM EXPERIMENTS WITH PuO_2 — POLYSTYRENE COMPACTS

H/Pu Atomic Ratio	Sphere Radius (cm)	Infinite Cylinder Diameter (cm)	Infinite Slab Thickness (cm)
5	PuO_2 - Polystyrene, 2.30 g Pu/cm^3, 11.46% ^{240}Pu:[a]		
	14.15 ± 0.4	17.9 ± 0.7	5.9 ± 0.2
5	^{239}Pu - Water, 4.16 g Pu/cm^3, 0% ^{240}Pu:[b]		
	8.4 ± 0.1	10.2 ± 0.4	2.7 ± 0.1
5	$^{239}PuO_2$ - Water, 3.46 g Pu/cm^3, 0% ^{240}Pu:[b]		
	9.3 ± 0.1	11.5 ± 0.5	3.24 ± 0.1
15	PuO_2 - Polystyrene, 1.12 g Pu/cm^3, 2.2% ^{240}Pu:[a]		
	13.5 ± 0.2	17.2 ± 0.4	6.0 ± 0.1
15	PuO_2 - Polystyrene, 1.05 g Pu/cm^3, 8.0% ^{240}Pu:[a]		
	15.6 ± 0.2	20.1 ± 0.4	7.4 ± 0.1
15	^{239}Pu - Water, 1.62 g Pu/cm^3, 0% ^{240}Pu:[b]		
	10.6 ± 0.2	13.4 ± 0.3	4.5 ± 0.1
15	$^{239}PuO_2$ - Water, 1.50 g Pu/cm^3, 0% ^{240}Pu:[b]		
	11.0 ± 0.2	14.0 ± 0.3	4.8 ± 0.1
50	PuO_2 - Polystyrene, 0.367 g Pu/cm^3, 18.35% ^{240}Pu:[a]		
	20.0 ± 1.	26.5 ± 0.2	10.9 ± 0.1
50	^{239}Pu - Water, 0.521 g Pu/cm^3, 0% ^{240}Pu:[b]		
	11.6 ± 0.1	14.8 ± 0.2	5.2 ± 0.1
50	$^{239}PuO_2$ - Water, 0.508 g Pu/cm^3, 0% ^{240}Pu:[b]		
	11.8 ± 0.1	15.0 ± 0.2	5.3 ± .1
50	Pu - Water, 0.521 g Pu/cm^3, 18.35% ^{240}Pu:[b]		
	17.2 ± 0.1	22.8 ± 0.2	9.4 ± 0.1
50	PuO_2 - Water, 0.508 g Pu/cm^3, 18.35% ^{240}Pu:[b]		
	17.4 ± 0.1	23.2 ± 0.2	9.6 ± 0.1

[a] Plexiglas reflector.
[b] Water reflector.

TABLE 16. CRITICAL DIMENSIONS OF REFLECTED SPHERES, INFINITE CYLINDERS, AND INFINITE SLABS DEDUCED FROM EXPERIMENTS WITH PuO_2 COMPACTS AT H/Pu = 0.04

Composition	Pu (g/cm^3)	^{240}Pu (wt%)	Sphere Radius (cm)	Infinite Cylinder (cm)	Infinite Slab (cm)
PuO_2-water[a]	5.8	18.35	11.2 ± 0.4	13.4 ± 0.6	3.34 ± o.10
^{239}Pu-water[b]	19.16	0	4.22 ± 0.14	4.76 ± 0.22	0.82 ± 0.02
$^{239}PuO_2$-water[b]	9.96	0	6.6 ± 0.2	7.7 ± 0.3	1.60 ± 0.04
PuO_2-water[b]	19.16	18.35	4.70 ± 0.16	5.4 ± 0.3	1.15 ± 0.03
PuO_2-water	9.96	18.35	7.4 ± 0.3	9.0 ± 0.4	2.31 ± 0.06

[a]Plexiglas reflector.
[b]Water reflector.

PLUTONIUM SOLUTIONS, UNLIMITED ^{240}Pu

The critical infinite slabs reported from Hanford in Ref. 94 apply to solutions of Pu containing 23.2% and 18.4% ^{240}Pu, as well as 4.7% ^{240}Pu. Because no high ^{240}Pu data exist for spheres, reflected slab thicknesses were converted to equivalent sphere radii by means of the extrapolation distances, δ_{sl} for slabs and δ_s for spheres, that are listed in Table 17. Critical radii of spheres, r_s, were obtained from S_n calculations that reproduced corresponding critical slab thicknesses, t_{sl}. The listed values of δ_s then satisfy the relation

$$r + \delta_s = t_{sl} + 2(1.06\ \delta_s),$$

where $\delta_{sl} = 1.06\ \delta_s$ for consistency with Fig. 6. Values of δ_{sl} measured in finite slabs but attributed to infinite slabs are shown parenthetically in Table 17 to suggest that the calculated values are reasonable.

In addition to the Hanford data, Table 17 includes critical data from Saclay[97] for a solution of plutonium containing 19% ^{240}Pu. The system selected represents the high H/Pu extreme of a range of measurements. Not included is a Hanford critical experiment with solution of plutonium containing 43% ^{240}Pu with partial water reflector.[98]

Computed values that are tied to the points of Table 17, give the curves of critical mass vs percent ^{240}Pu that appear in Fig. 35. These curves apply to water-reflected metal-water spheres.

TABLE 17. CRITICAL WATER-REFLECTED PLUTONIUM SPHERES FROM INFINITE SLAB THICKNESSES OVER A RANGE OF ^{240}Pu CONTENT

Infinite Slab			Sphere		
Thickness (cm)	δ_{sl} Calc.(Obs.) (cm)	δ_s Calc.(Obs.) (cm)	Critical Radius (cm)	Critical Volume (L)	Critical Mass (kg Pu)
	4.69% ^{240}Pu, 58 g Pu/L, 2.3 N HNO_3, H/Pu = 425:				
9.13	5.79 (6.40)	5.46 (6.04)	15.25	14.9	0.86
	18.4% ^{240}Pu, 66.5 g Pu/L, 2.4 N HNO_3, H/Pu = 367:				
11.38	6.15 (6.49)	5.80 (6.12)	17.9	23.9	1.59
	18.4% ^{240}Pu, 240.8 g Pu/L, 4.1 N HNO_3, H/Pu = 89:				
11.64	7.25 (7.19)	6.84 (6.78)	19.3	30.1	7.25
	18.9%^{240}Pu, 13.2 g Pu/L, 2.3 N HNO_3, H/Pu = 1850:[a]				
49.8	5.94	5.6	56.1	740	9.8
	23.2% ^{240}Pu, 283.8 g Pu/L, 2.2 N HNO_3,H/Pu = 78:				
12.43	7.31 (7.67)	6.90 (7.24)	20.15	34.3	9.7

[a] From parallelepiped, 130 x 130 cm x 66.3 cm high.

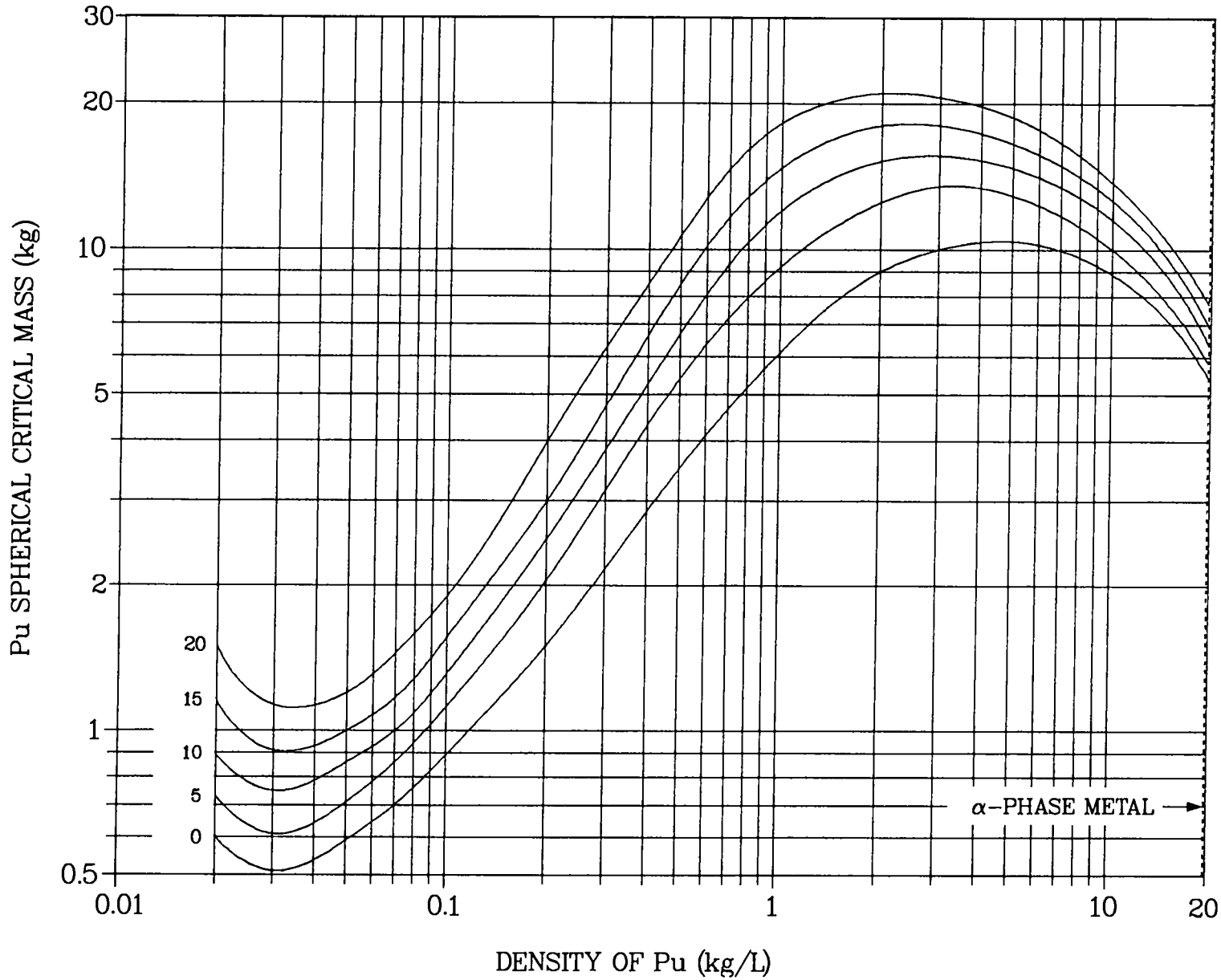


Fig. 35. Calculated effect of ^{240}Pu content on the critical mass of water-reflected homogeneous plutonium metal-water spheres. The ^{240}Pu content, in wt%, is indicated for each curve.

PLUTONIUM — POLYSTYRENE COMPACTS

Three reports from Hanford[99-101] describe critical experiments with homogeneous PuO_2-polystyrene mixtures at H/Pu ratios of 5, 15 and 50. Each set of experiments applies to bare and Plexiglas-reflected parallelepipeds ranging from elongated to squat shapes. For each composition, critical dimensions are extrapolated to the infinite cylinder and infinite slab and converted to the equivalent sphere. Calculations based on these dimensions also give corresponding dimensions for plutonium-water and PuO_2-water mixtures. Reported sphere radii, infinite cylinder diameters, and infinite slab thicknesses for the reflected systems are given in Table 15.

The PuO_2-polystyrene experiments were supplemented by similar measurements with slightly moderated compressed PuO_2 compacts.[102] The deduced critical dimensions of reflected spheres, infinite cylinders, and infinite slabs which were reported appear in Table 16.

HOMOGENEOUS PLUTONIUM-URANIUM SYSTEMS

Liquid Metal Fast Breeder Reactor (LMFBR) fuel cycle operations have led to critical experiments with moderated plutonium-uranium mixtures at Hanford and Aldermaston. Hanford measurements with bare and Plexiglas-reflected PuO_2-UO_2-polystyrene compacts, which were readily interpretable,[103-105] were supplemented by others with partially reflected plutonium-uranium nitrate solutions.[106] Aldermaston reported criticality of polyethylene-reflected PuO_2-UO_2-polyethylene compacts of various shapes,[107] and water-reflected plutonium-uranium nitrate solutions at various concentrations of plutonium.[108]

Table 18 gives critical dimensions derived from the above experiments for various reflected shapes. The Hanford reports also give critical sphere, cylinder, and slab dimensions for idealized $^{239}PuO_2$-$U(0.72)O_2$- water mixtures.

Calculations[109] based on the Hanford and Aldermaston data led to subcritical limits for plutonium-uranium mixtures that appear in the American National Standard for Nuclear Criticality Control and Safety of Plutonium-Uranium Fuel Mixtures Outside Reactors.[110]

TABLE 18. CRITICAL REFLECTED Pu-U SYSTEMS, DIMENSIONS DEDUCED FROM CRITICAL EXPERIMENTS

Pu in Pu+U (Wt %)	H/Pu	HNO_3 (N)	Pu Density (g/L)	^{240}Pu in Pu (Wt %)	Shape	Critical Dimension (cm)
			PuO_2 - U(0.151)O_2 - Polystyrene, Plexiglas Reflector:[103,105]			
7.6	259	—	64	23.0	cube	47.3 ± 0.2 edge
7.6	259	—	64	23.0	inf. slab	20.0
7.9	659	—	28.4	8.0	sphere	22.6 ± 0.1 r
7.9	659	—	28.4	8.0	inf. cyl.	31.2 ± 0.2 diam
7.9	659	—	28.4	8.0	inf. slab	14.8 ± 0.6
8.1	93	—	160	11.5	cube	47.8 edge[a]
14.6	210	—	85	8.0	sphere	19.7 ± 0.2 r
14.6	210	—	85	8.0	inf. cyl.	26.4 ± 0.2 diam
14.6	210	—	85	8.0	inf. slab	11.6 ± 0.1
15.0	20.2	—	510	11.5	cube	52.1 edge[a]
29.3	9.7	—	1010	11.5	cube	35.7 edge[a]
29.3	9.7	—	1010	11.5	inf. slab	12.9
30.0	158	—	112	8.0	sphere	18.5 ± 0.3 r
30.0	158	—	112	8.0	inf. cyl.	24.6 ± 0.2 diam
30.0	158	—	112	8.0	inf. slab	10.8 ± 0.1
			PuO_2 - U(0.72) - Polyethylene, Polyethylene Reflector:[107]			
25.1	18.6	—	950	5.9	cube	27.7±0.1 edge
			PuO_2 - U(0.72) Nitrate Solution, Water Reflector:[108]			
30.6	224	1.60	101	5.6	cyl.,h=d	300 ± 0.3 diam
30.6	795	1.10	31.6	5.6	cyl.,h=d	338 ± 0.3 diam
30.6	1372	0.95	18.6	5.6	cyl.,h=d	441 diam
30.6	1461	0.93	17.5	5.6	cyl.,h=d	456 diam

[a] From near-cubic parallelepiped.

WATER-MODERATED PLUTONIUM AND PLUTONIUM-URANIUM LATTICES

Consideration of the Plutonium Recycle Program led to Hanford critical experiments with water lattices of plutonium-aluminum rods.[111] To maintain a reasonable number of elements, 5 wt% Pu in aluminum was chosen instead of the 1.8 wt% Pu considered for recycle. Rods of 1.29-cm- diam and 61-cm length were clad with 0.076-cm-thick Zircaloy-2. Measured bucklings and extrapolation distances for various spacings of rods in a hexagonal lattice are given in Table 19, with critical volumes and masses of corresponding water-reflected equilateral cylinders.

TABLE 19. HEXAGONAL WATER LATTICES OF 1.29-cm- BY 61-cm- RODS OF 5 Wt% Pu (5% ^{240}Pu) - ALUMINUM CLAD WITH 0.076-cm-thick ZIRCALOY-2.

				Equilateral Cylinder	
Center Spacing (cm)	H/Pu	Buckling (10^{-4} cm^{-2})	Extrapolation Distance (cm)	Critical Volume (L)	Critical Mass (kg Pu)
1.90	218	96.05	8.10	60	3.45
2.16	354	108.39	7.92	47.7	2.14
2.29	427	112.61	7.93	44.1	1.76
2.54	580	108.53	7.73	48.8	1.58
2.79	760	101.07	7.34	60	1.65
3.05	940	88.40	7.03	82	1.83
3.30	1150	76.81	6.03	120	2.42
3.81	1610	53.32	5.80	234	3.35

Also at Hanford, Fast Test Reactor fuel pins became available for measurements of square-pitch water-flooded lattices.[112] The fuel of each pin consisted of a PuO_2-U(natural)O_2 mixture containing 19.84 wt% plutonium and had dimensions 0.494-cm diam by 91.4-cm length. The plutonium content of a pin averaged 34.2 g and contained 11.5% ^{240}Pu. The type 316 stainless-steel cladding was 0.038-cm thick. Lattice characteristics appear in Table 20.

TABLE 20. SQUARE WATER LATTICES OF 0.494-cm-diam BY 91.4-cm-long PuO_2+UO_2 AT 9.8 g/cm^3 CONTAINING 19.8 wt% PLUTONIUM CLAD WITH 0.038-cm-thick STAINLESS STEEL

Center Spacing (cm)	Water/Fuel Vol Ratio	Lattice Width No. of Pins	Critical Number of Pins
0.767 ± 0.013	1.67	36	1037 ± 1[a]
0.952 ± 0.013	3.33	28	605 ± 1
0.968 ± 0.013	3.49	28	579 ± 1
1.259 ± 0.013	6.87	18	279 ± 1
1.534 ± 0.013	10.88	18	205 ± 1
1.905 ± 0.013	17.53	14	162 ± 1

[a] Included 65 rods containing 24.4 wt% Pu, which was corrected to 1268 rods all of which contained 19.8 wt% Pu.

HOMOGENEOUS HYDROGEN-MODERATED ^{233}U

Unlike the survey of U(93) solution cylinders with a large height/diameter range, all critical ^{233}U solution cylinders were nearly equilateral.[40] Thus deduced infinite cylinder and slab dimensions are more dependent upon the calculations of Table 5 and Fig. 6. With one exception, the reflector, where present, was paraffin instead of water. A single measurement with water reflection gave essentially the same critical dimensions as paraffin. France also has reported data for partially reflected ^{233}U solution in cylinders.[59,92]

The initial ^{233}U solution survey was supplemented by spherical measurements, all at Oak Ridge.[48,51,113] Critical parameters for ^{233}U, similar to data presented earlier for U(93) and plutonium, appear in Figs. 36-39.

In addition to the $^{233}UO_2F_2$ solutions represented in Figs. 36-39, there were data for a similar range of critical $^{233}UO_2(NO_3)_2$ solutions. Ratios of uranyl nitrate to uranyl fluoride critical masses, influenced by nitrogen capture, appear in Table 21. Because of significant differences for concentrated solutions, ratios are given as functions of both ^{233}U density and $H/^{233}U$. Table 21 may be presumed to apply also to U(93) solutions, for which no direct comparison exists, but see Ref. 44.

TABLE 21. SPHERICAL WATER-REFLECTED CRITICAL MASS RATIO, $m_c[^{233}UO_2(NO_3)_2]/m_c[^{233}UO_2F_2]$ SOLUTIONS, vs ^{233}U DENSITY AND $H/^{233}U$

233 Density (kg/L)	Critical Mass Ratio	$H/^{233}U$	Critical Mass Ratio
0.0338	1.00	760	1.02
0.0492	1.00	510	1.01
0.064	1.03	394	1.03
0.100	1.06	247	1.02
0.160	1.04	151	1.00
0.274	1.14	84	1.09
0.381	1.23	58	1.11
0.491	1.43	42	1.26

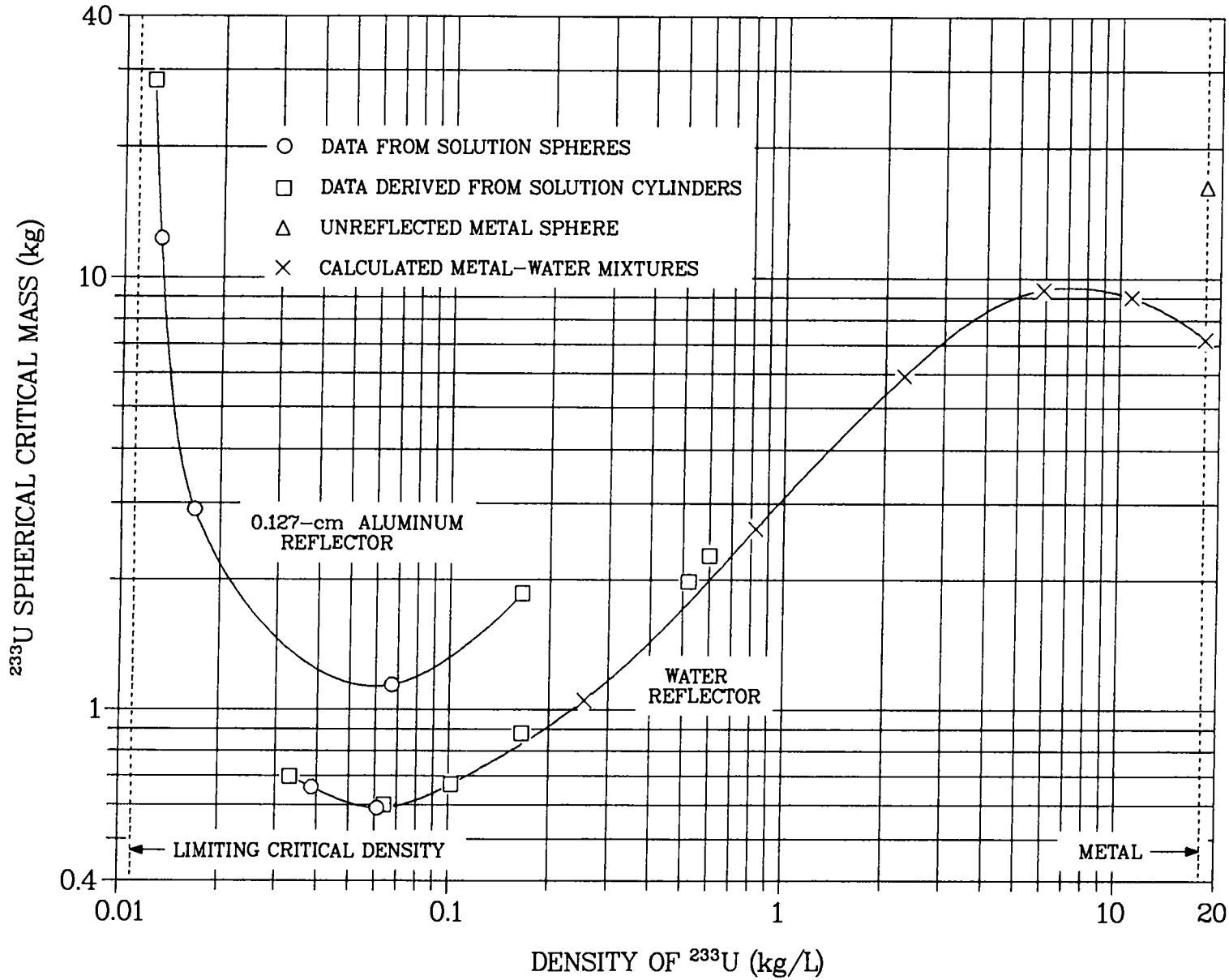


Fig. 36. Critical masses of homogeneous water-moderated ^{233}U spheres. Solution data appear unless indicated otherwise.

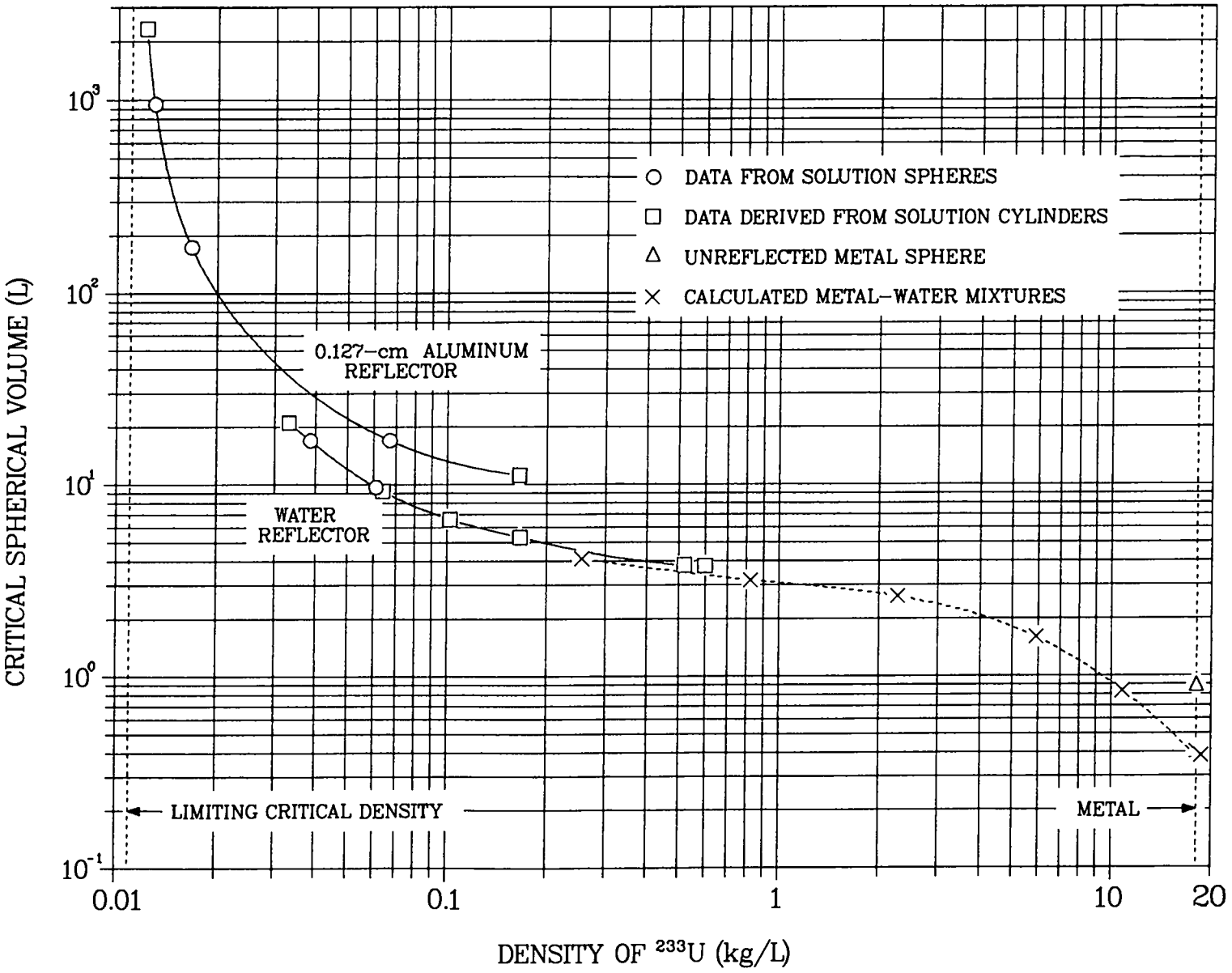


Fig. 37. Critical volumes of homogeneous water-moderated ^{233}U spheres. Solution data appear unless indicated otherwise.

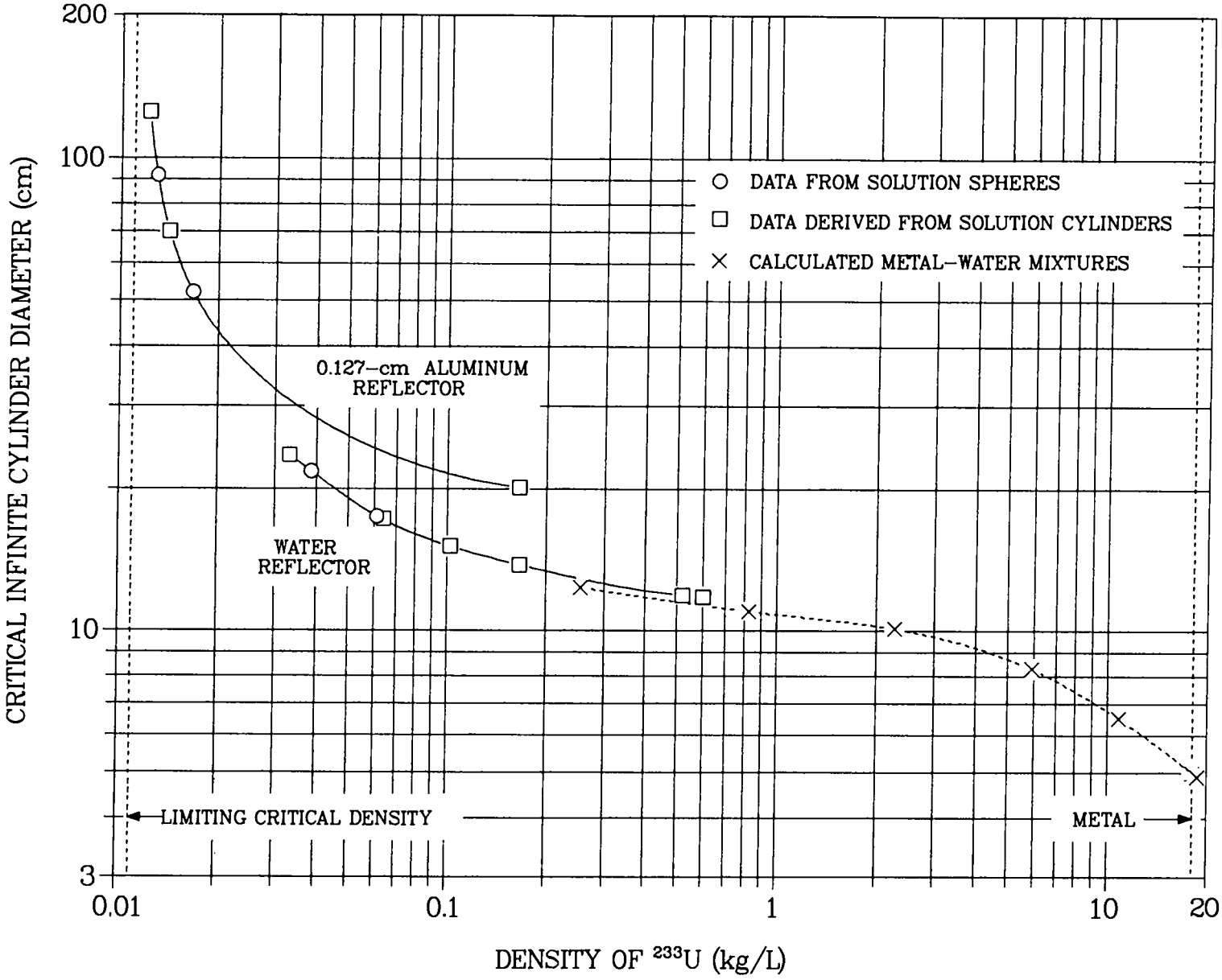


Fig. 38. Estimated critical diameters of infinitely long cylinders of homogeneous water-moderated ^{233}U. Solution data appear unless indicated otherwise.

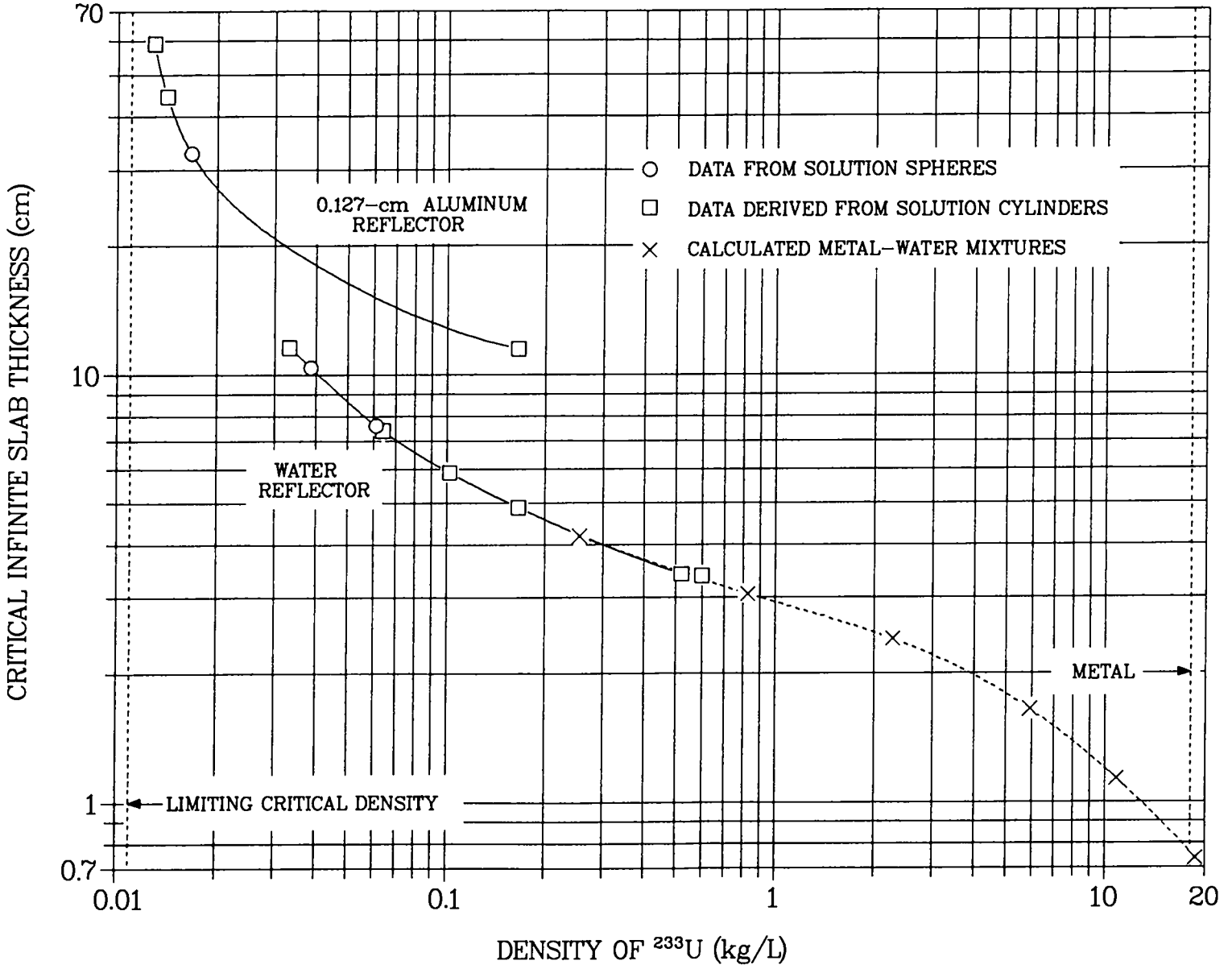


Fig. 39. Estimated critical thicknesses of slabs, infinite in other dimensions, of homogeneous water-moderated ^{233}U. Unreflected infinite slabs are fictitious. Solution data appear unless indicated otherwise.

POISONED SYSTEMS

HOMOGENEOUS ABSORBERS

Uranium 235

Changes in the concentration of uranium required to compensate for boric acid additions to certain critical $UO_2(NO_3)_2$ solutions have been established at ORNL.[51] All assemblies were essentially unreflected 59-cm-diam spheres. For U(93.2), initially critical at $H/^{235}U$ = 1380, the system remained critical when ~1.55 atoms of ^{235}U were added per atom of boron at concentrations up to 0.23 g B/L. Data also exist on poisoning by nitrate, phosphate, and bismuth.[44]

Hanford critical experiments[114] that determined the influence of cadmium nitrate dissolved in $U(85)O_2(NO_3)_2$ solution, or in the water reflector, supported the design of dissolvers at the Idaho Chemical Processing Plant. Solution was contained in a cylinder 24.2-cm-i.d. by 107-cm inside height. Water reflector extended to the top of the vessel and was underneath the tank. The observed critical conditions are summarized in Table 22.

ORNL also reported experiments related to the design of dissolvers. In one series[84], aluminum-clad, uranium-bearing, fuel plates were latticed in water, in dilute $U(92.6)O_2(NO_3)_2$ solution, and in uranyl-nitrate solution in which boron was also dissolved. In another series,[115] $U(62.4)O_2$-BeO fuel pins were latticed in water, in boron solution, in dilute $U(92.6)O_2(NO_3)_2$ solution, and in water containing both boron and uranyl nitrate.

TABLE 22. EFFECTS OF DISSOLVED CADMIUM NITRATE ON THE CRITICAL HEIGHT OF WATER-REFLECTED $U(85)O_2(NO_3)_2$ SOLUTION[a] IN A 24.2-cm-i.d. CYLINDER

Cadmium Concentration (g/L)		Critical Height
in Solution	in Reflector	(cm)
0	0	22.33
0	15	30.58
1.98	0	28.20
3.98	0	37.75
6.35	0	76.05

[a]The solution contained 0.194 M excess nitrate.

Additional information about homogeneously distributed poison is yielded by PCTR experiments, namely, the concentration required in a fissile mixture to reduce k_∞ to unity. Figure 40 gives results for the effect of boron in homogeneous $U(3.04)O_3$-polyethylene mixtures at various $H/^{235}U$ atomic ratios[116] and also includes a point for a $U(2.00)F_4$-paraffin mixture.[117] Figure 41 gives the boron concentration required to reduce k_∞ to unity for optimally moderated homogeneous mixtures of hydrogen and uranium as a function of the uranium enrichment. The values at 3.04 and 1.03 wt% were established experimentally;[118] the value at 2.00 wt% was estimated from the one measurement shown in Fig. 40. The extrapolation beyond 3 wt% is tentative.[14]

Plutonium Solutions

Hanford data on the influence of excess nitrate on the critical mass of water-reflected plutonium solutions[26] was shown in Fig. 28. There is less extensive information from the same source on poisoning by lithium, bismuth, and phosphate.

Effects of gadolinium as an absorber in plutonium-nitrate solutions were measured at Hanford in a 61-cm-diam cylinder reflected with water on the lateral surface and base.[119] For a solution containing 116 g Pu/L, 8.3% ^{240}Pu, and 1.85 $\underline{N}$ HNO_3, critical heights ranged from about 17 cm to 80 cm as gadolinium was added progressively to a concentration of about 2.4 g/L. According to calculations based on the experimental results, 4 g Gd/L would reduce k_∞ to unity. Similar measurements with solution at 363 g Pu/L, 8.3% ^{240}Pu, and 4.1 $\underline{N}$ HNO_3 extended the absorber concentration to 20 g Gd/L. In this case the deduced gadolinium concentration for $k_\infty = 1$ is about 70 g/L.

Critical radii of water-reflected spheres of Pu(4.7% ^{240}Pu)-U solution containing gadolinium were also established at Hanford.[120] Results are summarized in Table 23.

Other Hanford experiments with absorbers included effects of randomly packed glass or stainless steel Raschig rings in plutonium-nitrate solution,[121] and comparisons of plates of various absorbers extending through Plexiglas-reflected PuO_2-UO_2-polystyrene assemblies.[122,123] In the first case, glass Raschig rings, 3.8-cm-diam by 4.3-cm long and 0.32-cm wall thickness, contained either 0.5 or 4 wt% boron, and stainless-steel rings, 1.27-cm-diam by 1.27-cm long and 0.16-cm wall thickness, contained 1 wt% boron.

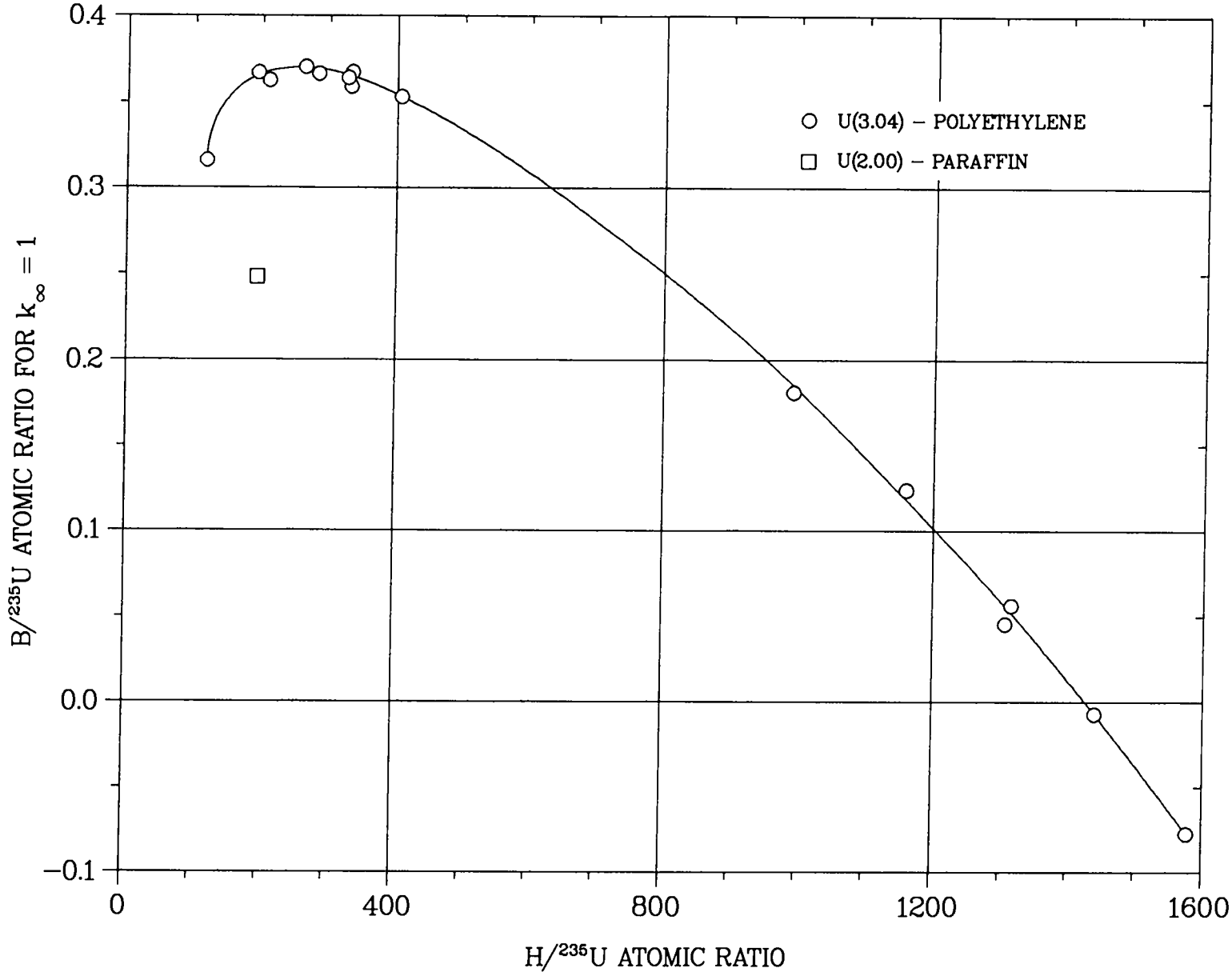


Fig. 40. Boron concentrations required for $k_\infty = 1$ in homogeneous hydrogen-moderated uranium enriched to 2% and 3% ^{235}U.

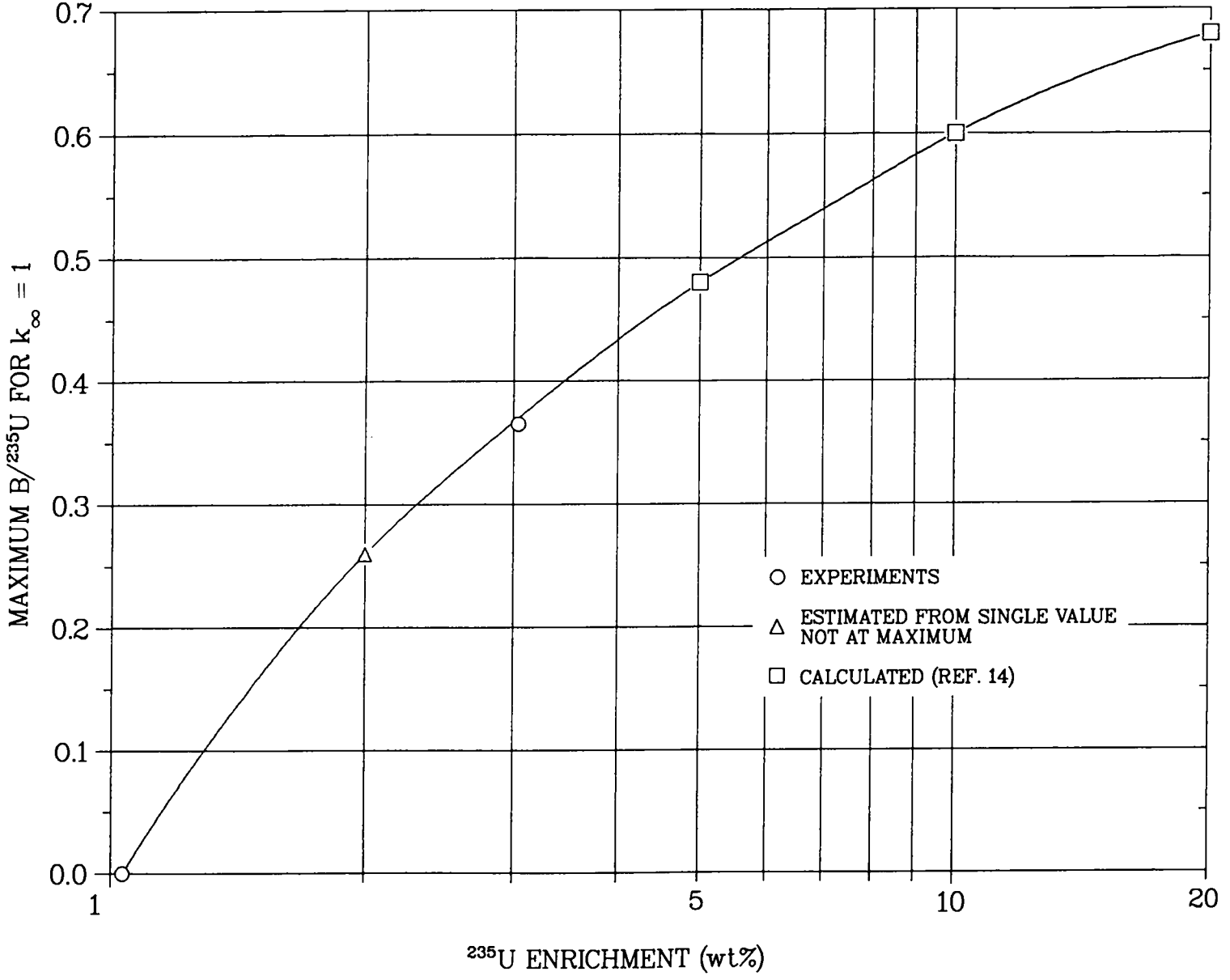


Fig. 41. Maximum boron-to-uranium atomic ratios required for $k_\infty = 1$ vs ^{235}U enrichment in homogeneous hydrogen-moderated uranium.

TABLE 23. CRITICALITY OF PLUTONIUM - URANIUM SOLUTIONS CONTAINING GADOLINIUM

Sphere Wall Thickness(cm)	wt% Pu in Pu + U	g Pu/L	g Gd/L	HNO_3 Normality	Critical Radius(cm) Observed	Calc.,no Gd
0.112	31	70.9	0.051	3.12	17.87	17.14
0.122	32	35.0	0.025	1.49	19.30	18.36
0.122	15	45.6	0.005	2.1	19.31	19.26

Plutonium-nitrate solutions ranging from 63 to 412 g Pu/L were critical in 30.5-, 45.7-, and 61-cm diam water-reflected tanks without Raschig rings but only in the 61-cm tank with 0.5 wt% borated rings. Exponential measurements with the 4 wt% borated rings indicated that bucklings were negative for all plutonium concentrations below 391 g/L. Similarly, with 1 wt% borated stainless steel rings, bucklings were negative for concentrations below 412 g Pu/L.

Two sets of measurements were reported for reflected PuO_2-UO_2-polystyrene assemblies at H/(Pu+U) = 30.6 with 14.6 wt% PuO_2 in the PuO_2+UO_2, and H/(Pu+U) = 2.8 with 29.3 wt% PuO_2 in the PuO_2+UO_2[122,123]. The first composition was selected to give a relatively soft neutron-flux spectrum, and the second composition a hard spectrum. In the first set either one or two plates of copper, copper plus 1% cadmium, or aluminum extended through the core near its midplane. Critical conditions were described adequately for comparison with Monte Carlo calculations. In the second set of experiments a single absorbing plate extended through assemblies of the same compositions. In order of effectiveness, the absorbers tested were boral, cadmium, stainless steel containing 1.6 wt% boron, stainless steel containing 1.1 wt% boron, stainless steel without boron, uranium depleted to 0.2 wt% ^{235}U, and lead. Again, the critical assemblies were well described.

Uranium 233 Solutions

The effect of nitrogen as an absorber has been indicated by the comparison of critical masses of $^{233}UO_2(NO_3)_2$ solutions with $^{233}UO_2F_2$ solutions in Table 21. Further, the influence of borosilicate glass Raschig rings in ^{233}U solution is included in Table 24. The only other experiments with poisoned ^{233}U solutions, also at Oak Ridge, gave the effect of boron on the critical concentration of ^{233}U in a 69-cm-diam sphere of $^{233}UO_2(NO_3)_2$ solution with little excess nitrate.[51] In summary, the addition of H_3BO_3 in an amount corresponding to 0.091 g B/L required an increase of ^{233}U concentration from 16.7 g/L ($H/^{233}U$ = 1530) to 19.4 g/L ($H/^{233}U$ = 1320) to maintain criticality. This represents about 0.75 atoms of boron per atom of added ^{233}U.

HETEROGENEOUS ABSORBERS

Experiments at ORNL resulted in definitive measures of the effectiveness of borosilicate-glass Raschig rings as fixed poisons in $U(93)O_2(NO_3)_2$ solutions[124] and $^{233}UO_2(NO_3)_2$ solutions.[113] All measurements were made in cylindrical geometry, with the amount of boron in the rings, the cylinder diameter, the ring dimensions, and the solution concentration as variables. The natural boron content of the rings varied from 0.5 to 5.7 wt%, and the portion of the vessel volume occupied by the glass ranged from 20.9 to 30 percent. Solutions containing between 415 and 63.3 g U/L were used. Results from exponential experiments, with the use of a critical layer of solution above the column of solution-ring mixture as a neutron source, provided estimates of the material buckling of the subcritical mixture as functions of solution concentration, boron content of the glass, and the glass volume present. These U(93) solution experiments indicated that $k_\infty \leq 1$ if the rings were uniformly distributed under the conditions of Table 24.

TABLE 24. EXPERIMENTALLY DETERMINED CONDITIONS FOR $k_\infty \leq 1$ WHEN RASCHIG RINGS ARE UNIFORMLY DISTRIBUTED IN $U(93)O_2(NO_3)_2$ OR IN $^{233}UO_2(NO_3)_2$ SOLUTIONS

Isotope	Minimum Volume Fraction Occupied by Glass (%)	Minimum Boron Content of Glass (wt%)	Maximum Solution Concentration (g U/L)
U(93)	24	3.3	415
	22	4.0	415
	24	0.5	72
^{233}U	38	3.9	333
	33	3.9	204

Two series of subcritical measurements were conducted at Rocky Flats for hydrogen-moderated $U(93)O_2(NO_3)_2$ systems that contained randomly packed borosilicate-glass Raschig rings. In one case the glass displaced 17.8 vol% of solution in a 107-cm-diameter tank that was surrounded by a close-fitting square concrete vault with open top.[125] The rings, 3.8 cm-long and 3.8-cm-o.d. with 0.20-cm-thick walls, contained 0.090 g B/cm^3. Results indicate that such a tank, if filled to infinite height, would not be critical with a solution containing ~325 g $^{235}U/L$. Transformation of dimensions gave 70 cm as the radius of the corresponding sphere and 67 cm as the thickness of a corresponding infinite slab.

In the other case, rings (4.5 cm long and 3.8-cm o.d. with 0.32-cm-thick walls and containing 0.135 g B/cm^3) were embedded in slabs of $U(93)O_2(NO_3)_2 \cdot 4H_2O$, which were then built into concrete-reflected parallelepipeds.[126] In some instances moderation was increased by the insertion of thin Plexiglas sheets. Typically, a 57.6-cm square core was judged to be critical at a height of 45 cm with H/U = 8.01, ρ(U) = 1.058 g/cm^3, and 18.8 vol% glass. The critical height was judged to be 40 cm with H/U = 11.14, ρ(U) = 0.934 g/cm^3, and 16.4 vol% glass. The corresponding estimated sphere radii were 31.0 cm for the first composition and 29.7 cm for the second. Infinite-cylinder diameters were about 39.9 and 38.6 cm, and infinite-slab thicknesses were about 20.8 and 19.8 cm. Deduced percentage of dimensional increase per volume percent of glass for spheres, infinite cylinders and infinite slabs were also given.

Additional ORNL critical data[127–130] and Rocky Flats[126] subcritical extrapolations on the effects of heterogeneous poisons on solutions containing ^{235}U at high enrichments are listed in Table 25. Shape conversions of the Rocky Flats results for parallel boron-steel plates in $U(93)O_2(NO_3)_2$ solutions lead to the following conclusions for essentially unreflected systems. At H/^{235}U = 82 and with 2.5-cm plate separation, the steel increased the critical sphere radius by a factor of 2.4. At H/^{235}U = 100 the corresponding factor is 4.0 with 2.5-cm plate separation and 1.9 with 3.8-cm separation. At H/^{235}U = 225, the factor is 3.7 with 3.8-cm separation and 1.4 with 5.4-cm separation. The increases of infinite-cylinder diameters are only slightly greater than the above ratios. Corresponding increases of infinite-slab thicknesses are 13 to 24% larger than the sphere-radius factors ($\leq$ 20% when the sphere factor is $\geq$ 2).

Data originating at Hanford,[131] BNL,[76,132] and Westinghouse Electric Corporation Atomic Power Department[133] from subcritical lattices of uranium rods of low enrichment poisoned with boron homogeneously distributed in the moderator water are given in Table 26. The amount of boron required for $B^2 = 0$ was determined by interpolation of bucklings measured with different amounts of boron in the moderator.

Other Hanford experiments[134] established the effect of soluble gadolinium or boron on the critical number of $U(4.3)O_2$ fuel pins in triangular lattices with pitches of 2.29, 2.79 (near optimum), and 3.30 cm. The fuel pins consisted of 1.26-cm-diam by 82-cm-long UO_2 clad with 0.08-cm-thick stainless steel. At a pitch of 2.79 cm, for example, 0.12 g Gd/L doubled the critical number of pins (about 167 without absorber) and 0.37 g B/L had about the same effect.

The effect of boron dissolved in the water moderating and reflecting lattices of U(4.95) metal rods was mentioned earlier. A more detailed account of 0.140 g B/L in triangular lattices is given in Table 27.[67]

TABLE 25. CRITICAL DIMENSIONS OF ^{235}U SOLUTIONS WITH HETEROGENEOUS POISONS

Container	Solution	Reflector	Poison	Critical Height (cm)
38.1-cm-diam stainless steel cylinder[129]	$U(93)O_2F_2$ $H/^{235}U=73.0$	Water	136 steel rods 2.22-cm-diam (49.2 vol% of core)	95.2
76.2-by 152.4-cm aluminum tank[128]	$U(93)O_2F_2$ $H/^{235}U=78.7$	Water (half reflected)	10 parallel 0.63-cm Boral plates containing about 0.3 g B/cm^2, clad with 0.16-cm stainless steel, and spaced 5.8-cm apart	17.6
25.4-cm-diam aluminum cylinder in 0.63-cm-thick copper[127]	$U(93)O_2F_2$ $H/^{235}U=52.6$	Water on sides and bottom	Close-packed copper tubing 4.2-cm o.d. and having an 0.56 cm effective wall thickness occupying 33.7 vol%	152.4
50.8-cm-diam stainless steel cylinder[130]	$U(87)O_2(NO_3)_2$ $H/^{235}U=81.4$	Water on sides and bottom	Pyrex tubing or rings $\leq$ 5.1-cm i.d. and containing $\sim$0.28 g B/cm^3	
			7.8 vol% glass	24.8
			9.45 vol% glass	29.5
			11.5 vol% glass	34.6
			13.3 vol% glass	49.8
			13.95 vol% glass	76.5
			16.7 vol% glass	Subcritical at 91.4
50.8-cm-diam stainless steel cylinder[130]	$U(87)O_2(NO_3)_2$ $H/^{235}U=141$	Water on sides and bottom	Pyrex tubing 5.1-cm-i.d. and containing $\sim$0.28 g B/cm^3 7.8 vol% glass	31.7

TABLE 25. (cont.)

Container	Solution	Reflector	Poison	Critical Height (cm)
50.8-cm-diam stainless steel cylinder[130]	$U(87)O_2(NO_3)_2$ $H/^{235}U=276$	Water on sides and bottom	Pyrex tubing 5.1-cm-i.d. and containing ~0.28 g B/cm^3 7.8 vol% glass	Subcritical at 91.4
90.5-cm-diam steel tank with a 0.48-cm-wall[126]	$U(93)O_2(NO_3)_2$	Concrete on base only	Parallel 0.351-cm-thick stainless steel plates containing 0.080 to 0.094 g B/cm^3 and extending 2.22 cm from bottom and 3.81 cm from wall	
	$H/^{235}U = 82$		2.54-cm plate separation	43.9
	$H/^{235}U = 100$		2.54-cm plate separation	∞
	$H/^{235}U = 100$		3.81-cm plate separation	31.5
	$H/^{235}U = 225$		3.81-cm plate separation	∞
	$H/^{235}U = 225$		5.33-cm plate separation	31.0

TABLE 26. HEXAGONAL LATTICES OF SLIGHTLY ENRICHED URANIUM IN BORON-POISONED MODERATOR WATER

Center Spacing (cm)	Water-to-Uranium Volume Ratio	B^2 Unpoisoned (10^{-4} cm^{-2})	Boron Concentration Required for $B^2 = 0$ (g/L)	$B/^{235}U$
U(1.007) metal (2.35-cm-diam and 112-cm-long clad with 2.53-cm-o.d. aluminum)[131]				
3.56	1.37	27.94	0.71	0.109
3.81	1.74	32.94	0.60	0.117
4.06	2.15	33.41	0.57	0.137
U(1.143) metal (1.52-cm-diam and 122-cm-long clad with 0.071-cm-thick aluminum)[132]				
2.17	1	21.33	0.82	0.080
2.40	1.5	40.23	1.01	0.147
2.61	2	48.22	0.83	0.162
2.98	3	47.12	0.49	0.145
3.32	4	36.03	0.28	0.111
U(1.299) metal (1.52-cm-diam and 122-cm-long clad with 0.071-cm-thick aluminum)[132]				
2.17	1	32.11	1.22	0.106
2.40	1.5	51.87	1.39	0.180
2.61	2	61.08	1.12	0.193
2.98	3	60.99	0.69	0.174
3.32	4	50.38	0.43	0.148
U(1.143) metal (0.98-cm-diam and 122-cm-long clad with 0.071-cm-thick aluminum)[77]				
1.44	1	12.03	~0.066	
1.59	1.5	31.21	0.91	0.134
1.72	2	42.26	0.86	0.171
1.96	3	46.18	0.60	0.176
2.17	4	40.14	0.39	0.154
$U(2.7)O_2$ [0.76-cm-diam clad with 0.78-cm i.d. 0.041-cm-thick stainless steel; $\rho(UO_2) = 10.18$ g/cm^3] (Ref. 133)				
1.03	2.2	40.75	1.6	0.149
1.10	2.9	53.23	1.5	0.182
1.19	3.9	63.26	1.3	0.212

TABLE 27. CRITICALITY OF U(4.95) METAL RODS LATTICED IN WATER AND IN WATER CONTAINING 0.140 g B/L

Description of Triangular Lattice[a]				Critical Mass (kg ^{235}U)	
Rod Diam. (cm)	Pitch (cm)	U Density (g/L)	Lattice Height(cm)	Water	Boron Solution
2.49	4.72	234.0	30	6.16	7.58
			60	8.42	10.02
2.07	4.45	180.6	30	5.15	6.87
			60	6.97	quad 8.91
1.31	3.21	140.6	30	3.58	4.59
			60	4.89	6.02
0.76	2.20	103.6	30	2.53	3.30
			60	3.47	4.33

[a]The minimum critical mass was attained at a rod diameter of about 0.17 cm.[67]

FISSILE UNITS WITH VARIOUS REFLECTORS

REFLECTORS ABOUT URANIUM AND PLUTONIUM METAL

Critical masses of unmoderated U(93.5) metal spheres are given for various reflectors as functions of reflector thickness in Fig. 42 and Table 28. These data originated at Los Alamos[20,38,135,136] and Livermore.[56] Table 29 adds critical masses of uranium metal at various ^{235}U enrichments and densities, also reported from Los Alamos.[20]

Experiments at Rocky Flats give critical conditions for various steel-oil (Texaco 522) composite reflectors about spheres of highly enriched uranium metal surrounding a steel ball.[137] Irregularities appear in the curve of critical mass vs thickness of steel reflector within oil at steel thicknesses between 2 and 4 cm. These irregularities are attributed to the resonance in the neutron elastic scattering cross section of iron.

From Aldermaston, the critical dimensions of U(45) metal surrounded by various nonhydrogenous reflectors are given in Table 30, and Table 31 gives critical data for U(37.67) metal cubes surrounded with various reflectors.[138,139]

The less-complete information about reflected spheres of ^{233}U metal, δ-phase ^{239}Pu, and α-phase ^{239}Pu can be represented, as in Fig. 43, in terms of the critical mass of U(93.5) metal enclosed in a reflector of the same composition and thickness. These correlations (based on data from Aldermaston,[139] Los Alamos,[20,135,136,140,141] Livermore,[41,142] and supported by Rocky Flats[42]), which show no systematic distinction between nonmoderating and moderating reflectors, provide a means of estimating critical masses of the other metals from the more abundant data for enriched uranium. Reasonably complete listings of critical masses of plutonium metal and ^{233}U metal appear in Tables 32 and 33.

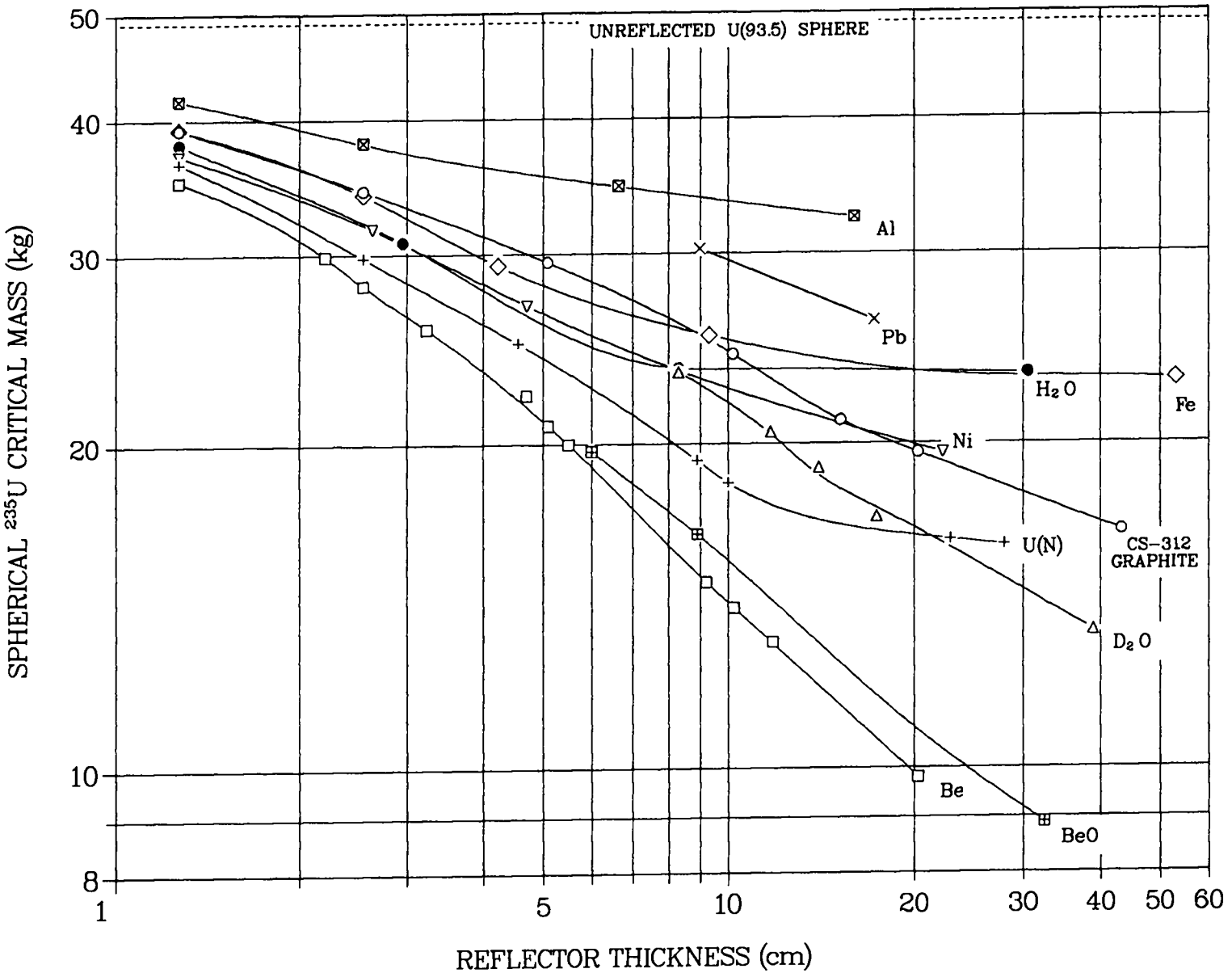


Fig. 42. Critical masses of U(93.5) metal spheres in various reflectors. Uranium density = 18.8 g/cm^3.

TABLE 28. CRITICAL MASSES OF SPHERICAL U(93.5) METAL WITH VARIOUS REFLECTORS

Reflector Material	Density (g/cm^3)	Critical Mass[a], kg of ^{235}U at a Density of 17.6 g/cm^3 Reflector Thickness (cm)				
		1.27[b]	2.54[b]	5.1	10.2	Infinite
Beryllium, type QMV	1.84	35.1	28.1	20.8	14.1	—
BeO (ρ=2.69 g/cm^3)	—	—	21.3	15.5	~8.9	
WC	14.7	—	—	21.3	16.5	~16.0
Uranium	19.0	35.6	29.3	23.5	18.4	16.1
Tungsten alloy (~92 wt% tungsten)	17.4	35.8	29.7	24.1	19.4	—
Molybdenum (99.8 wt%)	10.53	36.9	31.0	—	—	—
Paraffin	—	—	—	—	—	21.8
Polyethylene	0.921	38.9	30.8	—	—	—
H_2O	—	—	—	~24.0	22.9	22.8
D_2O	—	—	—	~27.0	21.0	~13.6
Cobalt	8.72	36.7	31.2	—	—	—
Copper	8.88	37.1	31.3	25.4	20.7	—
Nickel	8.88	36.4	31.2	25.7	~21.5	19.6
Al_2O_3	2.76	38.9	34.1	—	—	—
Graphite, type CS-312	1.67	39.3	34.4	29.5	24.2	~16.7
Iron	7.87	38.5	34.1	29.3	25.3	23.2
Zinc	7.04	—	—	29.8	25.0	—
Thorium	11.48	—	—	33.3	—	—
Lead	11.3	—	—	—	29.5	—
Aluminum, type 1100	2.70	41.7	38.1	~35.5	~32	<30.0
Titanium (96.5 wt%)	4.50	41.9	38.6	—	—	—
Magnesium	1.77	42.5	39.9	—	—	—

[a] Some of these masses were adjusted to the indicated reflector thickness by interpolation of the measurements.

[b] These masses were obtained by transformation of results from cylinders.

TABLE 29. CRITICAL MASSES OF URANIUM METAL AT VARIOUS ^{235}U ENRICHMENTS, LOS ALAMOS DATA

Uranium Core			Reflector			
Shape	Composition	Density (g U/cm^3)	Material	Thickness (cm)	Density (g/cm^3)	Critical Mass (kg ^{235}U)
sphere	U(93.71)	18.74	none			49.12 ± 0.15
sphere	U(97.67)	18.79	H_2O	>17	1.00	21.64 ± 0.04
sphere	U(93.9)	18.5	H_2O	>30	1.00	23.2 ± 0.2
sphere	U(93.7)	18.5	D_2O (99.8%)	39		13.4 ± 0.1
sphere	U(93.9)	18.5	graphite (CS-312)	43	1.66	17.0 ± 0.3
sphere	U(93.6)	18.6	Be (QMV)	11.8	1.84	13.1 ± 0.2
sphere	U(93.24)	18.62	U(natural)	18.0	18.62	16.63 ± 0.04
sphere	U(93.90)	18.69	U(natural)	10.0	19.00	18.61 ± 0.09
sphere	U(93.99)	18.67	U(natural)	4.42	18.67	25.0 ± 0.1
sphere	U(93.91)	18.70	U(natural)	1.73	19.00	34.3 ± 0.2
pseudosphere[a]	U(94.13)	18.70	U(natural)	22.9 av	19.0	16.39 ± 0.07
pseudosphere[a]	U(94)	16.0	U(natural)	22.2 av	19.0	19.7 ± 0.1
pseudosphere[a]	U(94)	15.8	U(natural)	22.2 av	19.0	20.1 ± 0.1
pseudosphere[a]	U(94)	13.1	U(natural)	21.0 av	19.0	25.3 ± 0.2
pseudosphere[a]	U(94)	9.35	U (natural)	18.4 av	19.0	37.0 ± 0.2
pseudosphere[a]	U(80.5)	18.7	U(natural)	22.2 av	19.0	18.3 ± 0.1
pseudosphere[a]	U(67.6)	18.75	U(natural)	21.6 av	19.0	20.8 ± 0.1
pseudosphere[a]	U(66.6)	18.75	U(natural)	21.6 av	19.0	21.2 ± 0.1
pseudosphere[a]	U(47.3)	18.8	U(natural)	19.7 av	19.0	27.1 ± 0.2
d=53.3 cm, h/d=0.65[b]	U(16.01)	18.68	none			232. ± 2.
d=53.3 cm, h/d=0.83[b]	U(14.11)	18.41	none			258. ± 2.
d=53.3 cm, h/d=1.14[b]	U(12.32)	18.64	none			312 ± 2.
d=53.3 cm, h/d=2.24[b]	U(10.90)	18.63	none			540. ± 13.
d=38.1 cm, h/d=0.83[c]	U(16.19)	18.75	U (natural)	7.6	19.0	111. ± 0.6

[a]Minimum 1.27-cm-cubic units.
[b]U(93.3) plates, 0.64-cm-thick, interleaved with natural uranium.
[c]Reference 69.

TABLE 30. CRITICAL PARAMETERS OF U(45.5) METAL WITH VARIOUS REFLECTORS

Reflector		Core	
Material	Thickness (cm)	Effective Radius (cm)	Critical Height (cm ± 0.1)
None	0	14.7	18.4
		21.5	13.9
Natural uranium (ρ = 16.3 g/cm^3)	15.2[a]	14.7	8.3
		21.5	6.0
		28.1	5.1
Graphite (ρ = 1.74 g/cm^3)	15.2[a]	14.7	7.6
		21.5	5.4
		28.1	4.52
Borated graphite (~5 wt% boron)	20.3[b]	14.7	9.6
		21.5	7.1
		28.1	6.1
Steel (ρ = 7.24 g/cm^3)	15.2[c]	14.7	10.5
		21.5	7.6
		28.1	6.5
Aluminum (ρ = 2.58 g/cm^3)	15.2[d]	14.7	11.2
		21.5	8.2
		28.1	6.9

[a] Critical heights corrected for the effect of additional 0.33 and 0.17-cm-thick steel reflector on top and bottom slab faces, respectively.

[b] 15.2-cm-thick graphite reflector without boron on sides; additional 0.95-cm-thick aluminum on top and on bottom.

[c] 16.7-cm-thick top reflector.

[d] 16.2-cm-thick top reflector.

TABLE 31. CRITICAL DIMENSIONS OF U(37.67) METAL CUBES WITH VARIOUS REFLECTORS

Reflector			Critical Dimensions[a]		
Material	Density (g/cm^3)	Thickness (cm)	Length (cm)	Volume (L)	Mass (kg ^{235}U)
Air			26.4 ± 0.2	18.4 ± 0.4	123.6 ± 3.0
Polyethylene	0.919	2.54	22.5 ± 0.2	11.4 ± 0.3	76.7 ± 2.0
Polyethylene	0.919	5.11	20.1 ± 0.2	8.08 ± 0.2	54.2 ± 1.4
Polyethylene	0.919	10.2	18.7 ± 0.2	6.53 ± 0.2	43.8 ± 1.4
Polyethylene	0.919	15.3	18.4 ± 0.2	6.27 ± 0.2	42.1 ± 1.8
Polyethylene	0.919	20.3	18.35 ± 0.2	6.17 ± 0.02	41.4 ± 0.2
Polyethylene plus Cadmium[b]	0.919	20.3	23.0 ± 0.2	12.15 ± 0.2	81.5 ± 2.0
Beech Wood	0.73	20.3	20.2 ± 0.2	8.20 ± 0.2	55.0 ± 1.4
Beech plus Cadmium[b]		20.3	22.9 ± 0.2	11.9 ± 0.2	80.1 ± 2.0
Concrete	2.37	20.3	18.5 ± 0.2	6.36 ± 0.2	42.6 ± 1.0
Concrete plus Cadmium[b]		20.3	20.5 ± 0.2	8.61 ± 0.2	57.8 ± 1.4
Water		20.3	19.0 ± 0.2	6.83 ± 0.2	45.8 ± 1.0

Note: ^{235}U density = 6.71 g/cm^3.

[a]Transformed from measurements with rectangular parallelepipeds.

[b]0.038-cm-thick cadmium between core and reflector.

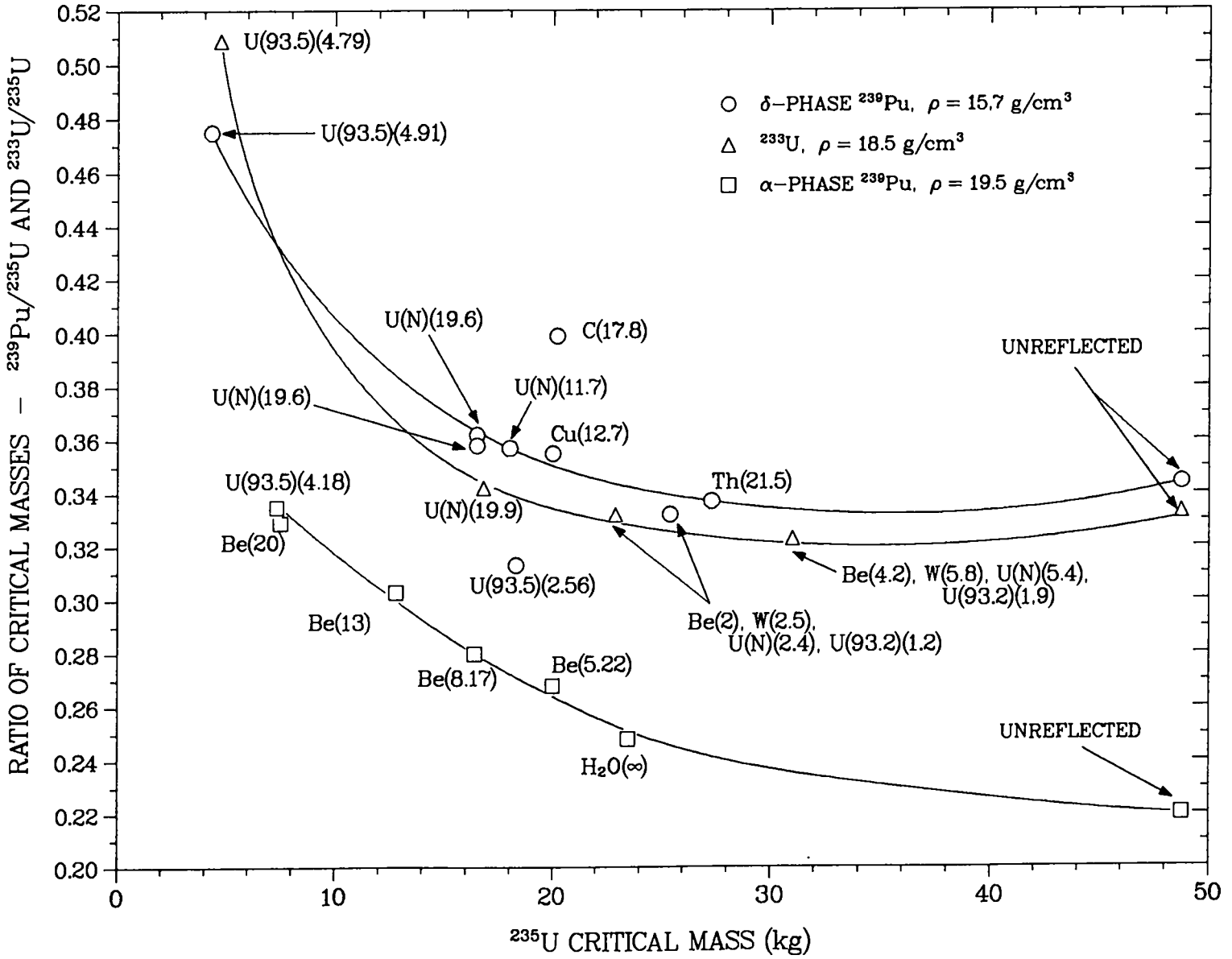


Fig. 43. Critical masses of spheres of ^{239}Pu and ^{233}U metal relative to those of U(93.5) metal with the same reflector. Indicated at each point are the reflector material and thickness in cm.

TABLE 32. CRITICAL MASSES OF PLUTONIUM METAL WITH OR WITHOUT VARIOUS REFLECTORS

Plutonium Core			Reflector			
Shape	^{240}Pu (wt%)	Pu Density (g/cm^3)	Material	Thickness (cm)	Density (g/cm^3)	Critical Mass (kg Pu)[a]
			Livermore[41,142]			
sphere	—	19.25[b]	Be	32 ± 4	1.84	2.47
sphere	—	19.25[b]	Be	21 ± 1	1.84	3.22
sphere	—	19.25[b]	Be	13.0 ± 0.1	1.84	3.93
sphere	—	19.25[b]	Be	8.17 ± 0.03	1.84	4.66
sphere	—	19.25[b]	Be	5.22 ± 0.02	1.84	5.43
sphere	—	15.8	Be	5.25	1.86	7.37
sphere	—	15.8	Be	1.77	1.86	10.8
sphere	—	15.8	C (pile grade)	3.83	1.63	10.8
sphere	—	15.8	Ti	8	4.46	10.8
sphere	—	15.8	U	6.74	18.8	7.37
sphere	—	15.8	U	1.93	18.8	10.8
d=8.23 cm, h/d=1.11	—	15.8	Be	5	1.86	7.62 ± 1%
d=8.23 cm, h/d=1.82	—	15.8	Be	2	1.86	12.5 ± 1%
d=8.23 cm, h/d=1.22	—	15.8	U	5	18.8	8.33 ± 1%
d=8.23 cm, h/d=1.88	—	15.8	Be	2	1.86	12.9 ± 1%
d=9.87 cm, h/d=1.75	—	15.8	none			17.3 ± 4%
d=9.87 cm, h/d=0.71	—	15.8	polyethylene	10	0.92	8.4 ± 4%
d=9.87 cm, h/d=0.64	—	15.8	Be	5	1.86	7.6 ± 1%
d=9.87 cm, h/d=0.90	—	15.8	Be	2	1.86	10.6 ± 1%
d=9.87 cm, h/d=0.86	—	15.8	C (pile grade)	5	1.63	10.1 ± 1%
d=9.87 cm, h/d=1.08	—	15.8	C (pile grade)	2	1.63	12.7 ± 1%
d=9.87 cm, h/d=0.70	—	15.8	U	5	18.8	8.2 ± 1%
d=13.85 cm, h/d=0.577	—	15.8	none			18.7 ± 1%

TABLE 32. (cont.)

Plutonium Core			Reflector			
Shape	^{240}Pu (wt%)	Pu Density (g/cm^3)	Material	Thickness (cm)	Density (g/cm^3)	Critical Mass (kg Pu)[a]
d=13.85 cm, h/d=0.284	—	15.8	Be	5	1.86	9.2 ± 1%[c]
d=13.85 cm, h/d=0.392	—	15.8	Be	2	1.86	12.7 ± 1%[c]
d=13.85 cm, h/d=0.316	—	15.8	U	5	18.8	10.25 ± 1%[c]
d=13.85 cm, h/d=0.400	—	15.8	U	2	18.8	13.0 ± 1%[c]
			Los Alamos[20,136,141]			
sphere	4.5	15.6	none			16.8 ± 1%
sphere	20.1	15.7	none			19.3 ± 1%
sphere	5.2	19.74[b]	water	>30	1.00	5.79 ± 0.6%
sphere	4.9	15.6	Be	3.69 ± 1%	1.83	8.4[c]
sphere	4.9	15.7	Al	7.9 ± 1%	2.82	11.0[c]
sphere	4.9	15.6	W alloy[d]	4.70 ± 1%	17.2	8.4[c]
sphere	5.1	15.25	Th	>21	11.6	9.2
sphere	1.5	15.6	U(Natural)	~24	19.0	5.7 ± 0.02[c]
sphere	4.8_2	15.4	U	19.6	19.0	6.0 ± 0.03[c]
sphere	1.35	15.6	U	11.7	19.0	6.2
sphere	4.9	15.6	U	4.13 ± 1%	18.9	8.4
sphere	4.90	15.62	Be	3.69	1.83	8.39
sphere	4.90	15.62	U(natural)	4.13	18.92	8.39
sphere	4.90	15.62	W alloy[d]	4.70	17.21	8.39
d=5.6 cm, h/d=12.5	5	15.4	water	>30	1.0	27. ± 1.5[c]
d=5.7 cm, h/d=7.1	5	15.4	graphite	17.8	1.60	16.3 ± 0.1[c]
d=5.6 cm, h/d=8.8	5	15.4	U(0.3)	7.6	18.7	20.0 ± 0.1[c]
d=15.2 cm, h/d=0.280	5	14.3	water	>30	1.00	11.1 ± 0.2[c]
d=15.2 cm, h/d=0.273	5	14.3	graphite	17.8	1.60	10.8 ± 0.1[c]

TABLE 32. (cont.)

Plutonium Core			Reflector			
Shape	^{240}Pu (wt%)	Pu Density (g/cm^3)	Material	Thickness (cm)	Density (g/cm^3)	Critical Mass (kg Pu)[a]
d=15.2 cm, h/d=0.390	5	14.3	graphite	2.54	1.60	15.4 ± 0.1[c]
d=15.2 cm, h/d=0.258	5	14.3	U(Natural)	7.6	16.7	10.1 ± 0.1[c]
d=28.0 cm, h/d=0.095	5	13.1[e]	water	>30	1.00	21.4 ± 0.8[c]
d=41.0 cm, h/d=0.049	5	13.1[e]	water	>30	1.00	34 ± 1.2[c]

Note: Plutonium is δ-phase containing 1 wt% gallium and parts are coated with 0.013-cm-thick nickel unless noted otherwise.

[a] Corrected for nickel coating unless noted otherwise.

[b] α-phase plutonium.

[c] Uncorrected for 0.013-cm-thick nickel on pieces.

[d] Composition 91.3 wt% W, 5.5 wt% nickel, 2.5 wt% Cu, 0.7 wt% Zn.

[e] Average diameter and density of overlapping layers of close-packed Pu disks 15.1-cm-diam, 0.312-cm-thick, in 0.013-cm- thick nickel cans with 15.16-cm-diam, 0.343-cm-high outside dimensions.

TABLE 33. CRITICAL MASSES OF ^{233}U METAL SPHERES

^{233}U core		Reflector			
Composition	U density (g/cm^3)	Material	Thickness (cm)	Density (g/cm^3)	Critical Mass (kg ^{233}U)
98.13% ^{233}U 1.24% ^{234}U 0.03% ^{235}U 0.60% ^{238}U	18.42	none			16.2 ± 0.06
same	18.42	U (natural)	19.9	19.0	5.74 ± 0.03
98.25 wt% ^{233}U	18.62	Be	4.20	1.83	7.47
98.25 wt% ^{233}U	18.62	U (natural)	5.31	18.92	7.47
98.25 wt% ^{233}U	18.62	W alloy (91.3 wt% W)	5.79	17.21	7.47
98.25 wt% ^{233}U	18.62	Be	2.04	1.83	9.84
98.25 wt% ^{233}U	18.62	U (natural)	2.30	18.92	9.84
98.25 wt% ^{233}U	18.62	W alloy (91.3 wt% W)	2.44	17.21	9.84

Source: Refs. 20, 140, 141.

REFLECTORS ABOUT HYDROGEN-MODERATED UNITS

The reflector savings effected by several materials on assemblies of $U(1.42)F_4$-paraffin mixtures were measured at Dounreay.* The results led to Fig. 44. Figure 45 shows the ratio of the critical volumes of water-reflected spheres to those of unreflected spheres from the appropriate curves of Figs. 42 and 44 and, from ORNL, reflector savings measurements on $U(93)O_2F_2$ solutions.[33]

Rocky Flats experiments with critical cylinders of $U(93.2)O_2(NO_3)_2$ solution included effects of positioning the cylinders within 122-cm^3 cavities with walls of 26-cm-thick concrete or 21-cm-thick Plexiglas.[143] (Walls of the plastic enclosure were of normal Plexiglas, but top and bottom were of fire-retardant Plexiglas containing 6.5 wt% bromine.) Typical critical heights of solutions in 28.0- and 33.0-cm-i.d. aluminum cylinders with and without reflector shells are given in Table 34. For some diameters, effects of stainless steel instead of aluminum containers were measured.

Additional information about reflected and moderated systems includes the Aldermaston subcritical observations[43] that 20.3-cm-thick Perspex (Plexiglas) is more effective about $U(30.14)O_2$-paraffin at $H/U^{235} = 8.14$ and 16.3 than is polyethylene of the same thickness. This comparison is shown in Table 10. According to ORNL critical results[50] for $U(93)O_2(NO_3)_2$ solution at $H/U^{235} = 62$ to 490, there is little influence on critical size if the inner 6.4-cm of a thick water reflector is replaced by stainless steel. An infinite reflector of steel is more effective than one of water.

A series of experiments, also at ORNL,[144] with $U(5)O_2F_2$ solution at $H/^{235}U$ of 488 investigated the effects of steel reflectors of thicknesses between 0.64- and 5.08-cm on the criticality of cylinders of 33.02- and 39.09-cm-diam. It was shown that 1.27-cm-thick steel surrounding each of the cylinders produced the maximum decrease in critical mass; a 5.08-cm-thick steel reflector was not as effective as was an infinitely thick water reflector.

* J. G. Walford and J. C. Smith, Dounreay Experimental Reactor Establishment, Dounreay, United Kingdom Atomic Energy Authority, 1963.

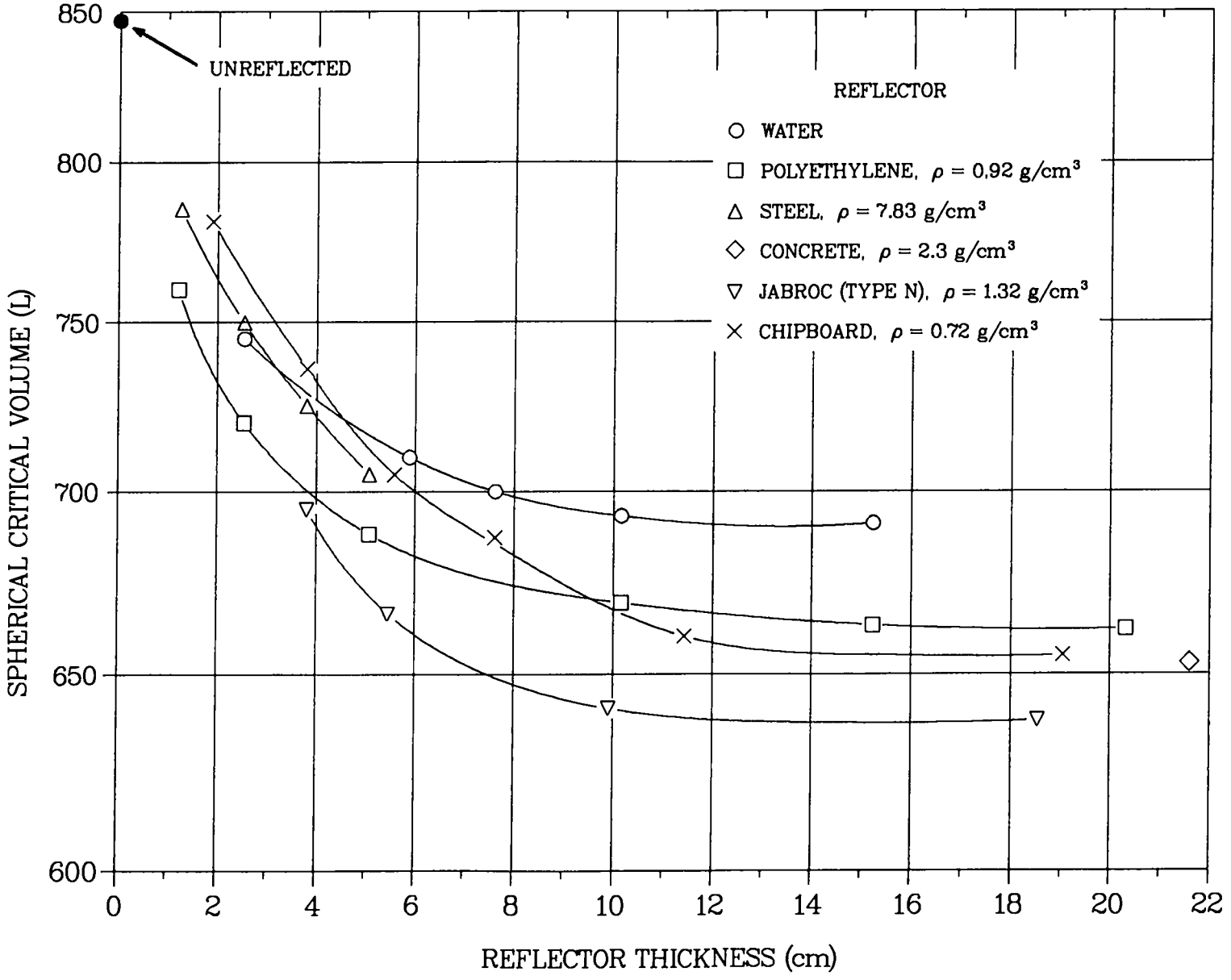


Fig. 44. Spherical critical volumes vs thickness of various reflectors for $U(1.42)F_4$-paraffin at $H/^{235}U = 422$ and $\rho(U) = 2.5$ g/cm³. Jabroc is a wood product containing 45% carbon, 6% hydrogen, and 37% oxygen.

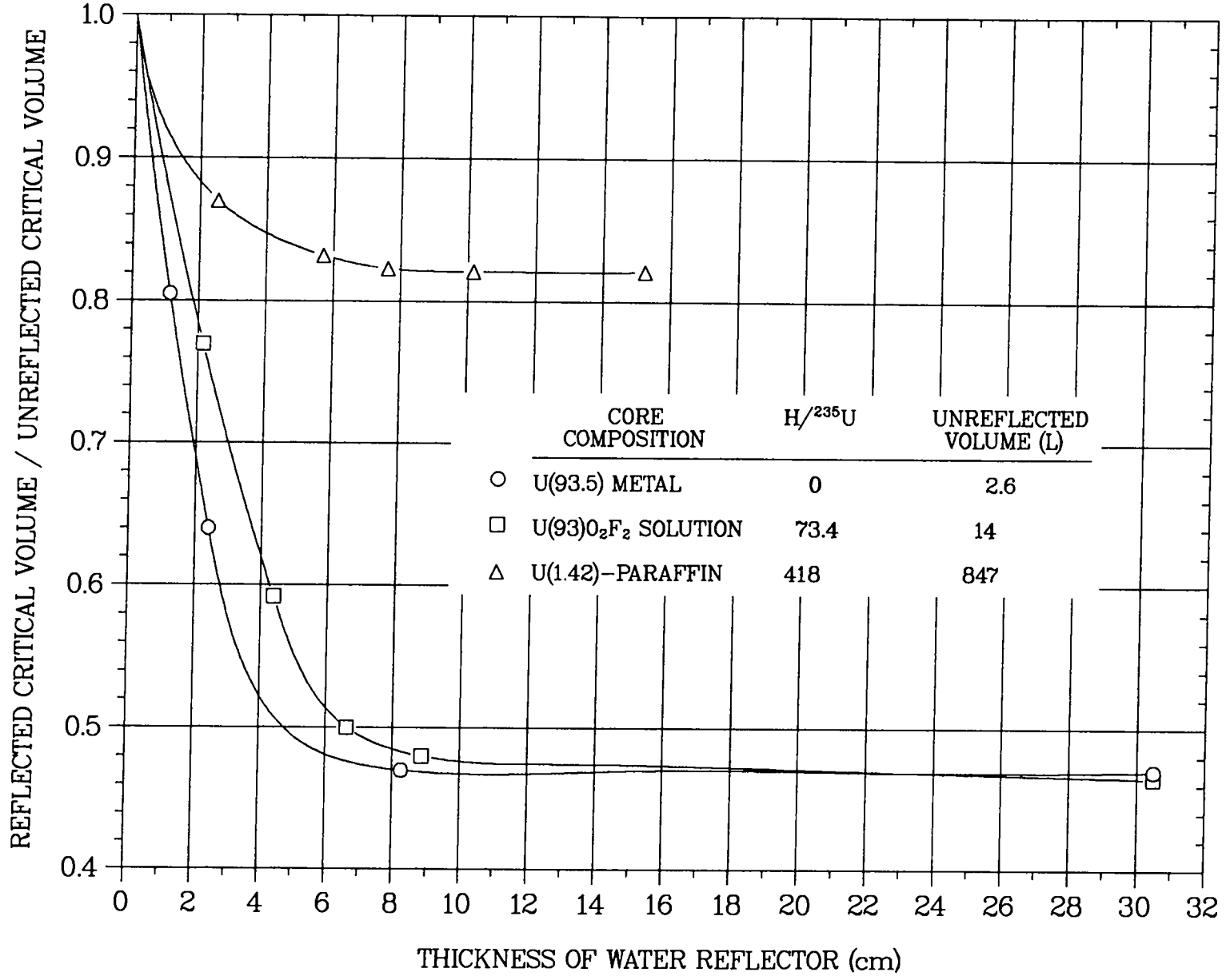


Fig. 45. Ratio of water-reflected to unreflected spherical critical volumes vs reflector thickness for enriched uranium.

TABLE 34. CRITICAL HEIGHTS (h_c) OF $U(93.2)O_2(NO_3)_2$ CYLINDRICAL SOLUTIONS WITHIN A CONCRETE OR PLEXIGLAS ENCLOSURE OR WITH ENCLOSURE REMOVED

Without Enclosure[a]		122-cm-Cubic Cavity in 26-cm-thick Concrete				in 21-cm-thick Plexiglas			
		at Center		in Corner		at Center		in Corner	
(g U/L)	h_c(cm)[b]	(g U/L)	h_c(cm)[b]	(g U/L)	h_c(cm)[b]	(g U/L)	h_c(cm)[b]	(g U/L)	h_c(cm)[b]
				Solution Diameter: 28.0 cm					
143	33.6	144	31.4	144	24.7	148	31.3	148	25.3
358	30.9	335	28.6	335	22.3	345	28.8	345	22.9
				Solution Diameter: 33.0 cm					
60	36.7	60	34.1	60	27.3	60	34.3	60	27.7
146	23.7	144	22.8	144	18.2	148	22.8	148	18.5
358	22.5	335	21.5	335	16.8	345	21.7	345	17.2

[a] Approximately centered in 10-m-cubic room with thick concrete walls.
[b] Standard deviations less than last quoted place.

SYSTEMS WITH NONHYDROGENEOUS DILUENTS

The dilution exponent, n(x), is defined such that the critical mass, m_c, of fissile material when diluted homogeneously with the volume fraction, F, of the material, x, is

$$m_c = m_{co} (1 - F)^{-n(x)},$$

where m_{co} is the critical mass of the undiluted system.

Dilution exponents for various elements in U(94) metal, both unreflected and reflected by ~23-cm-thick natural uranium, and unreflected δ-phase plutonium spheres have been deduced at Los Alamos from reactivity coefficients.[145] The "microscopic" dilution exponents in Table 35 are obtained in this manner. The "macroscopic" values in that table are from critical assembles consisting of 0.3-cm-thick plates of fissile material interleaved with plates of material x.[20,146] Microscopic dilution exponents also exist for plutonium reflected by U(0.3) of greater thicknesses than shown in Table 35. The 5.1-cm depleted uranium thickness shown in the Table was chosen as about equivalent to thick water.

For the efficient moderators, H_2O, D_2O, beryllium, beryllium oxide, and graphite, constant dilution exponents apply to smaller ranges than the apparent dilution exponent range for ^{238}U indicated in Fig. 19 (Livermore,[147–150] ORNL,[151] Los Alamos[152]). Critical masses of spheres of U(93.5) diluted by these materials appear in Fig. 46.

Additional experiments with alternating disks of δ-phase plutonium and graphite have been performed at Rocky Flats.[153,154] Although the data are applicable in instances where the conditions of the experiment are matched, the gross heterogeneity of the assemblies and their restricted geometry preclude generalization.

TABLE 35. DILUTION EXPONENTS n(x)

Diluent	Macroscopic n(x)	Range of F	Microscopic n(x)
	Unreflected 38.1-cm-diam U(93) Assembly[a]		
Al	1.37	0.48-1.00	1.51
Fe	1.26	0.49-1.00	1.29
Ni	1.14	0.48-1.00	1.22
Cu	1.15	0.51-1.00	1.42
Ta	1.35	0.49-1.00	
W	1.12	0.47-1.00	
Th	—	—	1.45
U(0.3)	0.67	0.50-1.00	0.75
	Unreflected 15.2-cm-diam δ-Pu Assembly[a,b]		
Fe(ss)	1.27	0.63-1.00	1.43
Th	1.43	0.63-1.00	1.60
	15.2-cm δ-Pu Core in 5.1-cm-thick U(0.3) Reflector[a,b]		
Al	1.32	0.48-1.00	—
Fe(ss)	1.23	0.48-1.00	—
Th	1.38	0.48-1.00	—

Note: "Microscopic" values are obtained by integrating reactivity coefficients of x; "macroscopic" values are from diluted critical assemblies.

[a] Undiluted assembly adjusted to same h/d as extreme diluted assembly.

[b] Plutonium disks were in thin Ni cans, reducing Pu density to about 90% of normal.

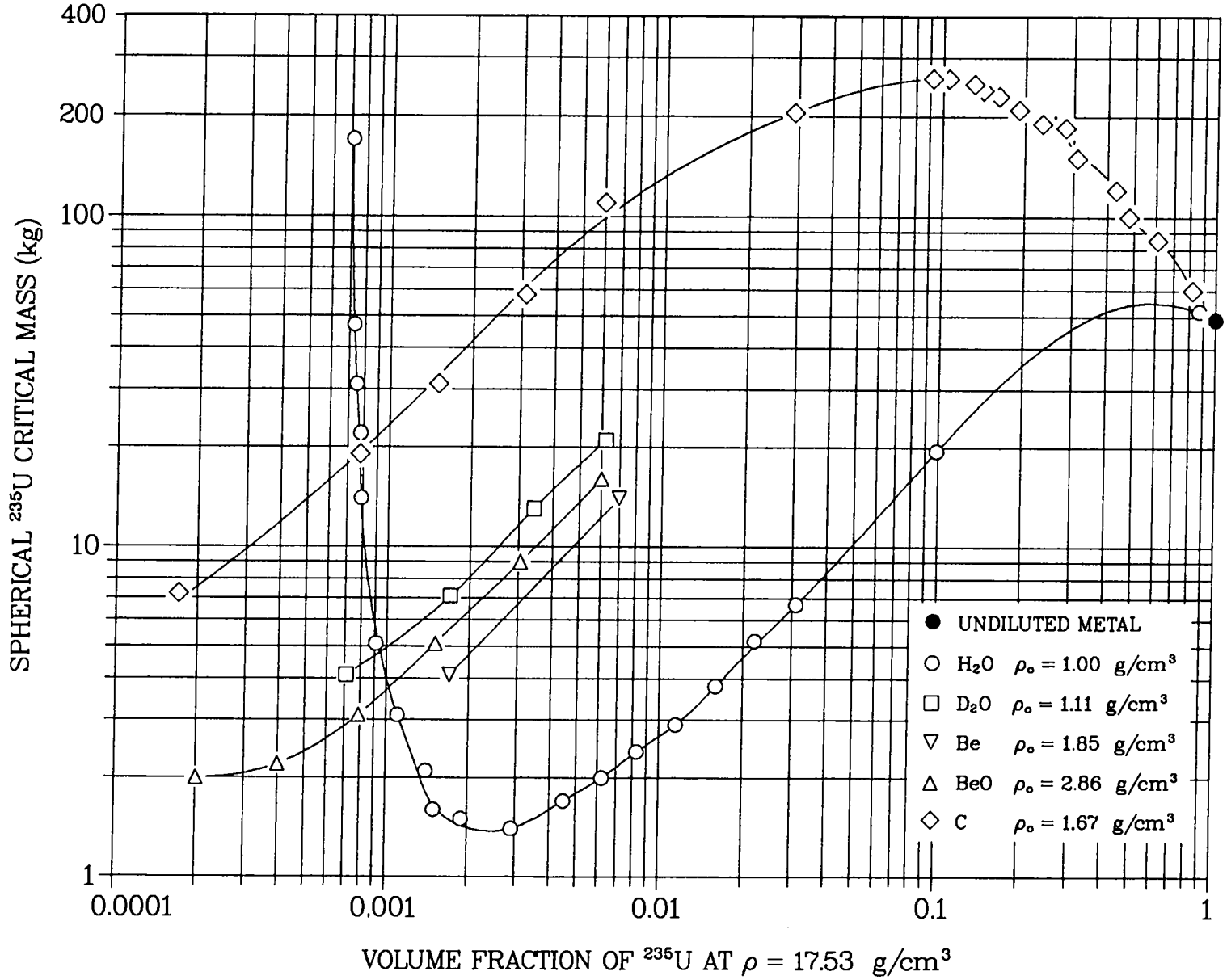


Fig. 46. Critical masses of unreflected spheres of U(93) diluted with various moderators.

COMPLEX SHAPES

ANNULI

The possibility of large-volume solution storage in annular cylinders prompted ORNL investigations that led to the critical data given in Figs. 47 and 48 for annuli of $U(93.2)O_2F_2$ solution with the inner cylinder lined with cadmium and filled with water.[33,155] From the results of tests with a solution having $H/^{235}U = 50.4$ in annuli of a range of dimensions, it was concluded that the critical, infinitely high, externally reflected annulus would have a minimum thickness of between 6.4- and 7.6-cm (see Fig. 47). Decreasing the solution concentration to $H/^{235}U = 309$ increased the minimum thickness of the infinitely high reflected annulus to between 8.9- and 10.2-cm (see Fig. 48). Information was also obtained for solutions at $H/^{235}U$ between 72.4 and 74.6 with the inner cylinder containing either air or water, both with and without a cadmium liner.

A series of Los Alamos experiments on a large annular tank was in support of upgraded fuel reprocessing at the Idaho Chemical Processing Plant.[156] The dimensions of the tank contents were 76-cm-o.d. and 57-cm-i.d. with an average height of 180 cm. Walls of 304-L stainless steel were 0.32-cm-thick radially and 0.64-cm-thick on top and bottom. The base sloped 6° for drainage.

All measurements were with $U(93.05)O_2(NO_3)_2$ solution at 283 g $^{235}U/L$, with 0.56 N HNO_3. The critical height was established with the unmodified tank, with 1.27-cm-thick 304-L steel added to the external surface of the tank, and with 0.95-cm-thick 304-L steel or various thicknesses of polyethylene against the inner surface of the tank. Illustrative solution heights were 129-cm in the unmodified tank, 87-cm with 1.27-cm-thick external steel, and 68-cm with 2.0-cm-thick internal polyethylene in addition to the external steel, increasing to 92 cm with 5.5-cm-thick polyethylene in addition to the external steel.

Critical experiments were performed at Valduc[157] with plutonium solutions in 50-cm-o.d. annuli with 20-, 30-, and 35-cm-i.d. Results for $Pu(NO_3)_4$ solution with 4 N HNO_3 in 20- and 30-cm-i.d. annuli with water reflection and cadmium-lined interior are reproduced in Fig. 49. The 35-cm-i.d. annulus was not sufficiently reactive for inclusion. As with the enriched uranium experiments, less reactive configurations had air or water without cadmium internally or were without water reflectors.

Limited results for critical U(93) metal annuli with various reflectors from Los Alamos,* from ORNL with no reflector, and a large number reflected with graphite[36,158] are listed in Table 36.

* D. P. Wood, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (1961).

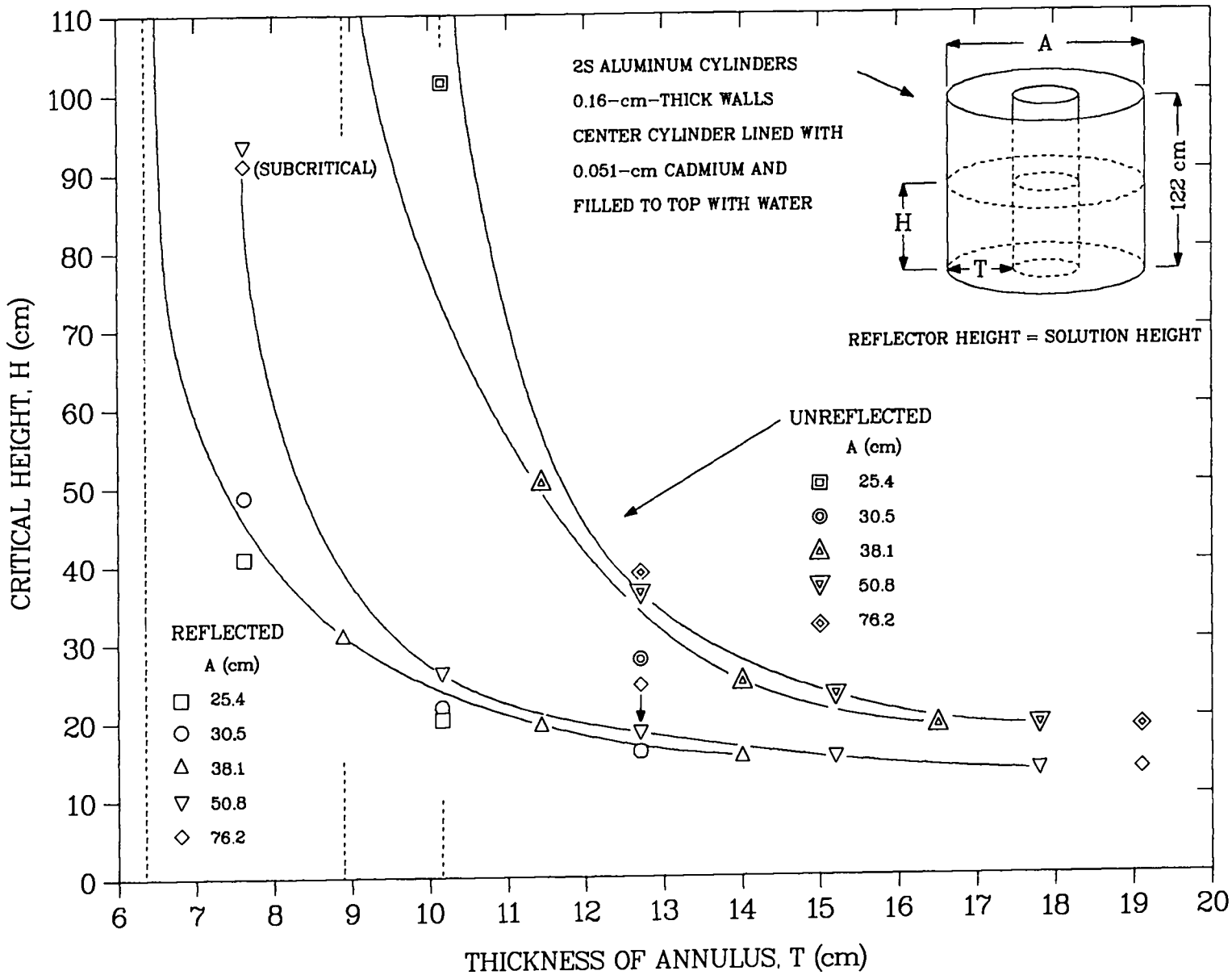


Fig. 47. Critical heights of cylindrical annuli containing $U(93)O_2F_2$ solution at $H/^{235}U = 50.4$ as functions of thickness and outside diameter of solution annulus.

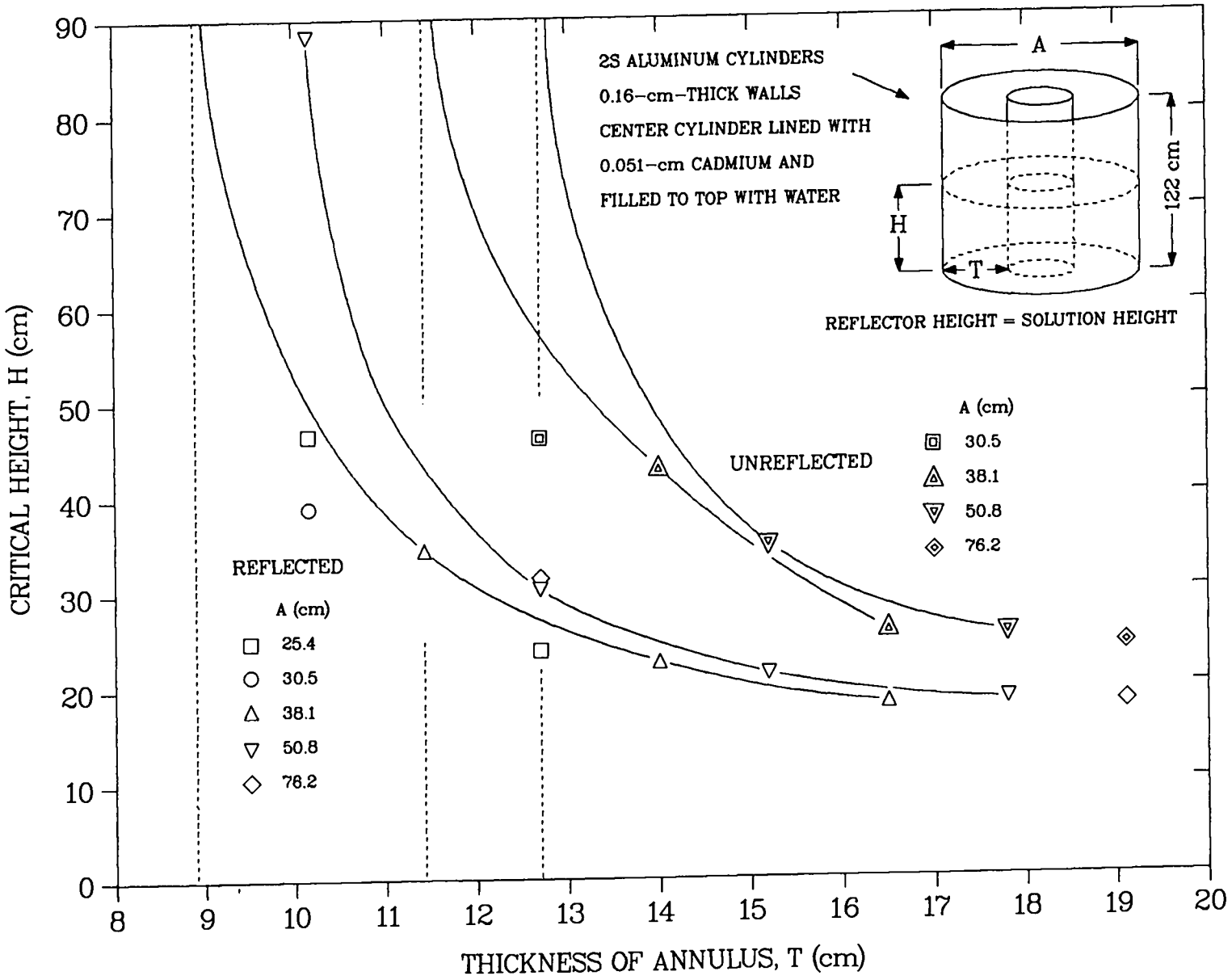


Fig. 48. Critical heights of cylindrical annuli containing $U(93)O_2F_2$ solution at $H/^{235}U = 309$ as functions of thickness and outside diameter of solution annulus.

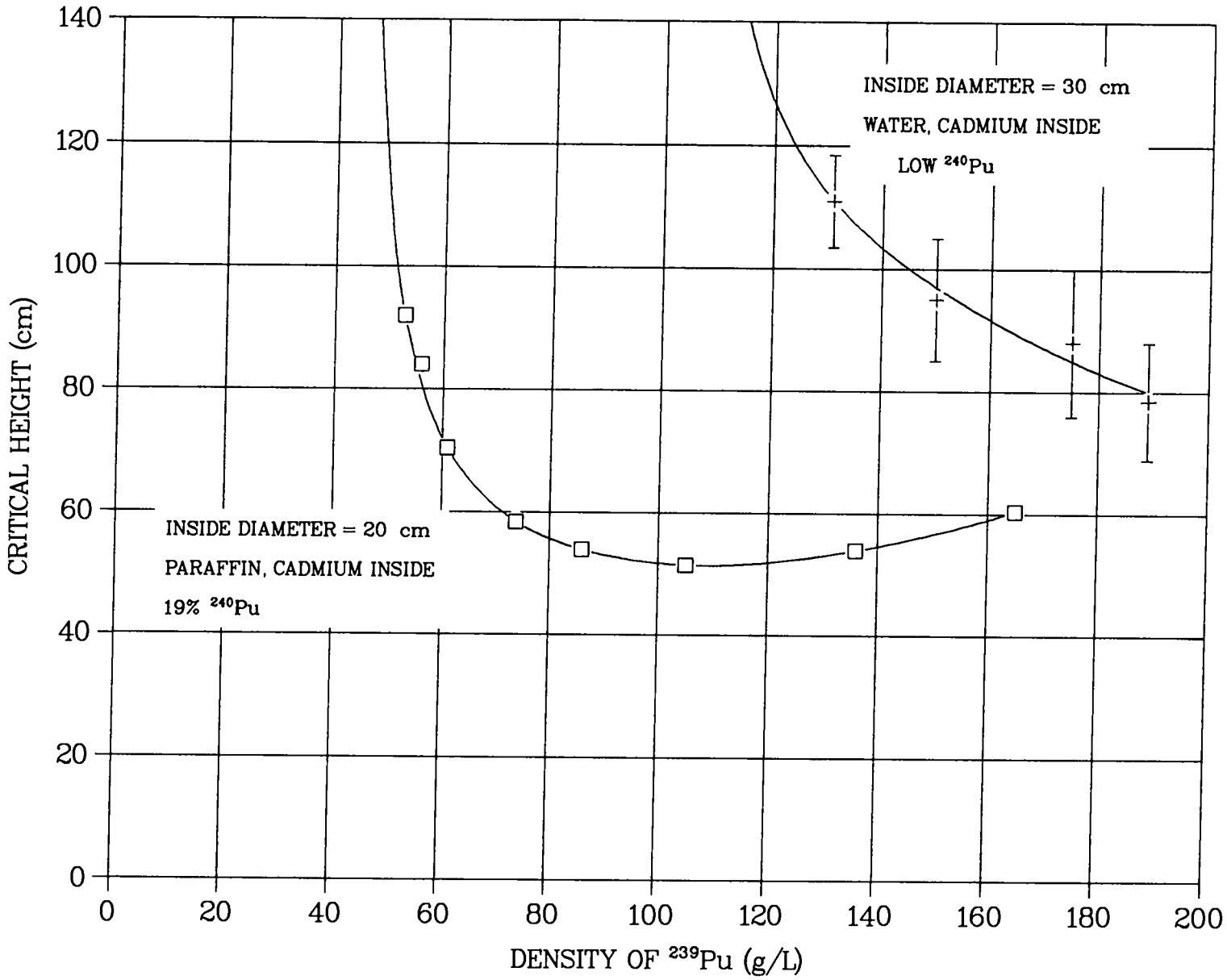


Fig. 49. Critical height of plutonium solution as function of plutonium density in 50-cm-o.d. annuli, water-reflected to height of solution.

TABLE 36. CRITICAL MASSES OF U(93) METAL ANNULI

Reflector[a]		Annular Core			
Material	Thickness (cm)	o.d.(cm)	i.d.(cm)	Critical Core Height(cm)	Critical Mass (kg ^{235}U)
Natural U	7.6	31.1	15.2	5.2	52 ± 0.3
Polyethylene	7.6	31.1	15.2	5.6	57 ± 0.3
Graphite	5.1	31.1	15.2	7.3	73 ± 0.3
Water	Thick	12.7	9.8	~168[b]	~150[b]
Water	Thick	15.6	12.7	~91[b]	~100[b]
Water	Thick	15.6	9.8	14.6 ± 0.3	29.7 ± 0.5
None	0	38.1	22.9	15.1 ± 0.2	192 ± 3
None	0	38.1	17.8	10.7 ± 0.2	167 ± 3
None	0	33.0	17.8	14.6 ± 0.3	155 ± 3
Graphite	25.4	38.1	33.0	11.5	57
Graphite	38.1	38.1	33.0	8.7	42.9
Graphite	2.53	38.1	27.9	16.1	147
Graphite	5.1	38.1	27.9	12.4	113
Graphite	7.6	38.1	27.9	10.3	95
Graphite	12.7	38.1	27.9	8.0	73
Graphite	25.4	38.1	27.9	5.5	50
Graphite	38.1	38.1	27.9	4.31	39.5
Graphite	2.54	38.1	22.9	9.9	125
Graphite	5.1	38.1	22.9	8.0	102
Graphite	7.6	38.1	22.9	6.8	87
Graphite	12.7	38.1	22.9	5.4	69
Graphite	25.4	38.1	22.9	3.83	48.5
Graphite	38.1	38.1	22.9	3.02	38.4
Graphite	2.53	38.1	17.8	7.8	121
Graphite	5.1	38.1	17.8	6.4	99
Graphite	7.6	38.1	17.8	5.5	85
Graphite	12.7	38.1	17.8	4.35	67
Graphite	17.8	38.1	17.8	3.66	57
Graphite	25.4	38.1	17.8	3.07	47.5
Graphite	38.1	38.1	17.8	2.42	37.4

TABLE 36. (cont.)

Reflector[a]		Annular Core		Critical Core	Critical Mass
Material	Thickness (cm)	o.d.(cm)	i.d.(cm)	Height(cm)	(kg ^{235}U)
Graphite	27.9	33.0	27.9	11.2	47.3
Graphite	40.6	33.0	27.9	8.9	37.7
Graphite	2.54	33.0	22.9	15.6	121
Graphite	5.1	33.0	22.9	12.2	94
Graphite	7.6	33.0	22.9	10.2	79
Graphite	10.2	33.0	22.9	8.9	69
Graphite	40.6	33.0	22.9	4.44	34.5
Graphite	2.54	33.0	17.8	9.7	102
Graphite	5.1	33.0	17.8	7.9	84
Graphite	7.6	33.0	17.8	7.0	72
Graphite	10.2	33.0	17.8	6.0	63
Graphite	15.2	33.0	17.8	5.0	53
Graphite	20.3	33.0	17.8	4.35	45.9
Graphite	40.6	33.0	17.8	3.15	33.3
Graphite	30.5	27.9	22.9	11.4	40.1
Graphite	43.2	27.9	22.9	9.4	33.3
Graphite	2.54	27.9	17.8	15.5	99
Graphite	5.1	27.9	17.8	12.0	76
Graphite	7.6	27.9	17.8	10.2	65
Graphite	10.2	27.9	17.8	9.0	57
Graphite	17.8	27.9	17.8	6.9	43.8
Graphite	22.9	27.9	17.8	6.1	38.8
Graphite	43.2	27.9	17.8	4.72	29.9
Graphite	33.0	22.8	17.8	12.0	33.7
Graphite	45.7	22.8	17.8	10.3	29.1

[a] Center of annulus also filled with reflector material.
[b] Uncertain extrapolation since only 43 kg of metal were available for the experiment.

INTERSECTIONS

Results of ORNL observations on several U(93) solution-filled "crosses" and Y-shaped pipe intersections appear in Table 37.[33] Later results for U(5) solution-filled crosses and Ys are summarized in Table 38.[159] Other critical intersecting pipes containing U(93) solution, reported from Rocky Flats, consist of arms spaced along a square central column.[160] Of a variety of configurations that include staggered arms and slanted arms, spaced crosses were chosen for representation in Table 39.

The data of Table 37 were generalized with conservative models by Newlon[161] and by Schuske and Morfitt.[162] More recently, all the data for intersecting pipes have provided the means of validating Monte Carlo Calculation of intersections.[163–165] The experimental data and calculations provided Thomas with a safe empirical model that has led to the Standard Nuclear Criticality Safety Criteria for Steel Pipe Intersections Containing Aqueous Solutions of Fissile Materials.[166,167]

TABLE 37. CRITICAL PARAMETERS OF U(~93) SOLUTIONS IN CYLINDRICAL 30° "Y" AND "CROSS" GEOMETRIES

Cylinder Diameter (cm)	Geometry	Solution $H/^{235}U$	Concentration (kg $^{235}U/L$)	Critical Height[a] (cm)
		Effectively Infinite Water Reflector Except at Top		
10.2	Cross	44.3	0.538	[b]
12.7	Cross	44.3	0.538	14.6
12.7	Cross	73.4	0.337	19.8
12.7	Y	73.4	0.337	39.6
		No Reflector		
12.7	Y	73.4	0.337	[b]
12.7	Cross	73.4	0.337	[b]
19.0	Cross	44.3	0.538	[b]
19.0	Cross	72.4	0.342	[b]

[a] Height above intersection of the axes.

[b] Extrapolation of the measurements from a height at least 36 cm above the intersection of the centerlines indicates that the solution in this geometry will not be critical at any height.

TABLE 38. CRITICAL INTERSECTING CYLINDERS OF $U(5.0)O_2F_2$ SOLUTION, WATER-REFLECTED EXCEPT ON TOP

Cylinder Inside Diameter (cm)	Uranium Concentration (g/L)	Liquid Height (cm)	
		Water Reflector	Solution
	Aluminum 30° "Y"[a]		
27.9	907	210	128.2
27.9	907	128.8	129.1
27.9	885	210	131.4
27.9	885	131.6	132.2
27.9	877	210	132.1
27.9	877	132.4	132.2
27.9	825	210	141.7
27.9	825	142.8	142.8
27.9	801	210	155.3
27.9	801	155.5	155.9
27.9	796	210	169.1
27.9	796	167.7	168.0
	Plexiglas Cross[b]		
26.7	905	210	115.6
26.7	905	115.9	115.9
26.7	896	117.3	117.2
26.7	856	210	133.9
26.7	856	134.1	134.1
27.3	906	210.	109.6
27.3	906	110.6	110.6
27.3	857	210	115.6
27.3	857	116.1	116.1

[a]The axis of the 30° member intersected the vertical axis 76 cm from its base.
[b]The axis of two side members intersected the vertical member 91 cm from its base.

TABLE 39. ARRAYS OF CROSSES (FOUR 137-cm-LONG ARMS) INTERSECTING A 17.8-cm-SQUARE COLUMN, $U(93)O_2(NO_3)_2$ SOLUTION AT 451 g $^{235}U/L$

Inner Diameter of Arms (cm)	Surface Spacing of Crosses (cm)	Critical Number of Arms	Critical Solution Height Above Top Arm (cm)
16.3	0	5.8	column full
16.3	13.2	8	117
16.3	16.8	12	column full
16.3	19.8(est.)	infinite	column full
13.6	0	8.0	column full
13.6	5.4	12	62
13.6	7.6	16	80
13.6	13.5(est.)	infinite	column full
11.0	0	16.6	column full

PART II — INTERACTING UNITS

HOMOGENEOUSLY MODERATED UNITS

Although the information presented thus far in this compilation describes the properties of critical single units of the fissile materials, this section concerns critical arrays of units which are individually subcritical under the conditions of the experiment. Unlike the correlations in Part I which, in many cases, are the results of generalization of data by reduction to common geometrical shapes, Part II reports, primarily, data obtained directly from experiment. The arrays are designated as being linear, planar, or spatial, depending upon whether the centers of the units establish a line, a plane, or a three-dimensional lattice. Within planar arrays the units were arranged with their centers at the corners of quadrilaterals or at the apexes of triangles, forming square and triangular patterns, respectively; in spatial arrays the units were arranged at the corners of rectangular parallelepipeds. Planar arrays are characterized by their boundaries as being hexagonal, square, rectangular, etc. In general, arrays of units in triangular patterns were hexagonal in outline; those in square patterns were quadrilateral in outline. Spatial arrays are cubes or parallelepipeds, depending upon their boundaries. Where there is the same number of units, n, in each dimension, the array is sometimes designated "n^3".

PLANAR AND LINEAR ARRAYS

Cylindrical Units of U(~90)

Oak Ridge provided most information about arrays of highly enriched uranium solution cylinders in which a common solution height was the controlling parameter. In a very extensive series of experiments, there were planar or linear arrays of two to seven identical solution cylinders in aluminum or stainless steel containers, at a variety of uranium concentrations, cylinder diameters and surface spacings between units.[168–170] Either the arrays were immersed in water except at the tops of solution cylinders, or were in air, i.e., without reflection or inter-unit moderation.

The more significant results of these experiments appear in Tables 40, 41, and 42 and Figs. 50, 51 and 52 for arrays in water, and in Tables 43 and 44 and Figs. 53 and 54 for arrays in air.

Descriptions of certain specialized experiments have been omitted. In one case, two of three solution cylinders remained fixed and the third was revolved about the axis of one of them, the critical height being measured as a function of the position of the third cylinder.[33] In another case, each of two interacting cylinders contained solution of a different concentration.[169] Another specialized experiment measured the critical height versus spacing of two interacting solution cylinders with a half-annulus of water on the outer surface of each.[169] Also omitted are effects of cadmium on three-unit and seven-unit arrays in air, which were minor relative to the effects in water[33] shown in Fig. 52.

TABLE 40. TWO EQUAL $U(93.4)O_2F_2$ SOLUTION CYLINDERS IN WATER; $H/^{235}U = 52.9$, 0.459 g $^{235}U/cm^3$ SOLUTION

Al Cylinder i.d. (cm)	Surface Separation (cm)	Critical Height (cm)
12.7	0.2	36.4
12.7	2.9	51.9
12.7	3.3	56.2
12.7	3.8	65.2
12.7	4.0	70.2
12.7	4.2	76
15.2	0.0	21.0
15.2	2.9	24.0
15.2	5.8	31.5
15.2	8.7	40.7
15.2	11.0	47.6
15.2	13.0	52
20.3	0.0	13.0
20.3	0.8	13.6
20.3	1.75	13.8
20.3	3.55	15.1
20.3	4.2	15.2
20.3	5.7	16.1
20.3	6.9	16.8
20.3	8.6	17.6
20.3	10.9	18.4
20.3	14.4	18.8
20.3	14.7	19.1
20.3	20.1	18.6
20.3	43.0	18.8

TABLE 40. (cont.)

Al Cylinder i.d. (cm)	Surface Separation (cm)	Critical Height (cm)
25.4	0.2	10.3
25.4	3.0	11.2
25.4	7.0	12.2
25.4	10.5	12.7
25.4	13.0	12.9
25.4	20.0	13.0
38.1	0.0	7.3
38.1	2.9	7.6
38.1	5.8	7.65
38.1	11.6	7.7

Source: Ref. 168.
Note: Aluminum containers with 0.15-cm walls.

TABLE 41. THREE- AND SEVEN-UNIT TRIANGULAR ARRAYS OF EQUAL SOLUTION CYLINDERS IN WATER. $U(93.2)O_2F_2$ $H/^{235}U = 44.3$, 0.538 g $^{235}U/cm^3$

Cylinder Inside Diameter (cm)	No. Cylinders, Configuration	Surface Separation (cm)	Critical Height (cm)
15.2	3 equilateral	0.38	17.8
15.2	3 equilateral	7.6	31.2
15.2	7 hexagonal	0.38	12.7
15.2	7 hexagonal	2.54	13.7
15.2	7 hexagonal	5.1	17.5
15.2	7 hexagonal	7.6	23.4
15.2	7 hexagonal	10.2	31.0
15.2	7 hexagonal	15.2	47.8
15.2	7 hexagonal	22.9	65.8
15.2	7 hexagonal	30.5	72.1
15.2	7 hexagonal	38.1	73.4
15.2	7 hexagonal	62.2	73.2
20.3	3 equilateral	0.38	14.5
20.3	3 equilateral	2.54	15.5
20.3	3 equilateral	5.1	17.8
20.3	3 equilateral	7.6	19.8
20.3	3 equilateral	10.2	21.3
20.3	3 equilateral	15.2	22.6
20.3	7 hexagonal	0.38	11.9
20.3	7 hexagonal	2.54	12.7
20.3	7 hexagonal	5.1	15.0
20.3	7 hexagonal	7.6	17.8
20.3	7 hexagonal	10.2	19.8
20.3	7 hexagonal	15.2	22.1
20.3	7 hexagonal	22.9	22.9

Source: Ref. 33.
Aluminum containers with 0.15-cm walls.

TABLE 42. SEVEN EQUAL $U(93.2)O_2F_2$ SOLUTION CYLINDERS IN WATER, $H/^{235}U = 50.1$

No.Cylinders, Configuration	Surface Separation[a] (cm)	Critical Height[a] (cm)
7 hexagonal	0.6	14.4
7 hexagonal	2.9	15.6
7 hexagonal	5.5	21.5
7 hexagonal	10.4	64
7 inline	0.6	23.4
7 inline	2.9	28.3
7 inline	5.5	40.8
7 inline	7.9	91
6 inline	5.5	40.8

Source: Ref. 171.
Note: 12.7-cm-i.d. Aluminum containers with 0.15-cm walls.
[a] Approximate values.

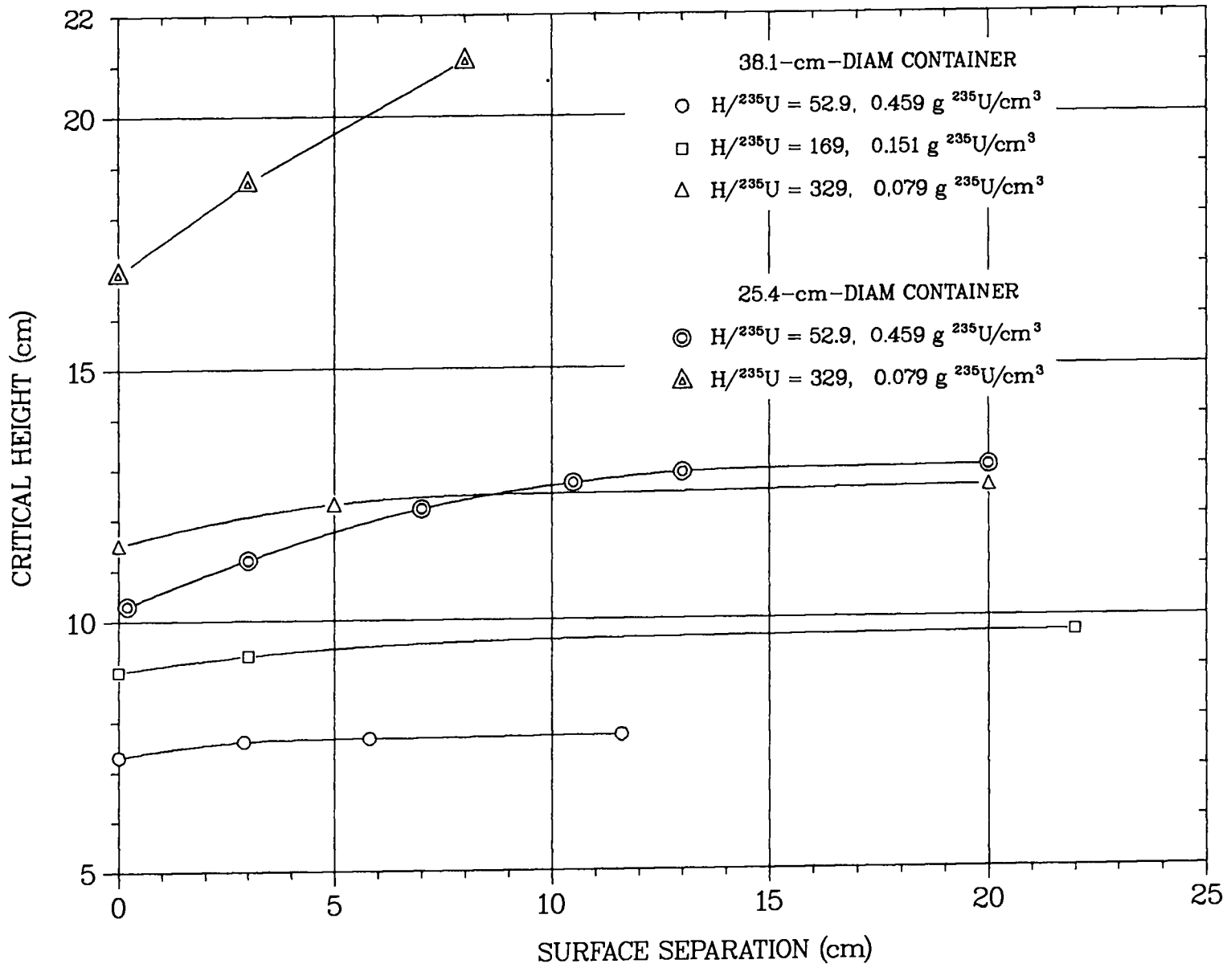


Fig. 50. Common critical height of two interacting $U(93.4)O_2F_2$ solution cylinders in water. Aluminum containers with 0.15-cm walls. Ref. 168.

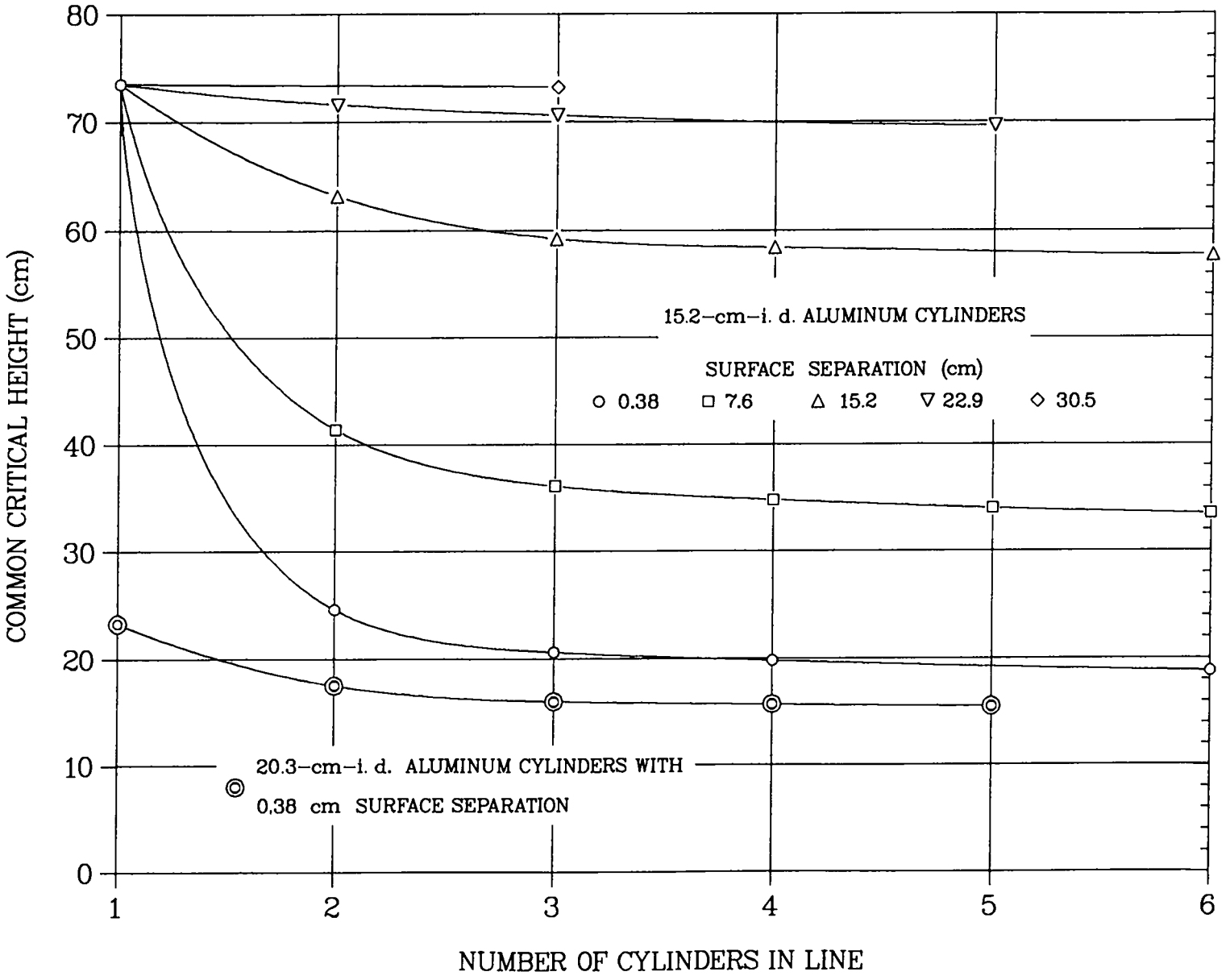


Fig. 51. Common critical height of interacting in-line $U(93.4)O_2F_2$ solution cylinders in water, $H/^{235}U$ = 44.3, 0.533 g $^{235}U/cm^3$. Aluminum containers with 0.15-cm walls. Ref. 33.

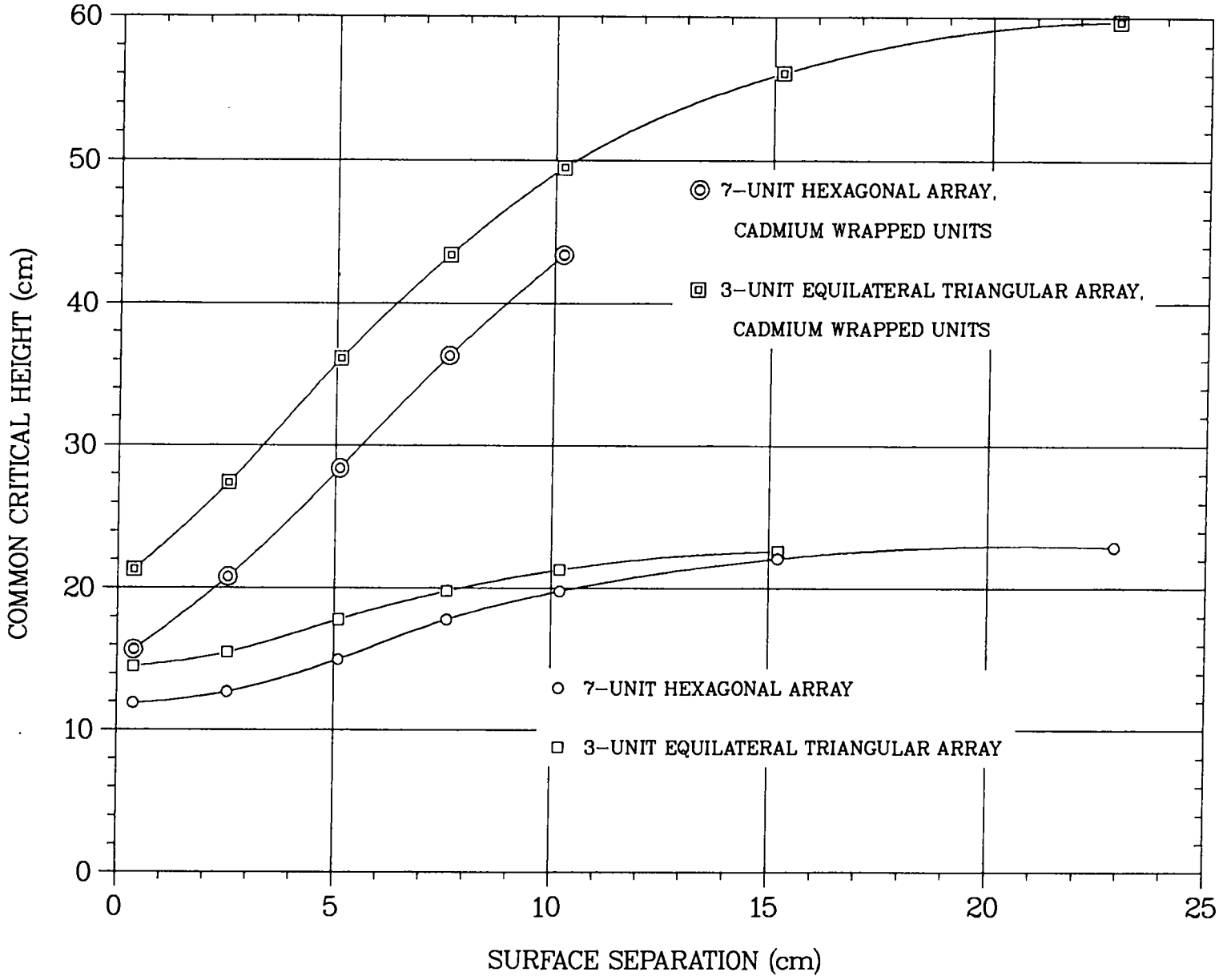


Fig. 52. Common critical heights of planar arrays of three- and seven-unit triangular patterns of $U(93.2)O_2F_2$ solution cylinders in water, $H/^{235}U = 44.3$, 0.538 g $^{235}U/cm^3$, 20.3-cm-i.d. aluminum containers with 0.15-cm walls. Ref. 33.

TABLE 43. TWO EQUAL U(~93) SOLUTION CYLINDERS IN AIR

Cylinder Inside Diameter (cm)	Solution Concentration g $^{235}U/cm^3$	Solution Concentration $H/^{235}U$	Surface Separation (cm)	Critical Height (cm)	Ref.
24.1[a]	0.087	297	2.54	61.2	170[b]
24.1[a]	0.087	297	7.6	80.5	170[b]
24.1[a]	0.087	297	15.2	113	170[b]
24.1[a]	0.087	297	20.3	137	170[b]
25.4[a]	0.0787	329	0.0	40.8	168[c]
25.4[a]	0.0787	329	1.9	44.9	168[c]
25.4[a]	0.0787	329	4.8	50.0	168[c]
25.4[a]	0.0787	329	8.0	55	168[c]
25.4[a]	0.0787	329	16.6	65	168[c]
25.4[a]	0.0787	329	31.3	74	168[c]
25.4[a]	0.0787	329	43.3	80	168[c]
30.0[d]	–	380	0.0	33	172[e]
30.0[d]	–	380	7.5	37.4	172[e]
30.0[d]	–	380	15	38.2	172[e]
30.0[d]	–	380	30	39.5	172[e]
30.0[d]	–	380	60	40.8	172[e]
30.0[d]	–	380	90	41.2	172[e]
30.0[d]	–	380	120	41.3	172[e]
38.1[a]	0.0787	329	0.2	20.1	168[c]
38.1[a]	0.0787	329	5.0	20.8	168[c]
38.1[a]	0.0787	329	9.7	21.0	168[c]
38.1[a]	0.0787	329	31.3	21.3	168[c]
38.1[a]	0.0787	329	50.0	21.5	168[c]
50.8[d]	0.0787	329	0.0	16.7	168[c]
50.8[d]	0.0787	329	10.0	17.0	168[c]
50.8[d]	0.0787	329	25.0	17.3	168[c]

[a] Aluminum containers with 0.15-cm walls.
[b] $U(93.2)O_2F_2$ solution.
[c] $U(93.4)O_2F_2$ solution.
[d] Stainless steel containers with 0.15-cm walls.
[e] $U(90)O_2(NO_3)_2$ solution.

TABLE 44. THREE- AND SEVEN-UNIT ARRAYS OF EQUAL $U(93.2)O_2F_2$ SOLUTION CYLINDERS IN AIR

Cylinder Inside Diameter (cm)	No. Cylinders, Configuration	Solution Concentration ($g\ ^{235}U/cm^3$)	$H/^{235}U$	Surface Separation (cm)	Critical Height (cm)	Ref.
12.7	7 hexagonal	0.480	50.1	0.6[a]	28.9[a]	171
12.7	7 hexagonal	0.480	50.1	2.9[a]	66[a]	171
15.2	7 hexagonal	0.538	44.3	0.38	22.6[a]	33
15.2	7 hexagonal	0.538	44.3	2.54	33.0	33
15.2	7 hexagonal	0.538	44.3	5.1	52	33
15.2	7 hexagonal	0.380	59	0.38	24.8	169[b]
15.2	7 hexagonal	0.380	59	2.54	39.1	169[b]
15.2	7 hexagonal	0.380	59	5.1	70	169[b]
15.2	7 hexagonal	0.380	59	6.4	99	169[b]
15.2	7 hexagonal	0.090	309	0.76	31.0	170
15.2	7 hexagonal	0.090	309	2.54	57	170
20.3	3 equilateral	0.090	309	0.38	41.4	170
20.3	3 equilateral	0.090	309	2.54	79	170
20.3	7 hexagonal	0.090	309	2.54	28.7	170
20.3	7 hexagonal	0.090	309	7.6	45.2	170
20.3	7 hexagonal	0.090	309	15.2	90	170
20.3	7 hexagonal	0.090	309	17.8	119	170
20.3	3 equilateral	0.538	44.3	0.38	27.2	33
20.3	3 equilateral	0.538	44.3	2.54	35.1	33
20.3	3 equilateral	0.538	44.3	5.1	45.2	33
20.3	3 equilateral	0.538	44.3	7.6	56	33
20.3	3 equilateral	0.538	44.3	10.2	69	33
20.3	7 hexagonal	0.538	44.3	0.38	18.3	33
20.3	7 hexagonal	0.538	44.3	2.54	21.6	33
20.3	7 hexagonal	0.538	44.3	5.1	25.7	33
20.3	7 hexagonal	0.54	44.3	7.6	29.7	33
20.3	7 hexagonal	0.54	44.3	10.2	33.5	33
20.3	7 hexagonal	0.54	44.3	15.2	41.9	33
24.1	3 equilateral	0.087	297	2.54	34.0	170
24.1	3 equilateral	0.087	297	10.2	52	170

TABLE 44. (cont.)

Cylinder Inside Diameter (cm)	No. Cylinders, Configuration	Solution Concentration ($g\ ^{235}U/cm^3$)	$H/^{235}U$	Surface Separation (cm)	Critical Height (cm)	Ref.
24.1	3 equilateral	0.087	297	20.3	71	170
24.1	3 equilateral	0.087	297	30.5	92	170
24.1	3 equilateral	0.087	297	45.7	126	170
24.1	3 equilateral	0.087	297	56	153	170
24.1	7 hexagonal	0.087	297	7.6	30.7	170
24.1	7 hexagonal	0.087	297	25.4	51	170
24.1	7 hexagonal	0.087	297	56	84	170

Note: Aluminum containers with 0.15-cm walls.
[a] Approximate values.
[b] $U(92.6)O_2(NO_3)_2$ solution.

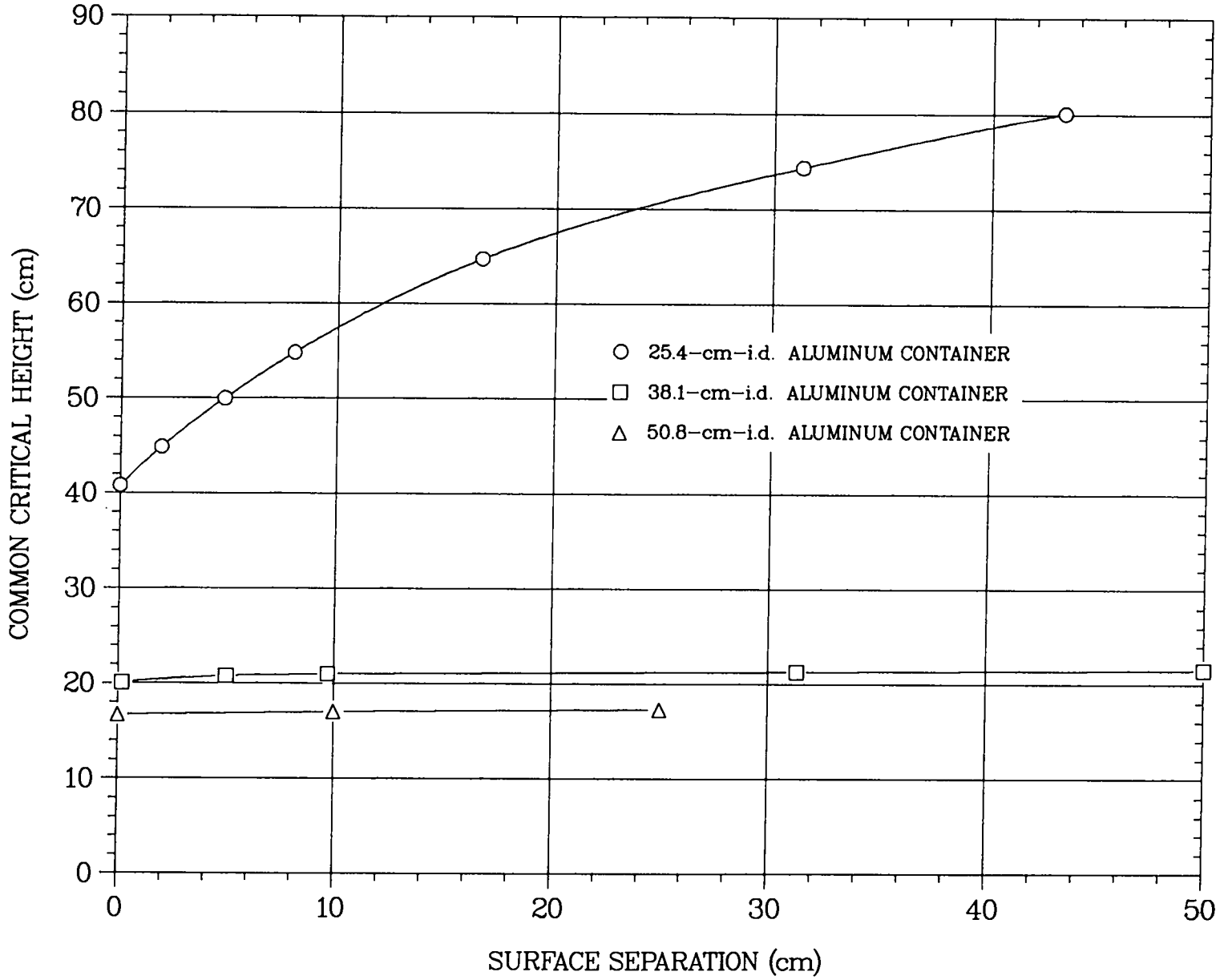


Fig. 53. Common critical height of two interacting $U(93.2)O_2F_2$ solution cylinders in air, $H/^{235}U = 329$, 0.0787 g $^{235}U/cm^3$). Ref. 168.

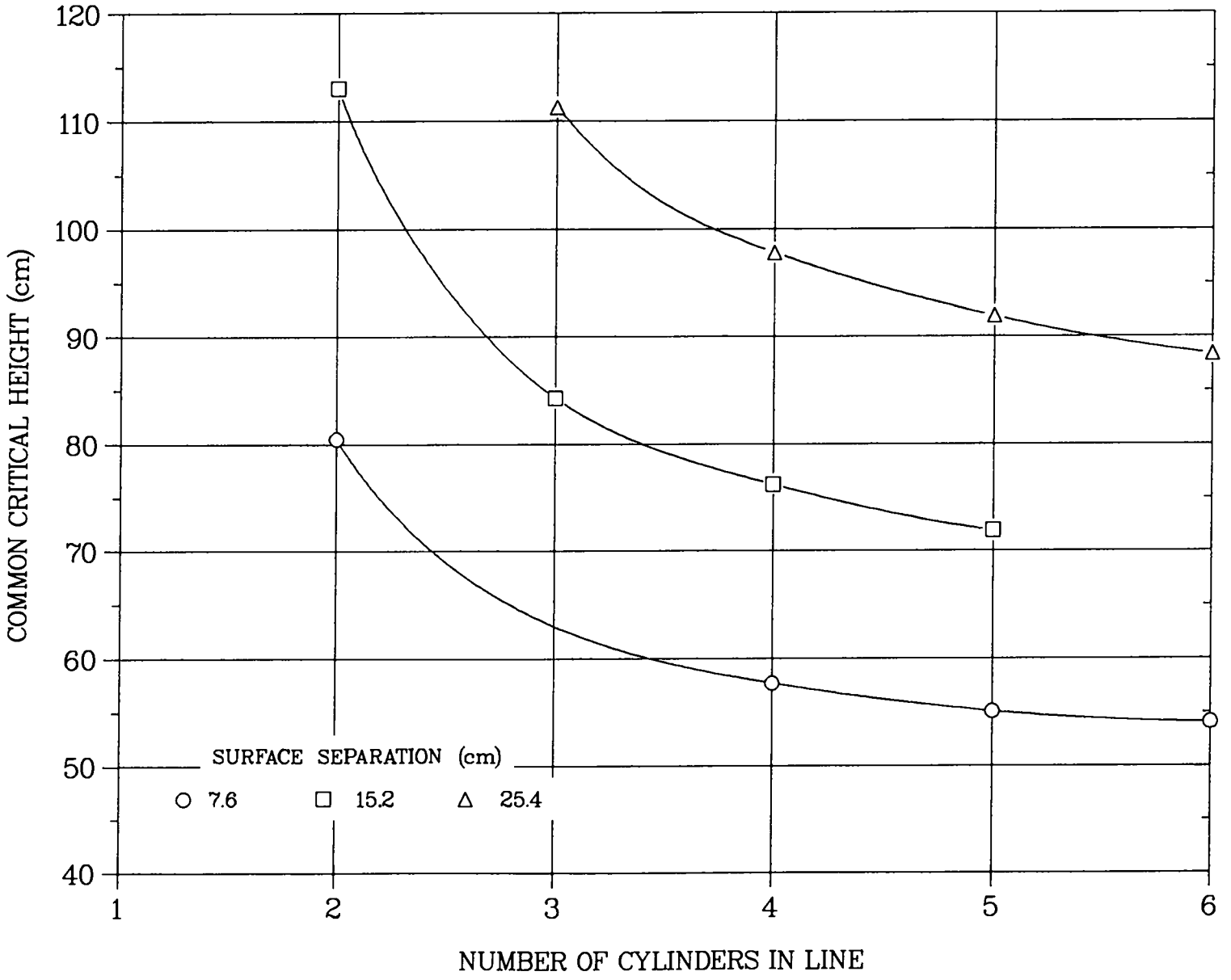


Fig. 54. Common critical height of interacting in-line $U(93.2)O_2F_2$ solution cylinders in air, $H/^{235}U = 297$, 0.087 g $^{235}U/cm^3$, 24.1-cm-i.d. aluminum containers with 0.15-cm-thick walls. Ref. 169.

A report from the USSR[172] describes critical volumes of two identical solution cylinders at various separations. Results are similar to those reported from Oak Ridge.

Further experiments at Oak Ridge dealt with larger arrays of solution in polyethylene containers or in aluminum containers that were not quite identical.[171] Generally, the number of cylinders at a fixed solution height, and spacing were the controlling parameters. The solution was $U(92.6)O_2(NO_3)_2$ at $H/^{235}U = 59$, 0.384 g $^{235}U/cm^3$, in polyethylene bottles about 122-cm high. One hundred of type A were 13.7-cm-o.d. with 1.14-cm-thick wall at the base and 0.51-cm-thick wall at the top and nominal capacity of about 13 L. Sixty-four of type B were 14.3-cm-o.d. with a uniform 0.64-cm-thick wall and a nominal capacity of about 15 L. Of 19 aluminum cylinders, seven were 15.2-cm-i.d. with 0.15-cm-thick wall and 183-cm high and the remainder were 15.2-cm-o.d. with 0.13-cm-thick wall and 152-cm high. Unreflected, unmoderated critical arrays of solution in type A polyethylene bottles are described in Table 45. The final three entries apply to double-tier planar arrays, more properly spatial arrays, illustrated in Fig. 55.

Table 46 shows the effect of Plexiglas between units and as a reflector for a 16-unit square array of type A bottles each containing 12.8 liters of solution. Clusters of three to five of these bottles in contact, some clusters subcritical, were also reported. Further, between 18 and 19 of the units in line and in contact against a 15.2-cm-thick Plexiglas wall were shown to be critical at a solution height of 112 cm.

Effects of perturbations on 6 x 6 arrays of 112-cm-high solution in type A containers were measured in terms of changes of solution height in five central units.* At 13.5-cm surface spacing a unit in a test position was somewhat more effective than the same volume of $U(37.1)O_2F_2$ solution at 0.52 g U/cm^3 of solution and much more effective than the same volume of H_2O or D_2O, 28 kg of $U(93)F_4$, 13 kg of $U(93)O_2$, 30 kg of U(93) metal scrap or pellets, or various arrangements of U(93.4) metal blocks totalling 24.5 kg. Only the same height of solution in a type B container was more reactive. Another series of measurements at 14.3-cm spacing in a tank showed a pronounced reactivity increase with a bottom water reflector, subcriticality with complete water immersion, and a modest effect of spray water at 66.8 liters per minute, the latter effect was more than compensated by the removal of one surface unit from the array.

* J. K. Fox and L. W. Gilley, Oak Ridge National Laboratory, 1961.

TABLE 45. CRITICAL ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION AT $H/^{235}U = 59$ IN TYPE A BOTTLES

Number of Units in Array	Configuration of Units Array Pattern		Surface Separation (cm)
	5.90 liters per Unit; Single Tier		
9	3 x 3	Square	3.00
16	4 x 4	Square	5.5
25	5 x 5	Square	7.6
36	6 x 6	Square	9.1
64	8 x 8	Square	11.3
100	10 x 10	Square	12.8
	9.30 liters per Unit; Single Tier		
9	3 x 3	Square	4.04
16	4 x 4	Square	7.3
25	5 x 5	Square	1.0
	2.76 liters per Unit; Single Tier		
9	3 x 3	Square	4.45
16	4 x 4	Square	8.4
25	5 x 5	Square	11.6
36	6 x 6	Square	14.3
81	9 x 9	Square	19.8
7		Triangular	3.89
19		Triangular	11.6
	12.76 liters per Unit; Double Tier[a]		
32	4 x 4	Square	9.4
50	5 x 5	Square	13.6
98	7 x 7	Square	21.2

[a]Vertical spacing between solution in two tiers was 14.2 cm.

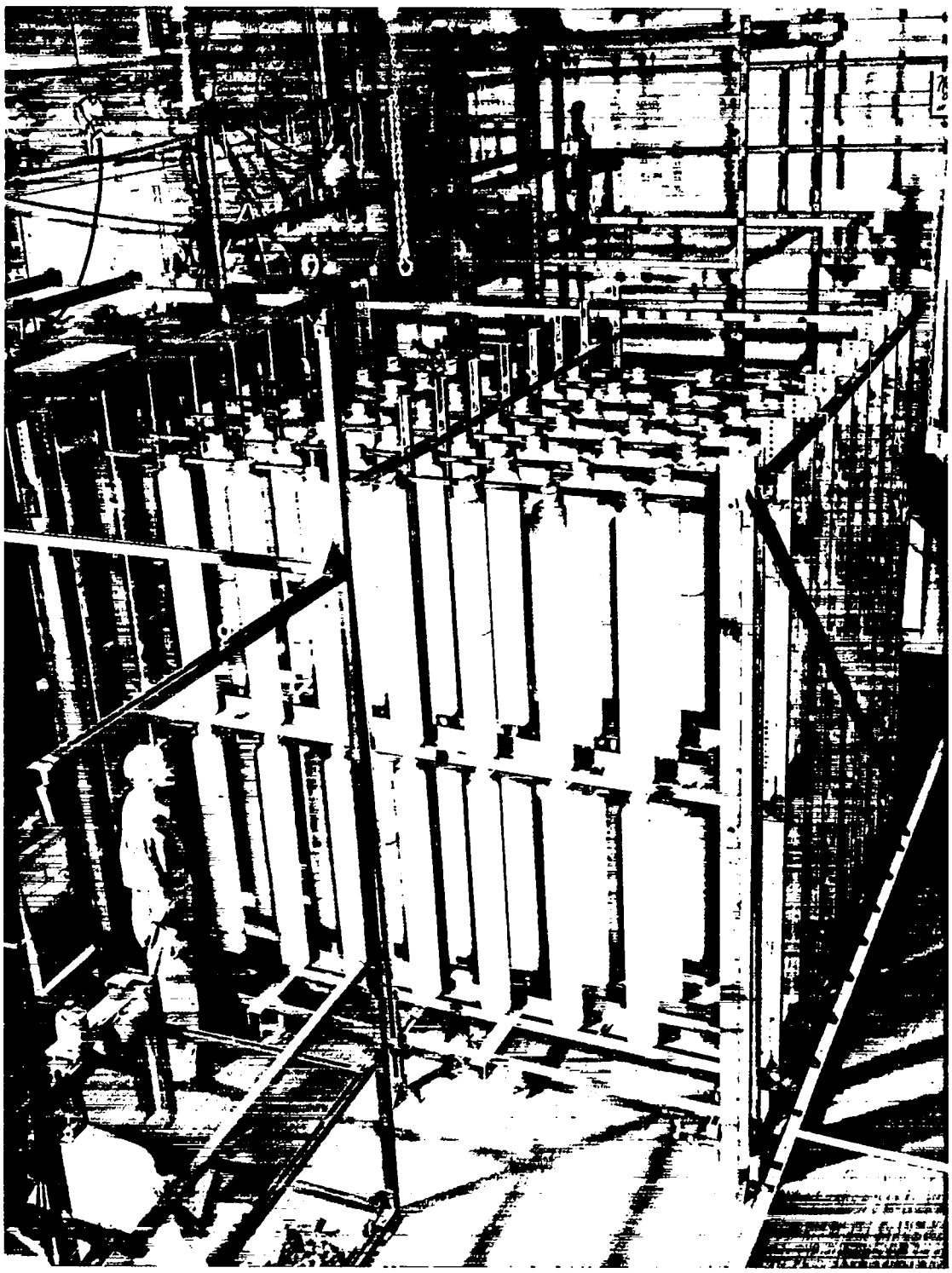


Fig. 55. Double-tier array of $U(92.6)O_2(NO_3)_2$ solution at $H/^{235}U = 59$ in Type A polyethylene bottles.

TABLE 46. COMPARISON OF PLEXIGLAS-MODERATED AND UNMODERATED ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION IN SIXTEEN 12.76-LITER UNITS IN 4 x 4 ARRAY

Plexiglas Moderator Thickness and Configuration	Plexiglas Reflector	Unit Surface Separation (cm)	Number of Units in Unreflected, Unmoderated Array, N	Ratio: N/16
1.27 cm closely surrounding each unit	None	9.4	19.2	1.20
1.27 cm centered between units	0.64 cm	12.5	28.5	1.78
2.54 cm centered between units	1.27 cm	13.7	33	2.06
3.81 cm centered between units	1.90 cm	13.4	31.8	1.99
3.81 cm centered between units	None	(12.2)[a]	27.3	1.71

[a] Interpolated value.

Some unreflected, unmoderated critical arrays of the same solution in type B polyethylene bottles are given in Table 47. Although type A bottles were much more common for solution storage or transfer, the type B containers with uniform walls were preferable for arrays intended to check calculations.

Reported with the seven-unit arrays of $U(92.6)O_2(NO_3)_2$ solution in 15.2-cm-i.d. aluminum containers listed in Table 44, there were various combinations of these units and others consisting of the same solution in the 15.2-cm-o.d. aluminum containers. In the largest array, 12 of the latter units surrounded the 7 original ones to form a 19-unit triangular array. In air, critical surface separations for various common solution heights are listed in Table 48. Combinations of the same solution units in aluminum cylinders were used to illustrate effects of concrete and water slabs as reflectors and within arrays, as shown in Fig. 56.

TABLE 47. CRITICAL ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION AT $H/^{235}U = 59$ IN TYPE B BOTTLES, 7.41 LITERS PER UNIT

Number of Units in Array	Configuration	Surface Separation (cm)
9	3 x 3	3.63
16	4 x 4	6.6
36	6 x 6	10.8
64	8 x 8	13.5
27	3 x 9	7.2
24	3 x 8	7.1
16[a]	4 x 4	5.9

[a] Plastic liner 0.51-cm thick inside bottle, resulting in a contained volume of 6.33 liters per unit.

TABLE 48. NINETEEN UNIT CRITICAL ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION CYLINDERS AT $H/^{235}U = 59$

Common Solution Height (cm)	Critical Surface Separation (cm)
51	8.9
76	12.6
102	15.1
127	16.9

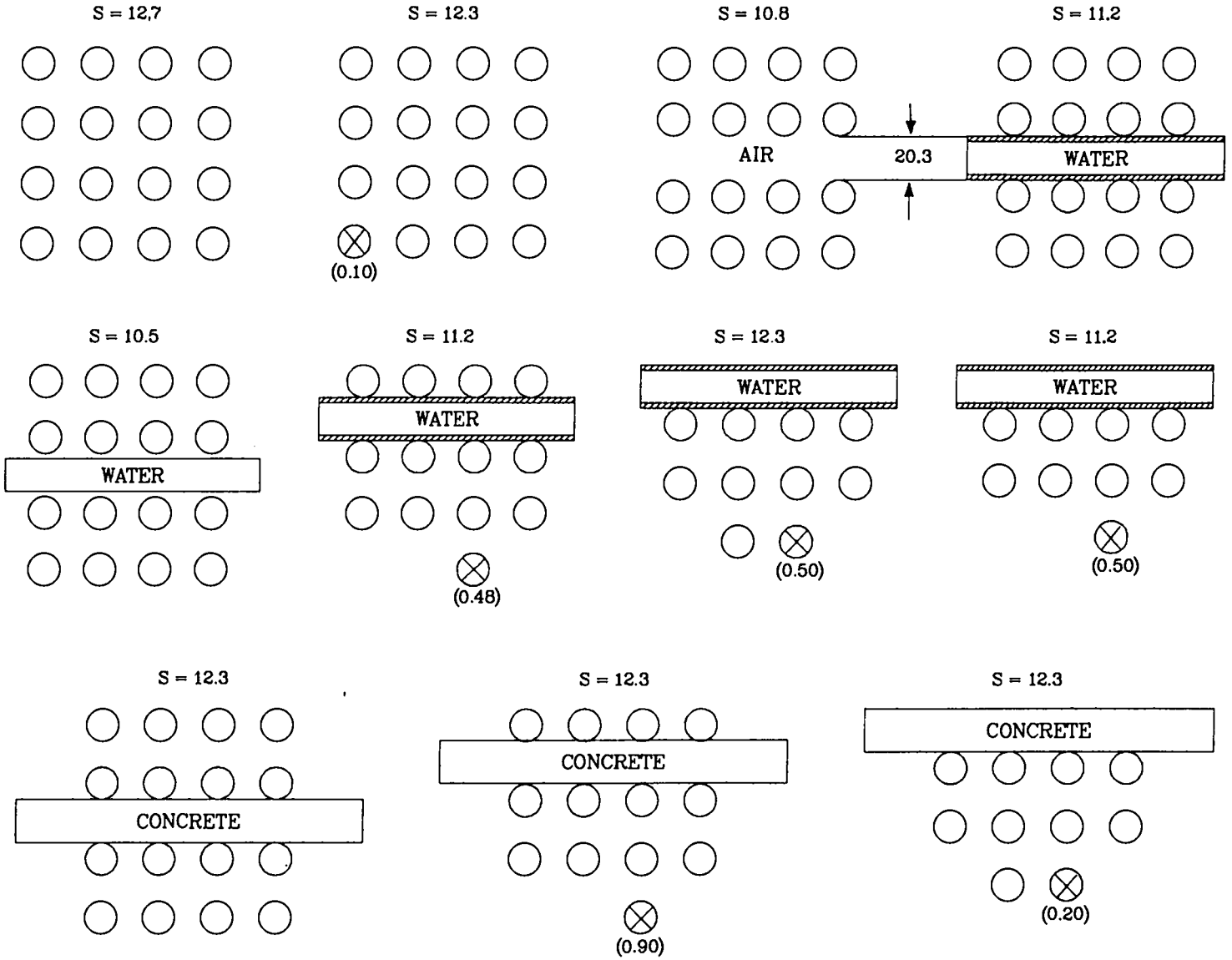


Fig. 56. Two-dimensional arrays of 15.2-cm-diam cylinders of $U(92.6)O_2(NO_3)_2$ solution with reflecting or moderating slabs of concrete or water; $H/^{235}U = 59$, 0.384 g $^{235}U/cm^3$. Solution containers 0.16-cm-thick aluminum, water containers, 0.32-cm-thick aluminum. The surface separation of containers in cm is noted above each configuration.

In addition to experiments with individual cylinders in 122-cm cubic cavities within 26-cm-thick concrete or 21-cm-thick Plexiglas (see Table 34), Rocky Flats reported arrays of $U(93.2)O_2(NO_3)_2$ solution cylinders within the same enclosures.[143] Critical heights of solution were measured in 4 x 4 arrays of full-height aluminum containers and of the containers in 0.32-cm-thick stainless steel sleeves. The cylinders were spaced uniformly within the surrounding reflector, apparently such that the closest axis-to-reflector distance is one-half the 30.5-cm axis-to-axis spacing of nearest units. Observed solution heights for such arrays of cylinders of two diameters are given in Table 49. Also reported were critical solution heights for 2 x 4, 2 x 3 and 2 x 2 arrays on the same lattice spacing.

TABLE 49. CRITICAL HEIGHTS OF SOLUTION IN FOUR-BY-FOUR ARRAYS OF CYLINDERS REFLECTED BY CONCRETE OR PLEXIGLAS

	122-cm-Cubic Cavity			
	in 26-cm-thick Concrete		in 21-cm-thick Plexiglas	
Solution Diameter (cm)	Concentration (g ^{235}U/L)	Critical Height (cm)[a]	Concentration (g ^{235}U/L)	Critical Height (cm)[a]
Without sleeve:				
21.1	63	27.2	56	31.8
21.1	339	17.1	332	18.8
With s.s. sleeve:				
21.1	63	28.6	56	34.8
21.1	345	17.2±0.1	332	19.3
Without sleeve:				
16.1	78	51	56	78
16.1	335	31.8	332	35.6
With s.s. sleeve:				
16.1	78	57	56	106
16.1	336	32.3	332	38.1

[a] Standard deviation less than last quoted place.

Subcritical experiments in the USSR were conducted on unreflected arrays of 6-L units of $U(90)O_2(NO_3)_2$ solution at 113 g U/L.[173] Each unit consisted of solution to a depth of 24-cm in a glass vessel of about 18-cm diameter with a wall thickness of 0.5-cm. An essentially critical array consisted of 39 units in three layers of 12, 15, and 12 units spaced 4.5-cm between solution surfaces within a layer and 12-cm between solution surfaces in adjacent layers. Two- and four-layer arrays are also described with extrapolations to criticality.

Slab Units of U(~90)

At Oak Ridge slabs of U(93.2)O_2F_2 solution were arranged with their bases in the same plane and their surfaces of maximum area either parallel or perpendicular in order to study interaction between large areas of fissile material.[171] The aluminum vessels containing the solution were 120.6-cm wide and either 7.6- or 15.1-cm-thick; the solution height was adjusted for criticality under the various experimental conditions. Data from several combinations and arrangements of these slabs are shown in Figs. 57 to 60; the environment varied from no moderator or reflector to complete submersion of the array to the height of the solution in the containers. Figure 61 shows the data resulting from the arrangement of these solution slabs in "T" and "L" geometries.

In experiments exploring the effect of hydrogenous material separating units of fissile material,[174] a pair of unreflected, parallel slabs of U(93.2)O_2F_2 solution, about 15-cm-thick, were separated 30.5-cm. Plexiglas of increasing thickness was first centered between the slabs and then placed on the inside facing surfaces. Figure 62 shows the common solution height of the two slabs as a function of the total thickness of moderator between the slabs. The two curves must meet, of course, when the space between the slabs is completely filled with the moderator.

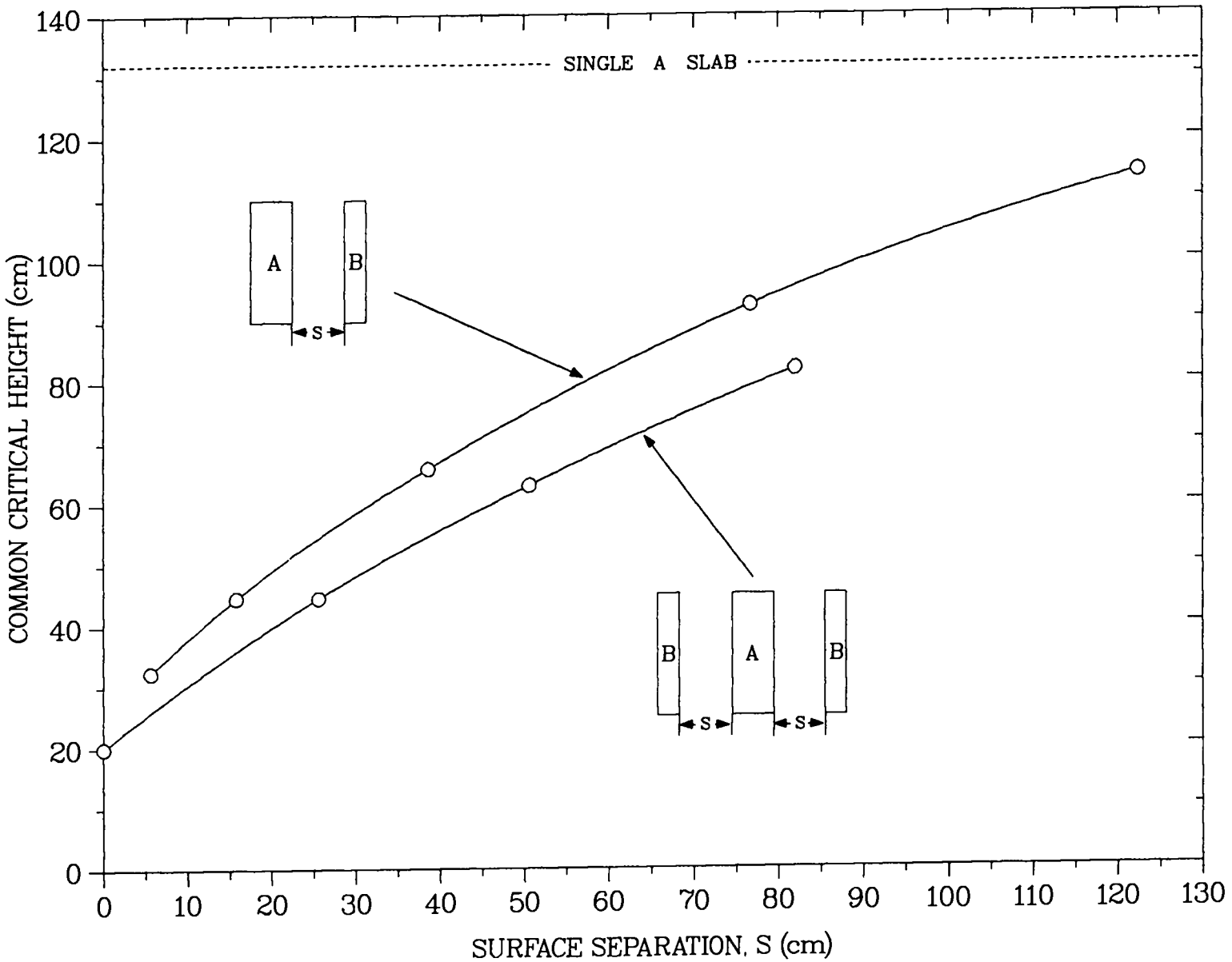


Fig. 57. Common critical heights of two and three parallel $U(93.2)O_2F_2$ solution slabs in air, 0.32-cm-thick aluminum containers.

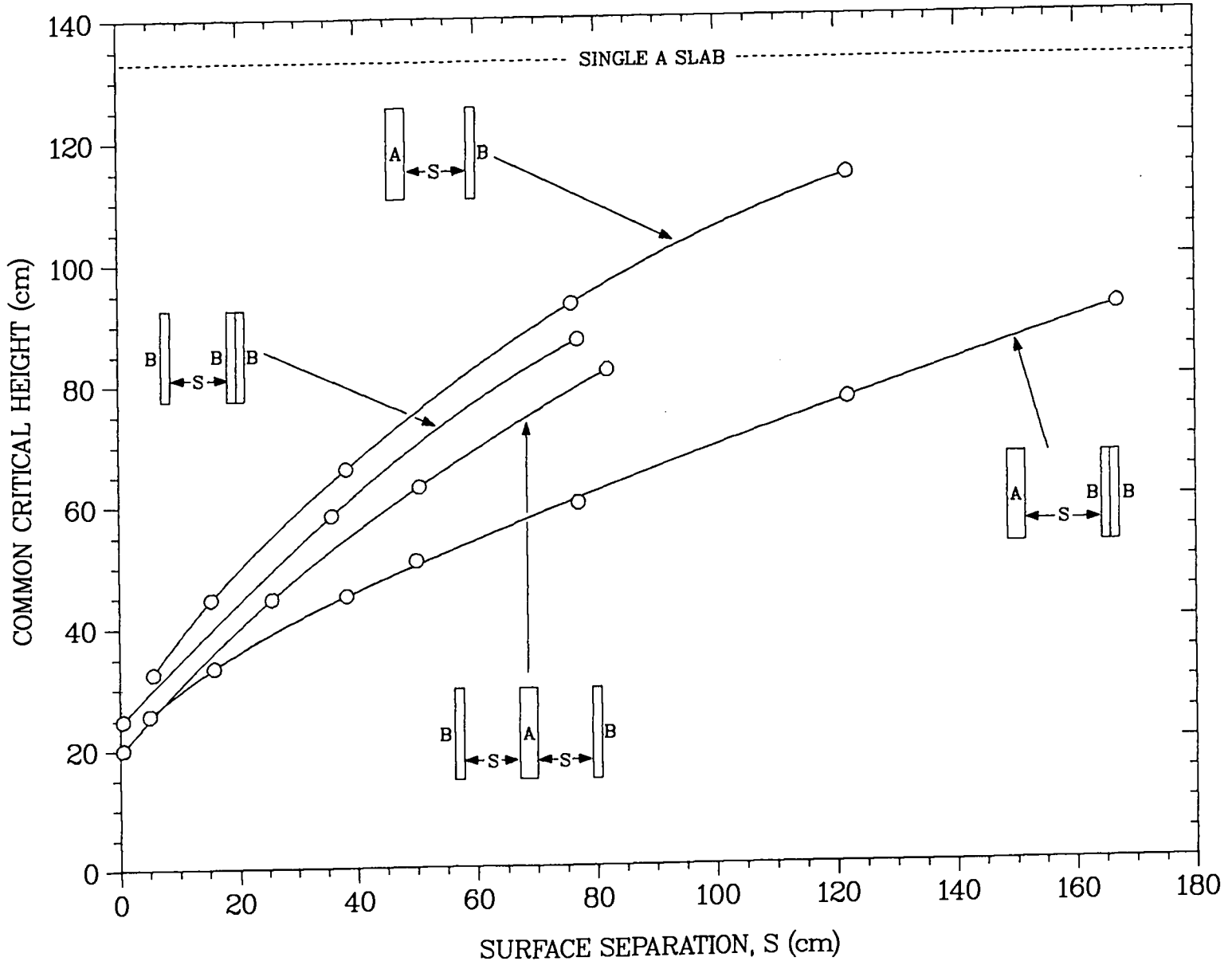


Fig. 58. Common critical heights of two and three parallel $U(93.2)O_2F_2$ solution slabs in air, $H/^{235}U = 337$, 0.076 g $^{235}U/cm^3$, 0.32-cm-thick aluminum containers.

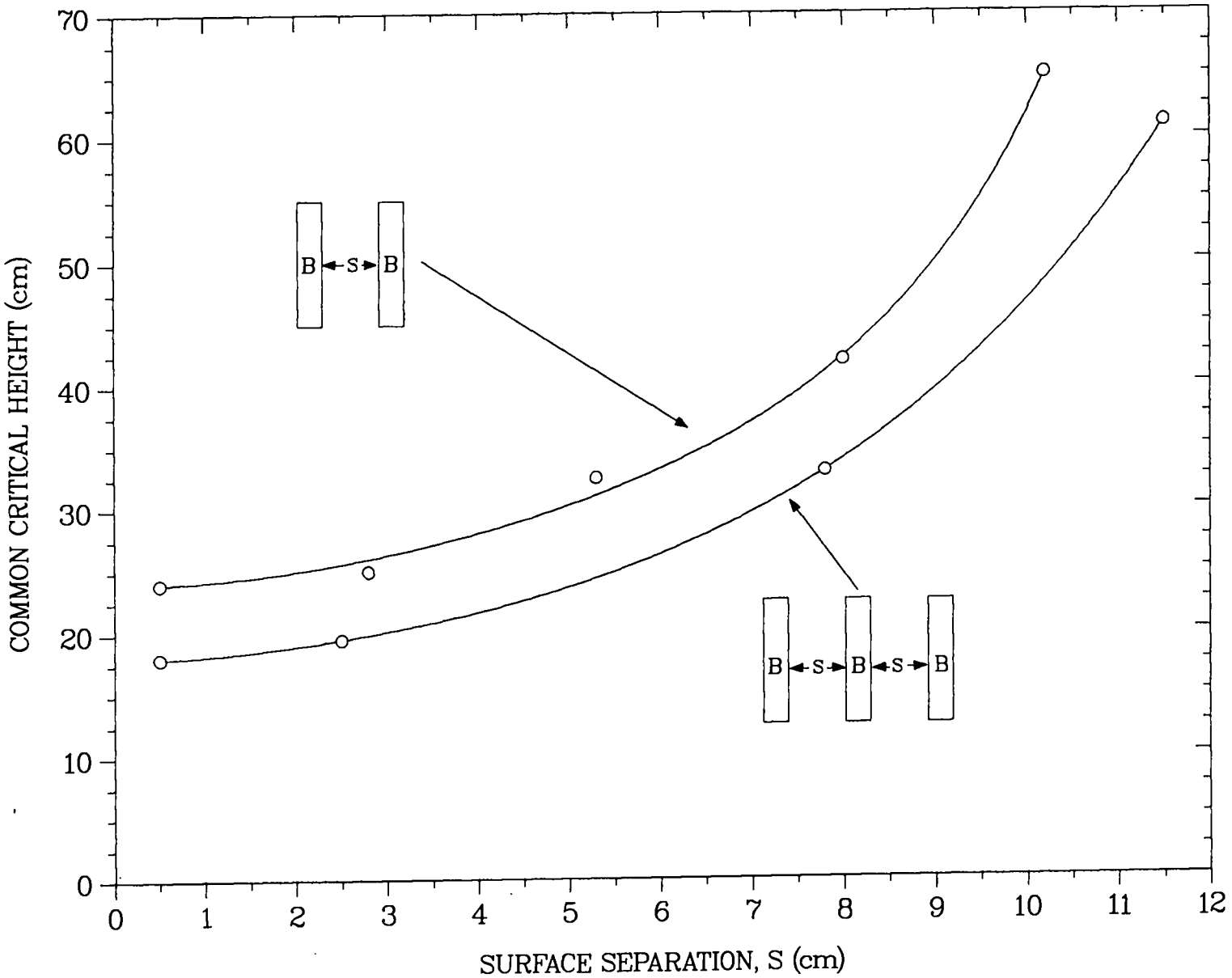


Fig. 59. Common critical heights of two and three parallel $U(93.2)O_2F_2$ solution slabs in water, H/235 = 337, 0.076 g $^{235}U/cm^3$. Aluminum containers with 0.32-cm-thick walls and 120.6-cm-wide by 7.6-cm-thick inside dimensions. Reflection consisted of immersion to height of solution.

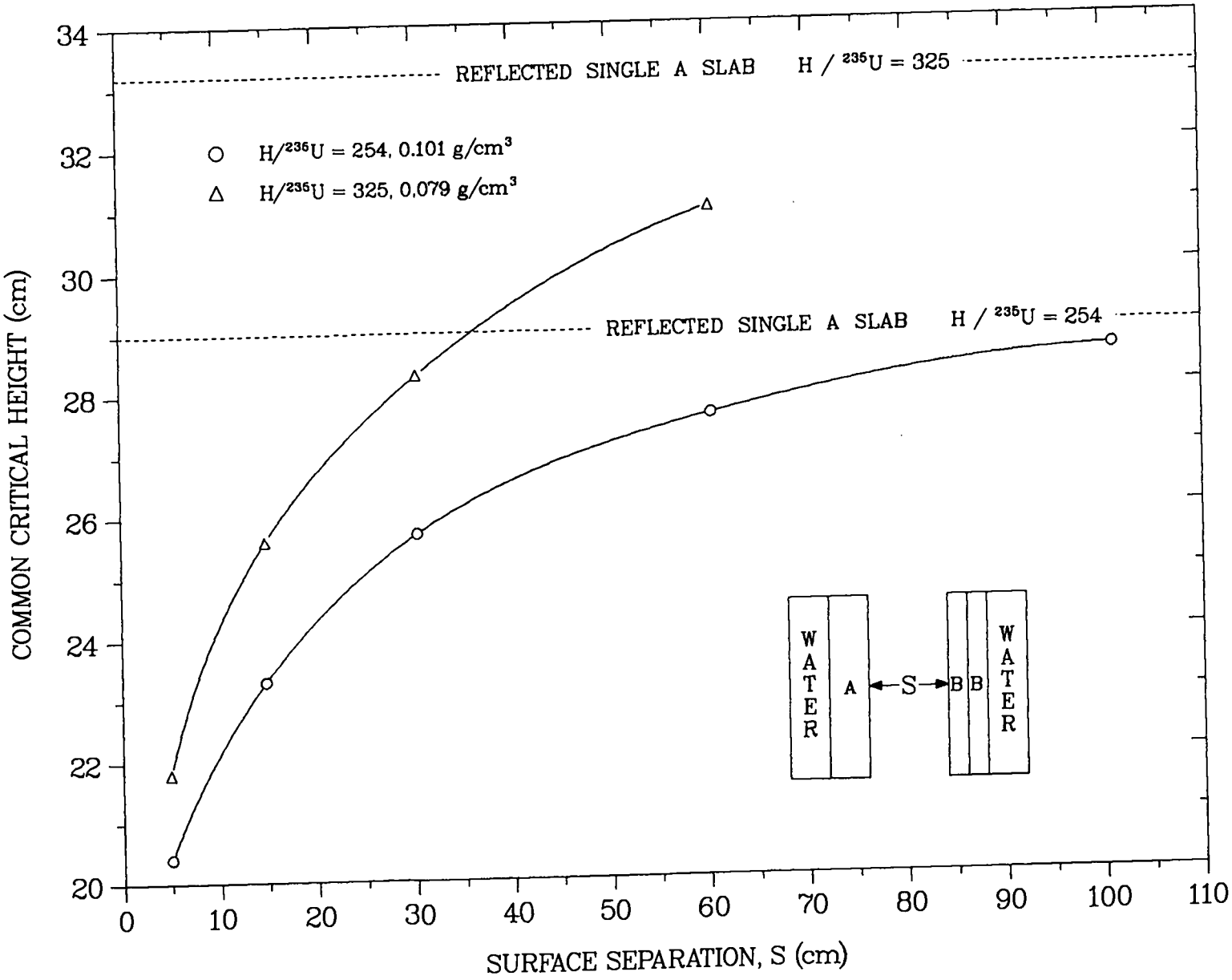


Fig. 60. Common critical height of two parallel $U(93.2)O_2F_2$ solution slabs with partial water reflection, 0.32-cm-thick aluminum containers.

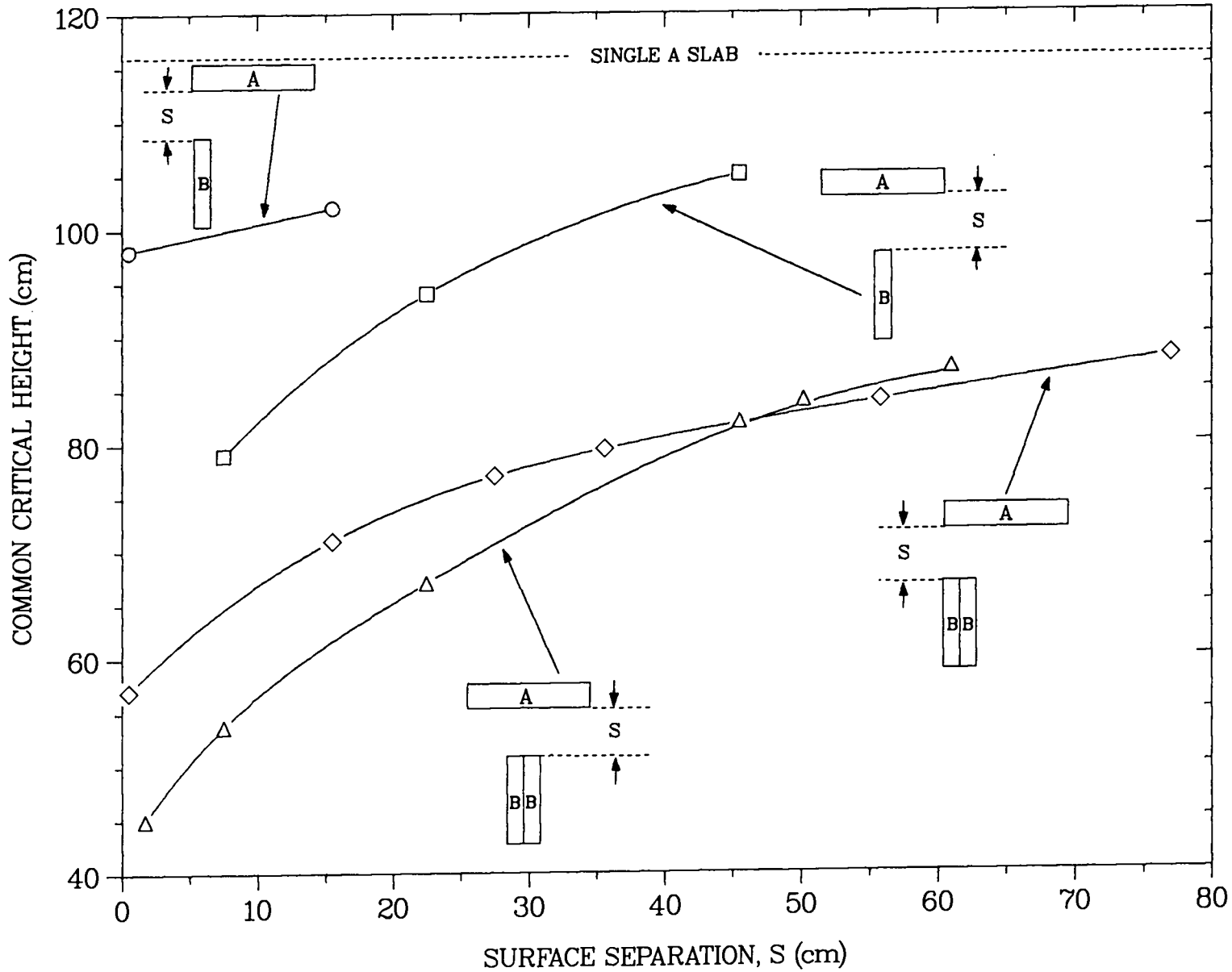


Fig. 61. Common critical heights of pairs of perpendicular $U(93.2)O_2F_2$ solution slabs in air; $H/^{235}U = 325$, 0.079 g $^{235}U/cm^3$ solution; 0.32-cm-thick aluminum containers.

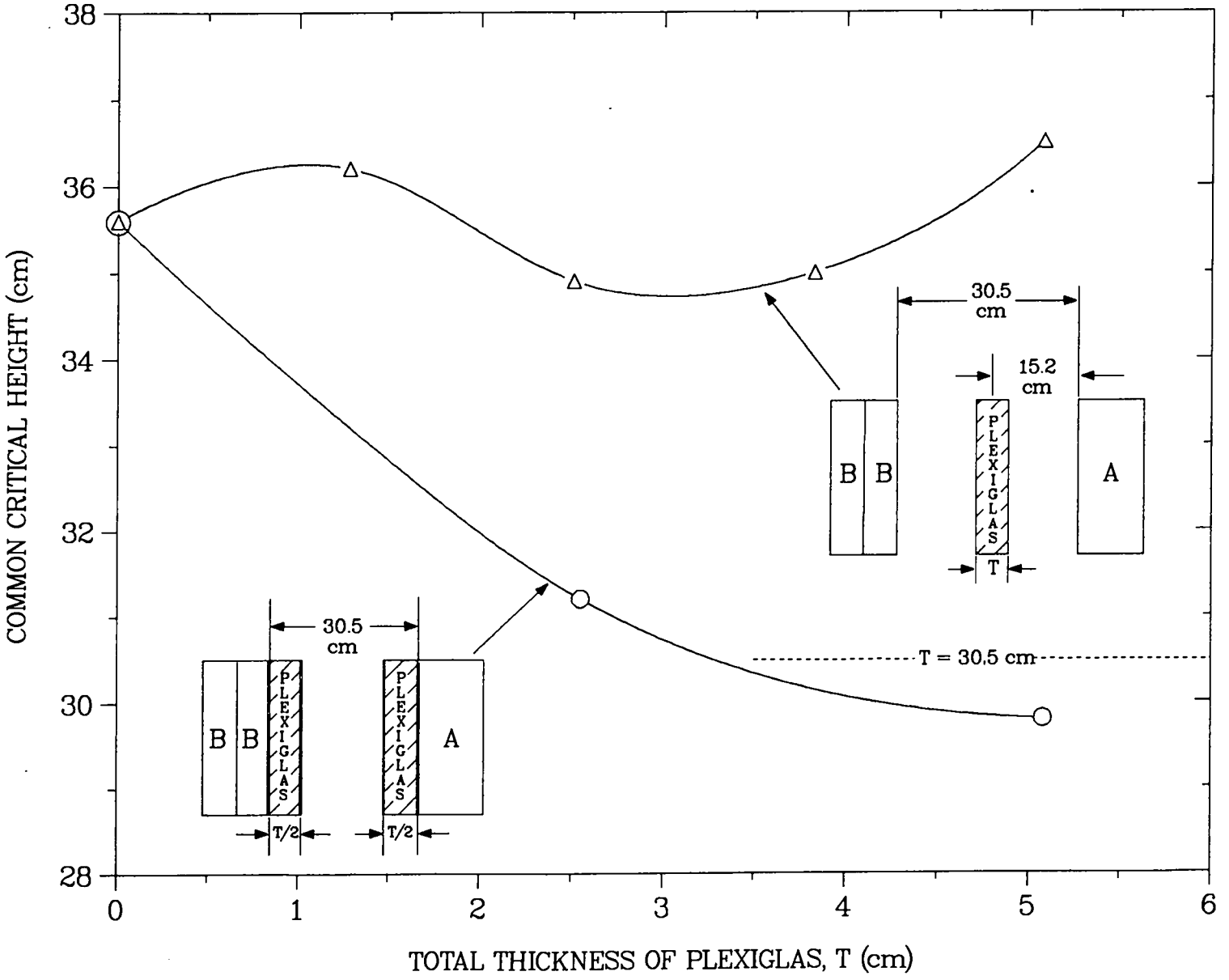


Fig. 62. Common critical height of two parallel $U(93.2)O_2F_2$ solution slabs separated by Plexiglas; $H/^{235}U$ = 293, 0.88 g $^{235}U/cm^3$; 0.32-cm-thick aluminum containers.

Combinations of Slab and Cylindrical Units of U(~90)

Data from Oak Ridge* describing critical configurations of a slab and a cylinder of $U(93.2)O_2F_2$ solutions, unreflected and unmoderated, are given in Table 50. The ^{235}U concentration of the cylinders remained constant throughout the experiments, while that of the slab was varied. Results of interaction between a slab and a cylinder of solution of the same concentration,[171] unreflected and partially reflected, are shown in Fig. 63.

TABLE 50. CRITICAL INTERACTING SLAB AND CYLINDER OF $U(93.2)O_2F_2$ SOLUTION[a]

Container Surface Separation (cm)	Cylinder[b]			Slab[c]		
	Concentration		Critical Height (cm)	Concentration		Critical Height (cm)
	g ^{235}U/L	H/^{235}U		g ^{235}U/L	H/^{235}U	
5.0	79	325	53.3	79	325	28.3
15.2	79	325	67.6	79	325	34.3
30.5	79	325	85.1	79	325	39.1
5.0	79	325	49.5	101	254	28.7
15.2	79	325	67.6	101	254	33.5
30.5	79	325	83.8	101	254	39.4

[a] The lower surfaces of the solution slab and cylinder were coplanar, and the projection of the cylinder was centered horizontally on the face of the slab.
[b] Solution was contained in a 25.4-cm-diam aluminum cylinder with 0.16-cm-thick walls.
[c] Solution was contained in a 15.1-cm-thick aluminum slab with 0.32-cm-thick walls.

* J. K. Fox and L. W. Gilley, Oak Ridge National Laboratory, 1961.

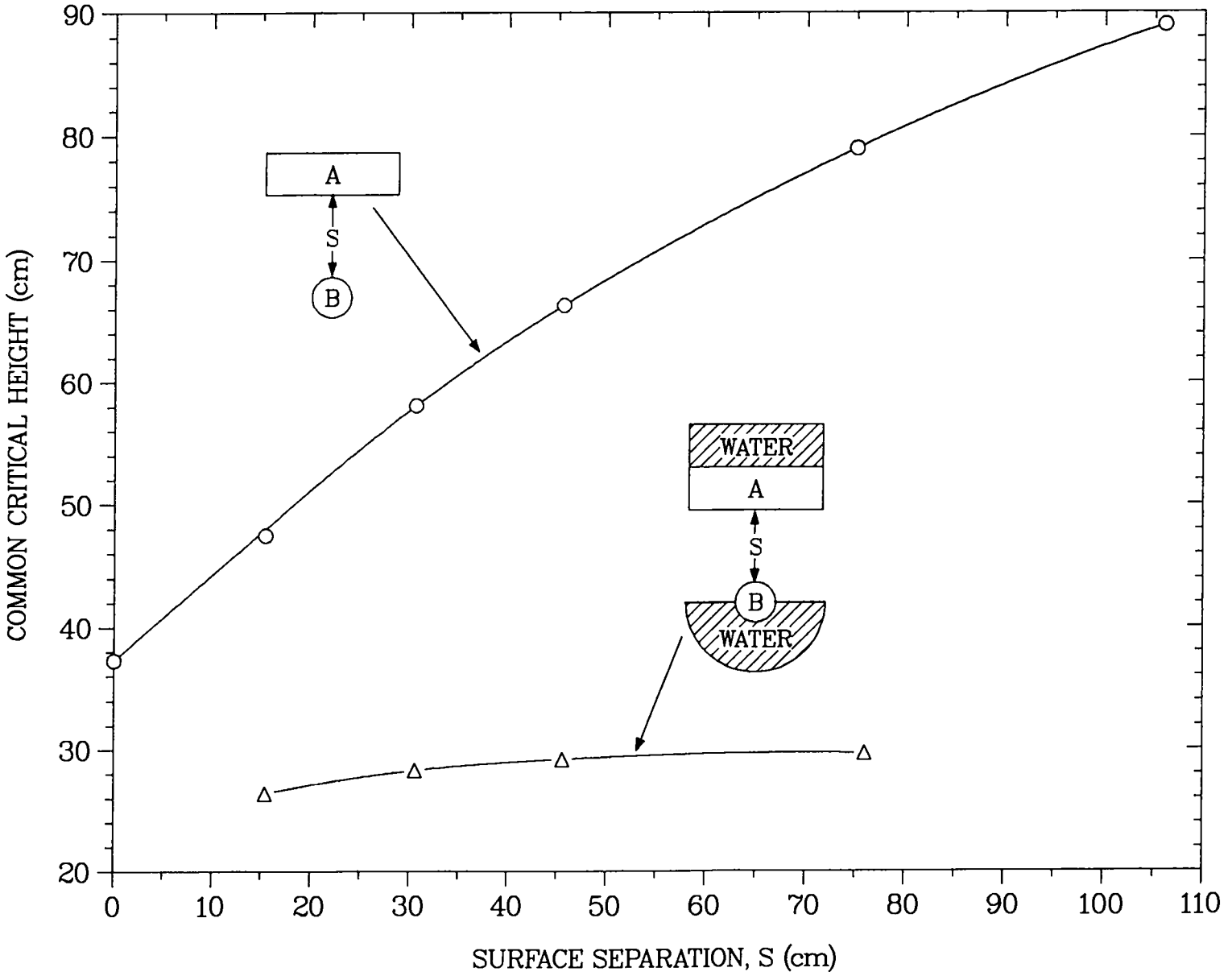


Fig. 63. Common critical height of parallel $U(93.2)O_2F_2$ solution slab and cylinder, unreflected and with partial water reflection; $H/^{235}U = 331$, 0.078 g $^{235}U/cm^3$.

Rocky Flats reported critical arrays of $U(93.2)O_2(NO_3)_2$ solution cylinders standing on a horizontal slab of the same solution.[175] This simulates plant conditions where some solution may have drained from vertical cylinders into a shallow horizontal sump. The stainless steel slab tank was 120.7-cm square by 20.3-cm high, with a 0.64-cm-thick base. Stainless steel cylinders of 11.0-cm-i.d. had wall and base thicknesses of 0.198-cm; those from 13.6-to 21.3-cm-i.d. had thicknesses of 0.280-cm and larger cylinders were 0.308-cm-thick. Square arrays of cylinders were arranged such that the minimum axis-to-slab-edge spacing was one-half the axis-to-axis spacing of nearest neighbors.

The following refer to minimally reflected systems unless stated otherwise. Without cylinders, the critical slab thickness was 12.7-cm at 465 g U/L and 13.0-cm at 510 g U/L. With the slab empty, the critical solution height in sixteen 16.3-cm-i.d. cylinders was 78-cm at 495 g U/L; for nine 21.3-cm-i.d. cylinders, it was 47-cm at 500 gU/L; and for a single 23.9-cm-i.d. cylinder it was 112-cm at 525 g U/L. With solution in both slab and cylinders, critical systems with maximum critical cylinder solution heights (above the top of slab solution) are described in Table 51.

With the system flooded and no solution in the slab tank, the critical solution height in an array of sixteen 13.6-cm-i.d. cylinders with 30.2-cm pitch was 92-cm at 505 g U/L.

TABLE 51. CRITICAL $U(93.2)O_2(NO_3)_2$ SOLUTION CYLINDER-ON-SLAB ARRAYS, CONFIGURATIONS WITH MINIMUM REFLECTOR

Cylinder Inside Diameter (cm)	Number of Cylinder in Array	Slab Thickness (cm)	Solution Height (cm)[a]	Common Concentration (g U/L)
11.0	1	12.6	87	465
11.0	4	12.3	88	465
11.0	9	12.1	88	465
11.0	16	11.4	88	465
13.6	16	10.0	108	520
16.3	1	12.2	88	470
16.3	4	12.0	88	480
16.3	9	10.5	90	485
21.3	1	11.1	88	500
21.3	4	8.8	91	500
22.4	1	10.8	108	505
22.9	1	10.1	110	505
23.4	1	8.9	111	525

[a] Data are available for critical combinations with smaller solution heights in arrays.

Units of U(<90)

The critical dimensions of an unreflected array of two slabs dissimilar both in ^{235}U enrichment and in chemical composition were determined at ORNL as a function of their separation.* One of the slabs was a solution of $U(93.2)O_2F_2$ having a ^{235}U concentration of about 480 g/L, corresponding to an $H/^{235}U$ of 50.1. The slab was vertical, 120.6-cm wide, 7.6-cm-thick, and of variable solution height. The second slab, parallel to the first, was constructed of blocks of an homogenized mixture of CF_2 and $U(37)F_4$. The uranium density was 3.1 g/cm^3, and absorbed moisture accounted for an $H/^{235}U$ of 0.1. In some experiments Plexiglas was placed between adjacent layers of blocks and between adjacent rows in one direction, producing an $H/^{235}U$ of 16. The critical height of the solution slab as a function of the surface separation of the two components is shown in Fig. 64 for various slabs of $U(37)CF_6$ blocks.

Dounreay** is the source of data on a pair of slabs of a solution of $U(30.45)O_2F_2$. Each slab was 6.09-cm-thick and 120-cm-wide. They were filled to the common height necessary for criticality when the space between the two was occupied by various materials. Figure 65 shows the data for a solution at a uranium concentration of 599 g/L (^{235}U concentration of 182 g/L) corresponding to an $H/^{235}U$ of 130, and Fig. 66 gives more limited results for a uranium solution concentration of 375 g/L corresponding to an $H/^{235}U$ of 214.

Neutron interaction between two identical parallelepipeds of homogeneous $U(30.14)O_2$-paraffin (CH_2) reflected by 20.3-cm-thick polyethylene on all sides except the facing ones was investigated at Aldermaston.[43] The mixture had a ^{235}U density of 0.331 g/cm^3 and an $H/^{235}U$ of 81.8. The parallelepipeds were 20.3- by 20.3-cm in cross section and of variable but common height. The critical separation was determined as a function of the separation of the two components and of the material between them. The experimental arrangement and data are shown in Fig. 67. It can be seen that with a separation of 10-cm, the mass in each component is 80% of the critical mass of a completely reflected single unit.

Measurements were also made at Dounreay*** on two identical rectangular parallelepipeds of $U(1.42)F_4$-paraffin, containing 2.5 g of U/cm^3, with an $H/^{235}U$ of 420. The parallelepipeds were reflected by 20.3-cm-thick polyethylene on all sides except the facing ones. The critical separation was measured as a function of the total uranium in both components under two different conditions: with the facing areas constant and the lengths of the two components of the array varied, and with the base areas constant and the heights varied equally. The critical separation and dimensions are given in Table 52. Figure 68 shows the influence of separation on the critical mass under the two conditions.

---

* J. K. Fox and L. W. Gilley, Oak Ridge National Laboratory, 1964.

** John G. Walford and J. C. Smith, Dounreay Experimental Reactor Establishment, Dounreay, United Kingdom Atomic Energy Authority, 1963.

*** J. G. Walford and J. M. Scott, United Kingdom Atomic Energy Authority, Dounreay Experimental Research Establishment, Dounreay, 1963.

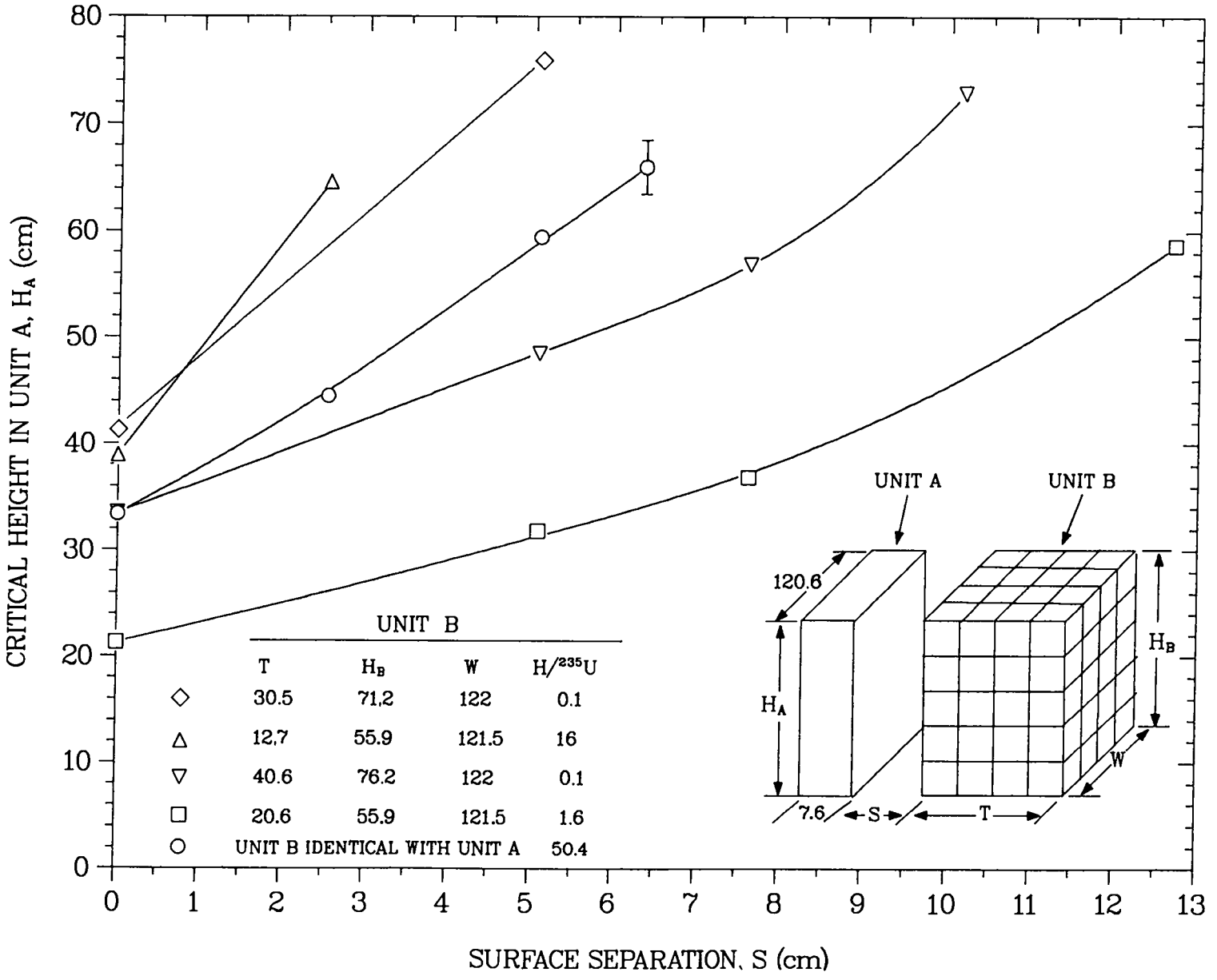


Fig. 64. Critical height of a $U(93.2)O_2F_2$ solution slab interacting with a parallel slab of $U(37)F_4$ - CF_2 blocks in air. Unit A was solution in 0.32-cm-thick aluminum container. All dimensions in cm.

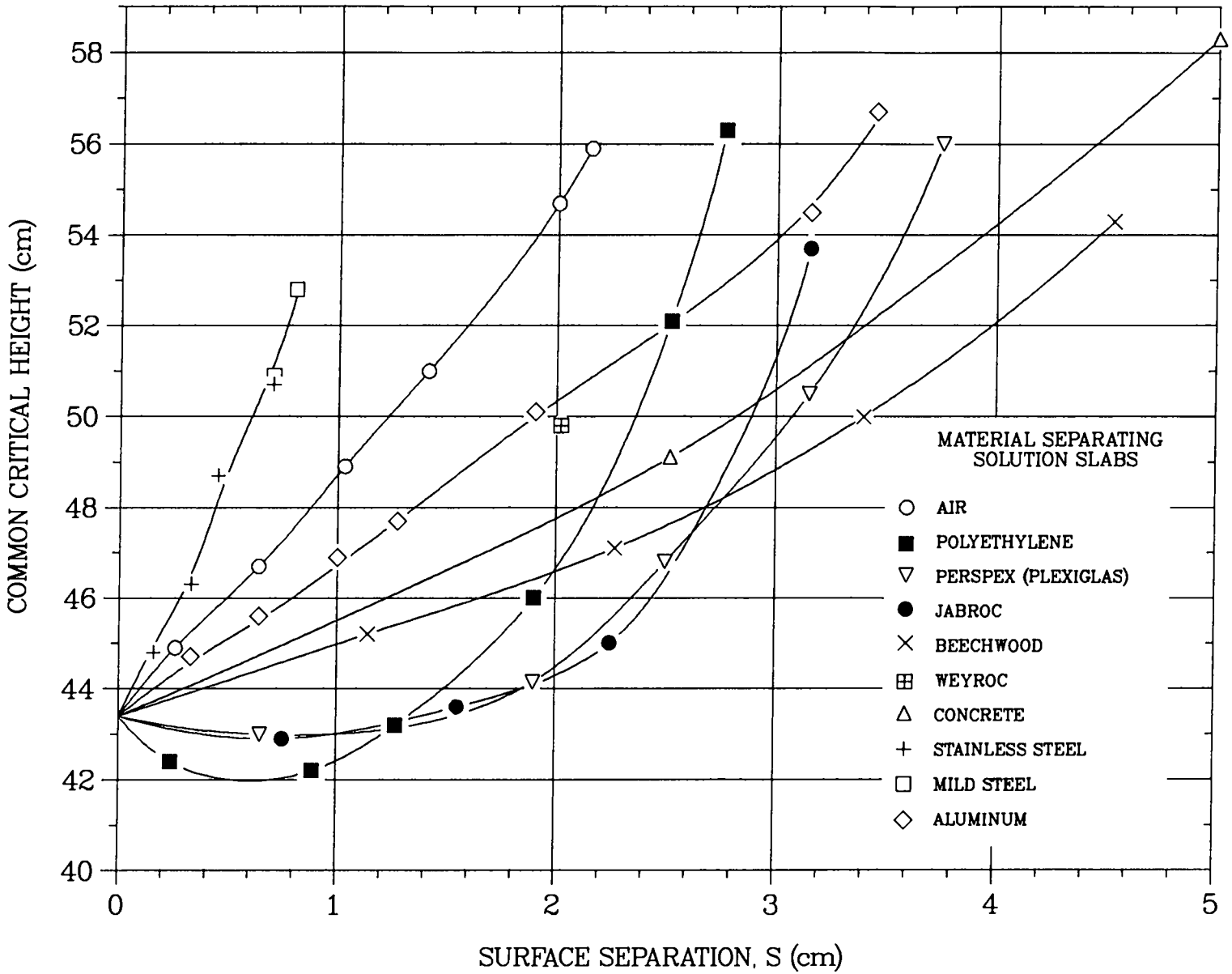


Fig. 65. Common critical height of two parallel $U(30.45)O_2F_2$ solution slabs with thick polyethylene reflectors on outer faces and separated by various materials; $H/^{235}U = 130$, 0.182 g $^{235}U/cm^3$. Slabs 120-cm wide by 6.1-cm thick. Jabroc is a wood product containing about 45% carbon, 6% hydrogen, and 37% oxygen, at a density of 1.32 g/cm^3. Weyroc is also a wood product with a density of 0.72 g/cm^3.

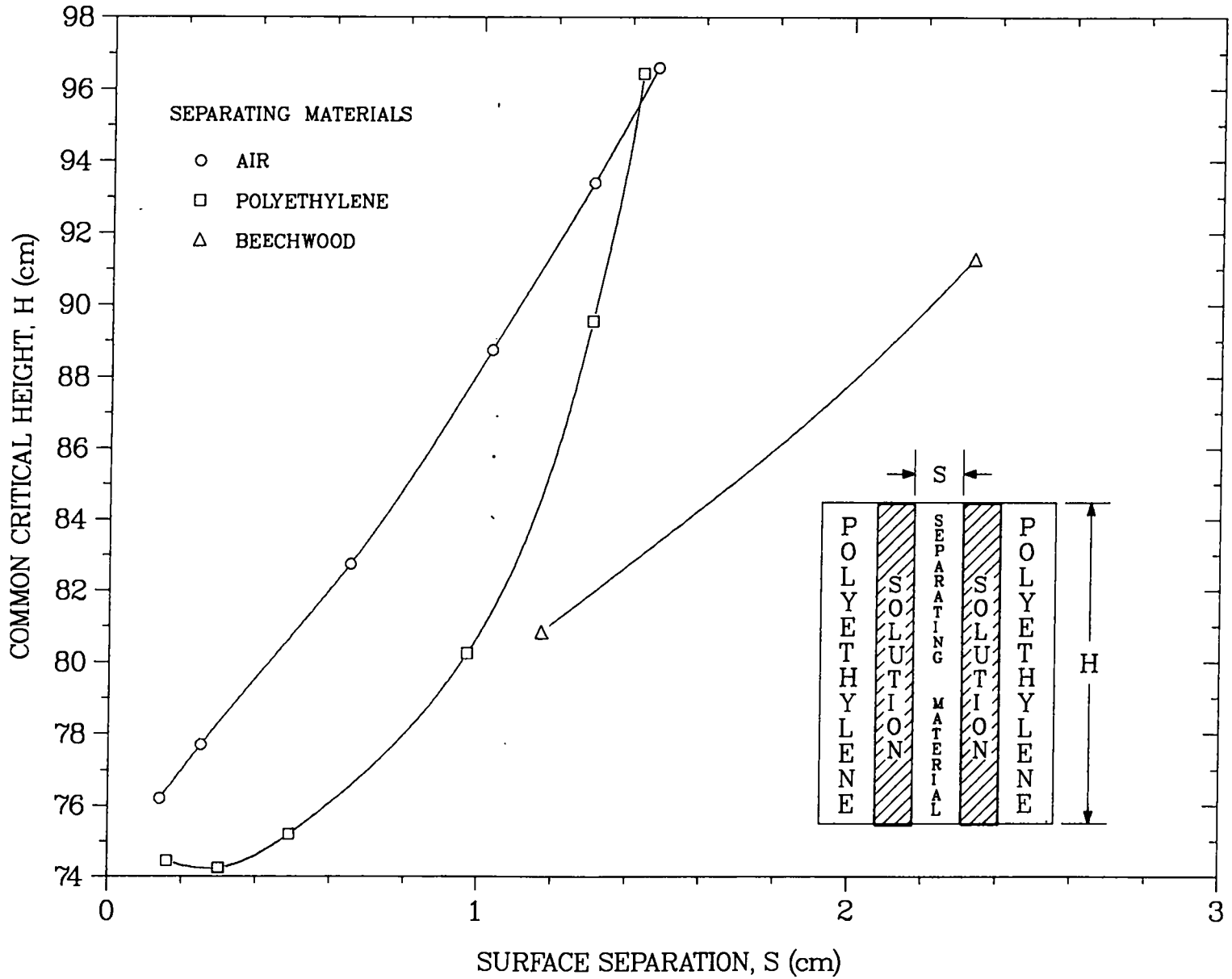


Fig. 66. Common critical height of two parallel $U(30.45)O_2F_2$ solution slabs with thick polyethylene reflectors on outer faces and separated by air, polyethylene, or beechwood; $H/^{235}U = 214$, 0.117 g $^{235}U/cm^3$ solution. Slabs 120-cm wide by 6.1-cm thick.

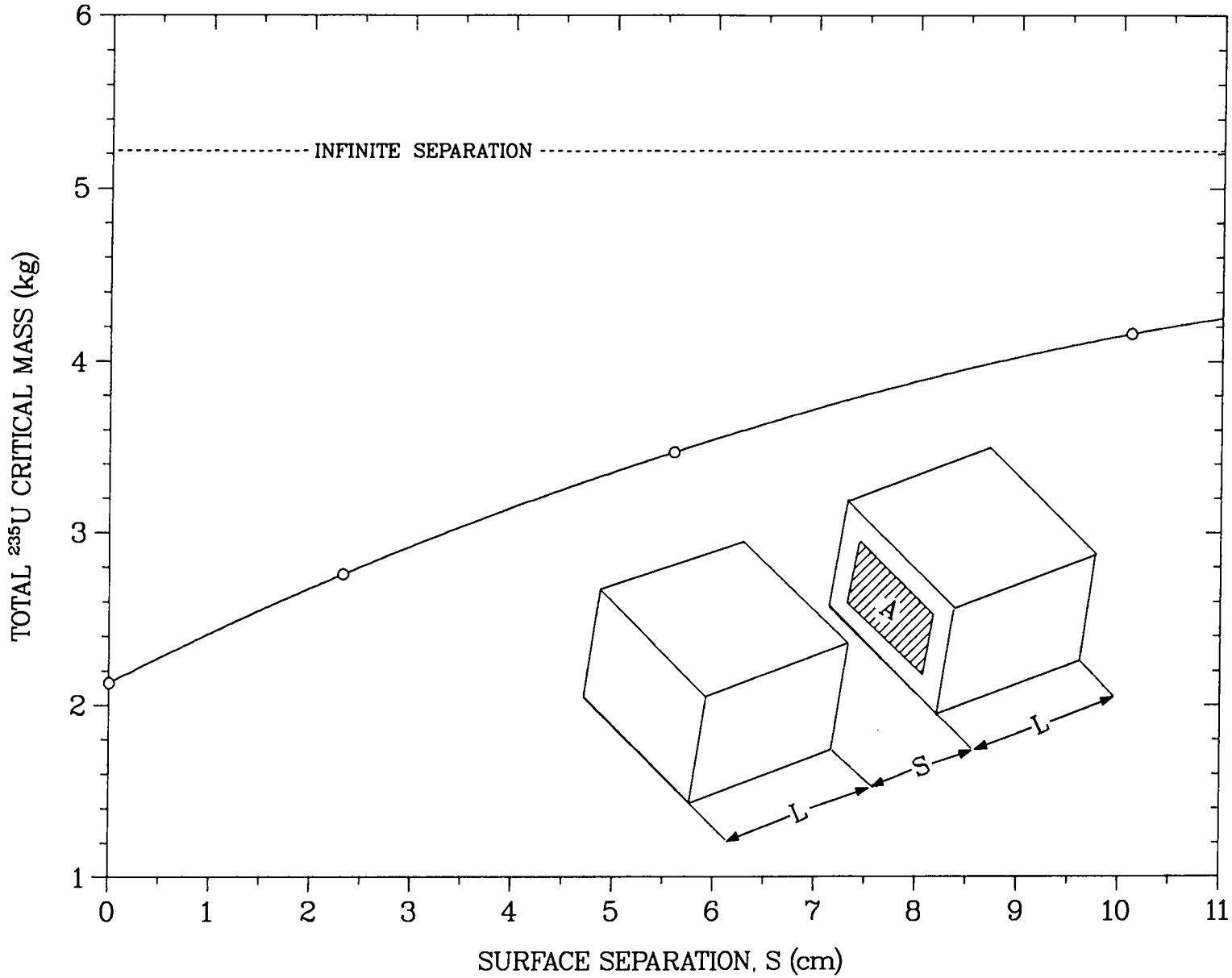


Fig. 67. Total critical mass of two identical $U(30.14)O_2$-paraffin parallelepipeds with external polyethylene reflection and separated by air, $H/^{235}U = 81.8$, 0.331 g $^{235}U/cm^3$.

TABLE 52. CRITICAL DIMENSIONS OF TWO PARALLELEPIPEDS OF HOMOGENEOUS U(1.42)F_4-PARAFFIN REFLECTED BY 20.3 cm OF POLYETHYLENE EXCEPT ON FACING AREAS

	Dimensions of Each Parallelepiped					
Separation (cm)	Width (cm)	Height (cm)	Length (cm)	Total Volume (L)	Total Mass (kg) Uranium	^{235}U
0.000	92.1	92.3	45.3	771	1920	27.3
0.533	92.1	92.3	46.2	785	1950	27.7
0.998	92.1	92.3	46.2[a]	808	2010	28.5
			48.7[a]			
1.443	92.1	92.3	48.7	830	2070	29.4
2.121	92.1	92.3	51.3	873	2170	30.8
2.771	92.1	92.3	53.9	917	2280	32.4
3.899	92.1	92.3	53.9[a]	982	2450	34.8
			61.5[a]			
4.943	92.1	92.3	61.5	1048	2610	37.1
6.541	92.1	92.3	66.7	1134	2820	40.1
8.438	92.1	92.3	71.8	1222	3040	43.2
10.363	92.1	92.3	75.7	1288	3210	45.6
0.000	46.1	90.6	92.3	771	1920	27.3
0.254	46.1	91.3	92.3	777	1930	27.4
0.508	46.1	92.5	92.3	787	1960	27.8
0.742	46.1	93.8	92.3	798	1990	28.3
0.935	46.1	95.1	92.3	809	2010	28.5
1.511	46.1	98.9	92.3	842	2100	29.8
2.731	46.1	109.2	92.3	929	2310	32.8
3.576	46.1	116.9	92.3	995	2480	35.2

[a] In these cases the lengths of the two parallelepipeds were unequal.

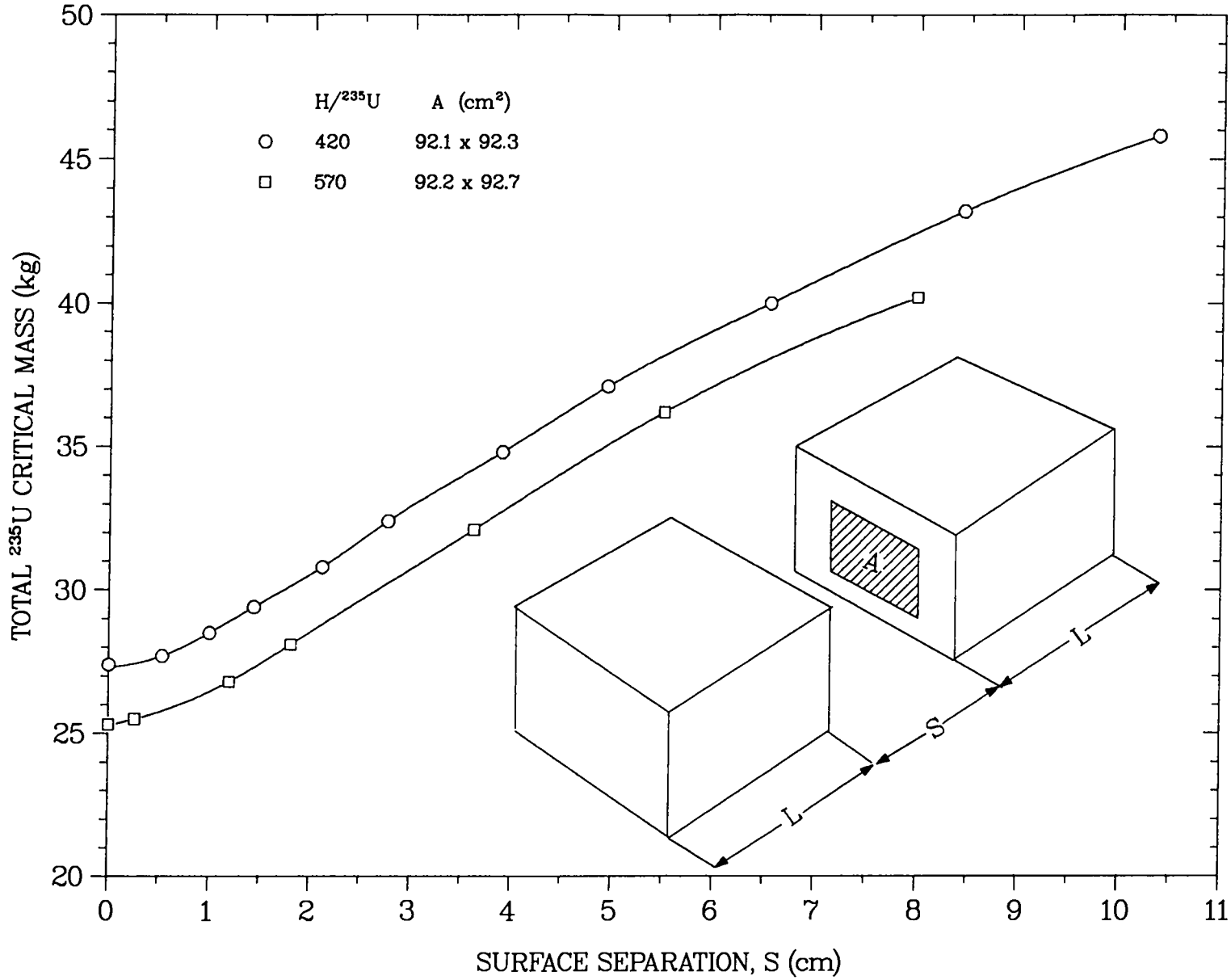


Fig. 68. Total critical mass of two identical U(1.42)F_4-paraffin parallelepipeds with 20.3-cm-thick polyethylene reflector except on facing areas.

Also shown in Fig. 68 are similar results obtained with a $U(1.42)F_4$-paraffin mixture containing 2.22 g U/cm^3 with an $H/^{235}U$ of 570. This moderation is approximately that yielding minimum critical mass. These parallelepipeds were constructed with a constant facing area and with a 20.3-cm-thick polyethylene reflector on all surfaces except the facing areas.

Data on the neutron interaction of two homogeneously moderated units, one a slab of $U(30.45)O_2F_2$ solution 6.1-cm thick and the other a parallelepiped of $U(1.42)F_4$-paraffin, separated by various materials, originated at Dounreay.* Figure 69 shows the experimental arrangement and Table 53 gives the results.

* J. G. Walford and J. C. Smith, United Kingdom Atomic Energy Authority, Dounreay Experimental Research Establishment, Dounreay, 1963.

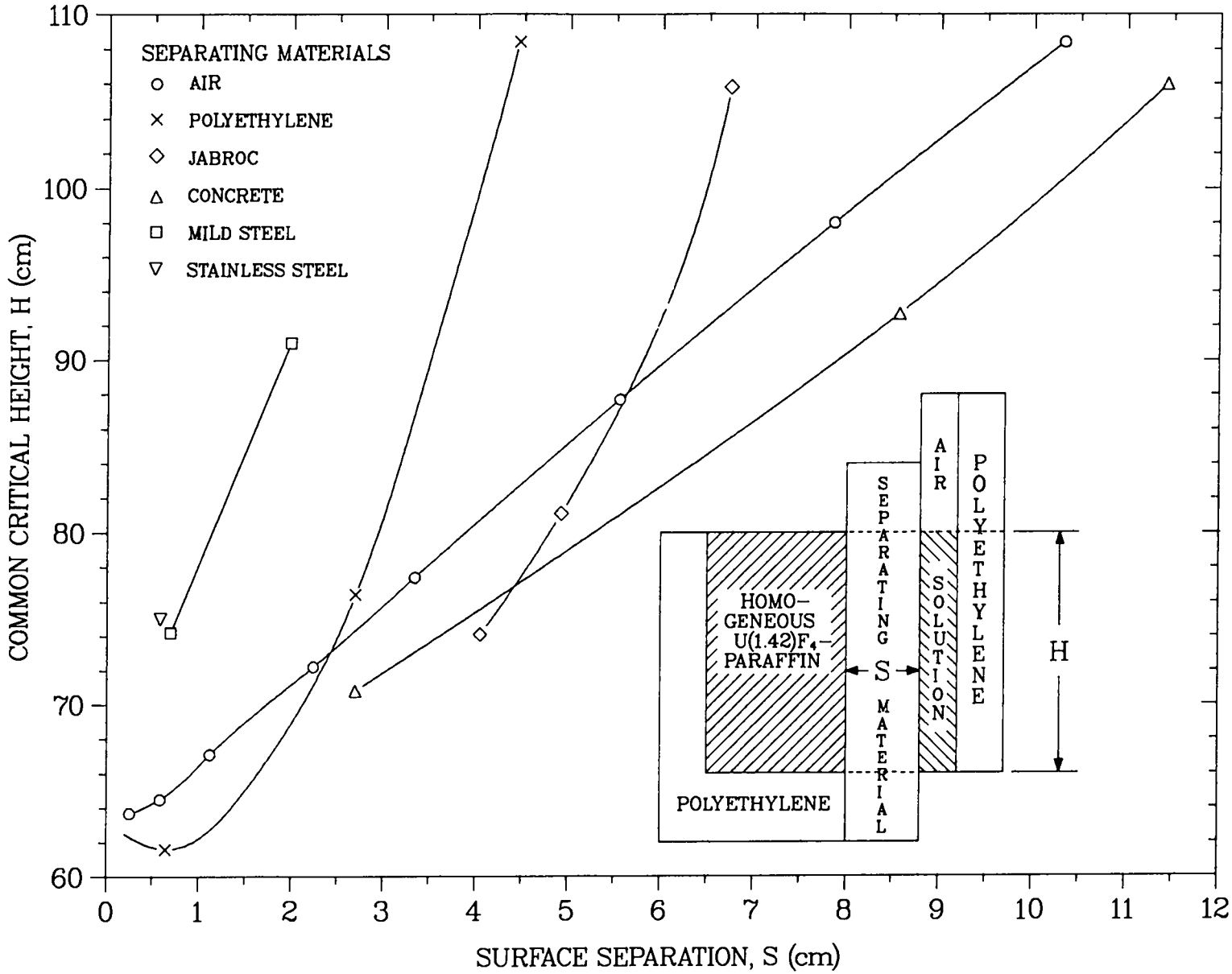


Fig. 69. Common critical heights of a $U(30.45)O_2F_2$ solution slab and a facing $U(1.42)F_2$-paraffin parallelepiped, external polyethylene reflectors, and various materials between components; solution at $H/^{235}U$ = 112, 0.204 g $^{235}U/cm^3$ in the 6.09-cm-thick by 120-cm-wide slab; parallelepiped 62-cm thick and 123-cm wide.

TABLE 53. CRITICAL DIMENSIONS OF PARALLELEPIPEDS OF $U(1.42)F_4$-PARAFFIN AND $U(30.45)O_2F_2$ SOLUTION

Material Separating Components	Critical Height (cm)	Critical Separation (cm)	Mass (kg ^{235}U)		Volume, (L)	
			$U(1.42)F_4$-Paraffin	$U(30.45)O_2F_2$	$U(1.42)F_4$-Paraffin	$U(30.45)O_2F_2$
Air	63.7	0.25	145	10.68	478	46.2
	64.5	0.58	148	10.90	488	47.1
	67.1	1.12	154	11.32	509	49.0
	772.2	2.24	165.5	12.20	546	52.8
	77.4	3.34	177.8	13.08	586	56.5
	87.7	5.55	201.5	14.81	664	64.1
	98.0	7.85	225.0	16.55	742	71.6
	108.4	10.32	248.5	18.35	820	79.2
Polyethylene	61.6	0.64	141.2	10.40	466	95.0
	76.4	2.70	175.5	12.90	578	55.8
	108.4	4.44	248.5	18.35	820	79.2
Jabroc[a]	74.1	4.05	170.0	12.55	561	54.2
	81.1	4.92	186.5	13.74	614	59.9
	105.8	6.73	242.5	17.89	800	77.2
Concrete	70.8	2.70	162	11.91	535	51.6
	92.7	8.56	214	15.70	705	67.8
	106	11.43	244	17.90	805	77.4
Mild steel	74.2	0.69	170.5	12.55	562	54.2
	91.0	1.98	209	15.40	689	66.5
Stainless steel	75	0.58	172	12.70	567	54.8

[a] Jabroc is a wood product containing about 45% carbon, 6% hydrogen, and 37% oxygen; $\rho = 1.315$ g/cm^3.

Critical planar arrays of tall cylinders containing $U(5.0)O_2F_2$ solution were studied at Oak Ridge.[176,177] The aluminum containers were 24.1-cm-i.d. with 0.32-cm lateral wall thickness, 0.64-cm-thick bottom, and 152-cm height. The solution concentration was 901 gU/L at $H/^{235}U = 497$. Units were of three different solution heights: 61-cm at a volume of 27.8 L, 122-cm at 56 L, and 142-cm at 65 L. All arrays were unreflected. Critical conditions are given in Table 54.

TABLE 54. PLANAR ARRAYS OF $U(5.0)O_2F_2$ SOLUTION CYLINDERS AT $H/^{235}U = 497$

Unit Volume (L)	Number of Units	Array Pattern	Critical Surface Separation (cm)
27.8	7	triangular	1.89
27.8	9	square	1.32
27.8	16	square	3.62
27.8	25	square	5.2
56	5	triangular	1.37
56	7	triangular	4.44
56	19	triangular	13.2
56	9	square	4.85
56	16	square	9.2
56	25	square	12.4
65	5	triangular	1.47
65	7	triangular	5.1
65	19	triangular	14.8
65	9	square	5.6
65	16	square	10.4
65	25	square	14.1

Plutonium Solutions

An experiment to determine the safety of spaced cylinders of plutonium solution was performed at Rocky Flats.[178] The $Pu(NO_3)_4$ solution, containing 5 N excess HNO_3, at a concentration of 400 g Pu/L, was held in stainless steel pipes 14.1-cm-o.d. and 12.8-cm-i.d. Five 19-L vertical cylinders were arranged in a linear array with a 61-cm center spacing and located about 5-cm above a thick concrete slab. Benelex (pressed wood chips impregnated with a hydrocarbon plastic), with a density of 1.44 g/cm^3 and 10.2-cm-thick, was located 15.3-cm from the sides of the array and 150-cm from the top of the solution. The array was apparently quite subcritical.

The neutron interaction between two identical annuli of plutonium nitrate solution in stainless steel containers was studied at the Station de Criticalité[157] in France. The internal and external diameters of the containers were 30.0- and 50.0-cm, respectively. Common critical height versus solution separation was reported for concentrations of 41, 56 and 74 g Pu/L without reflector and with water reflector in various locations.

Another report of the French Commissariat a l'Energie Atomique gives experimental critical data on interacting unreflected cylinders containing plutonium solutions, and a few enriched uranium solutions for comparison.[179] The $Pu(NO_3)_4$ solution contained about 2 N HNO_3 and the plutonium consisted of 95.5% ^{239}Pu, 4.2% ^{240}Pu and 0.3% ^{241}Pu. Plutonium solutions were contained in as many as four 30-cm-o.d. stainless steel cylinders with 0.3-cm-thick walls and in as many as nine 25.6-cm-o.d. cylinders, also with 0.3-cm-thick walls. The $U(89.9)O_2(NO_3)_2$ solution was contained in a pair of 30-cm-o.d. stainless steel cylinders with 0.3-cm-thick walls.

Figures 70-72 give critical heights of solution in various configurations of 30-cm-o.d. cylinders as functions of surface separation of cylinders. Figure 72 compares relations for two cylinders containing plutonium or uranium solution at roughly equivalent concentrations. Curves for two, three, and four interacting cylinders of plutonium solution are given for two different concentrations in Figs. 70 and 71. For as many as nine interacting 25.6-cm-o.d. cylinders of plutonium solution, critical heights at various spacings are listed in Table 55.

Also reported were results for pairs of dissimilar units and critical heights as functions of concentration for isolated 30-cm-o.d. cylinders containing either plutonium or uranium solutions.

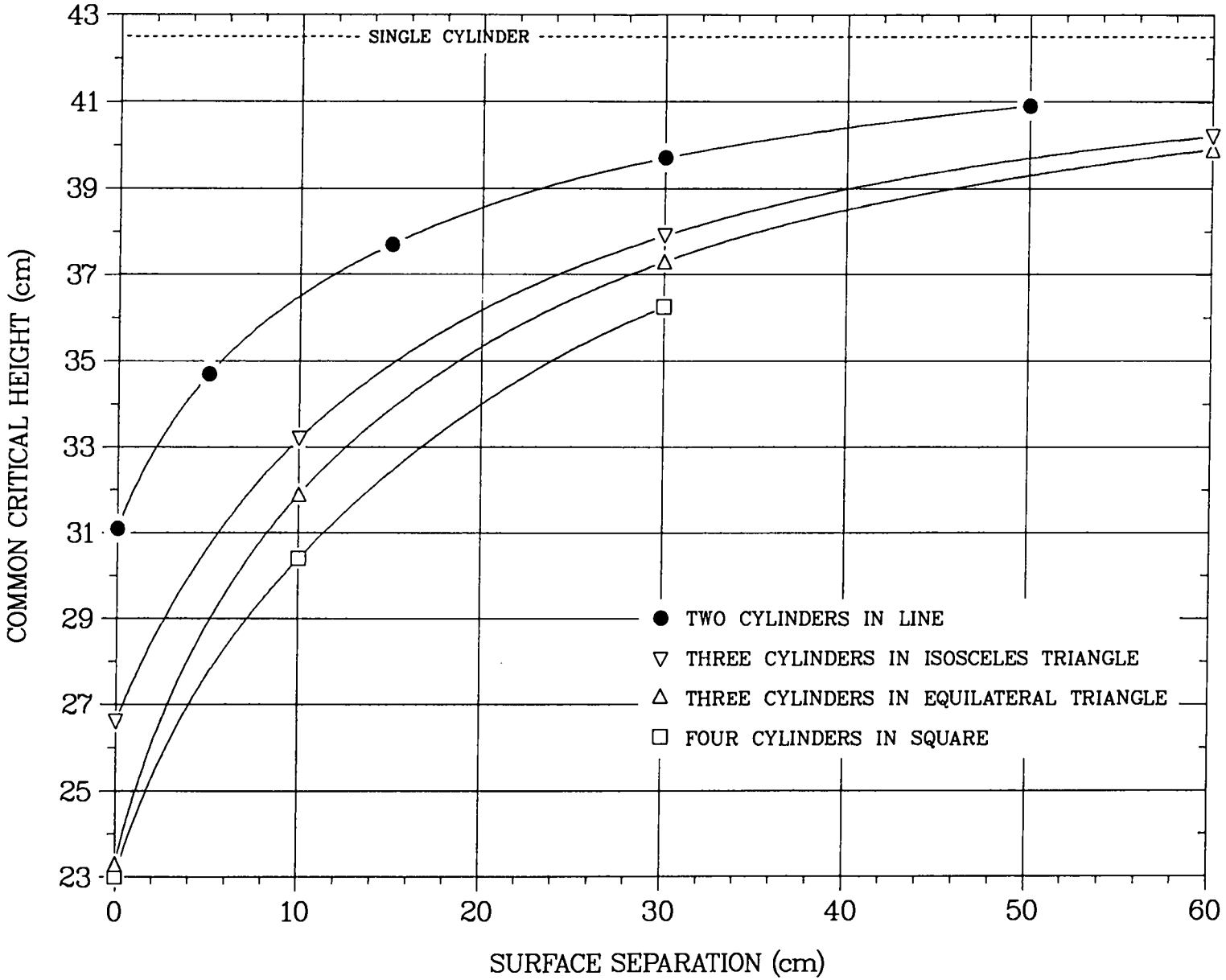


Fig. 70. Common critical heights of two, three, or four unreflected $Pu(NO_3)_4$ solution cylinders at 0.146 g/cm^3 of $^{239}Pu + {}^{241}Pu$; 30-cm-o.d. stainless-steel containers with 0.3-cm-thick walls.

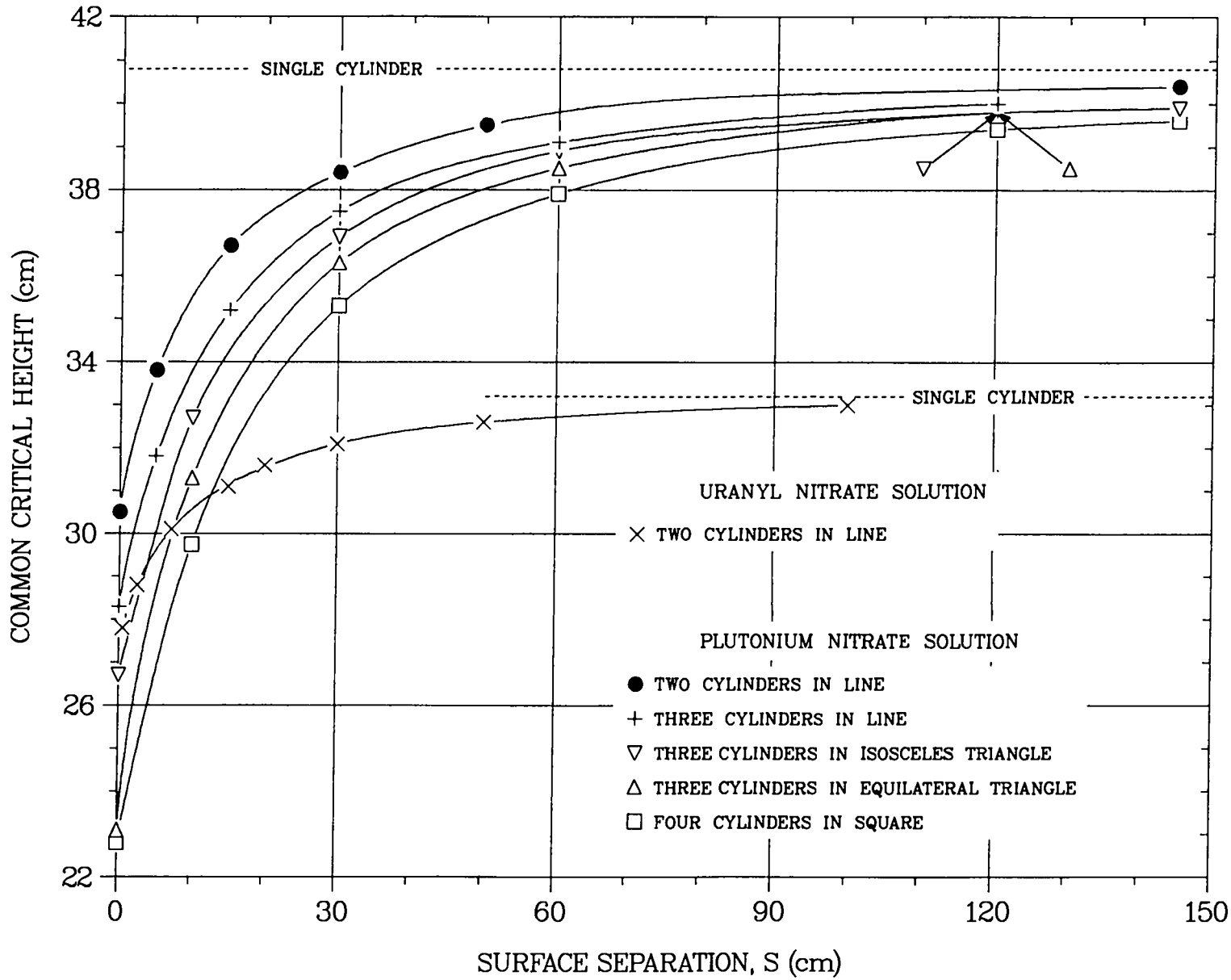


Fig. 71. Common critical heights of two, three, or four unreflected $Pu(NO_3)_4$ solution cylinders or two $U(90)O_2(NO_3)_2$ solution cylinders; concentration of fissile element = 0.110 g/cm^3. Stainless steel containers 30-cm-o.d. with 0.3-cm-thick walls.

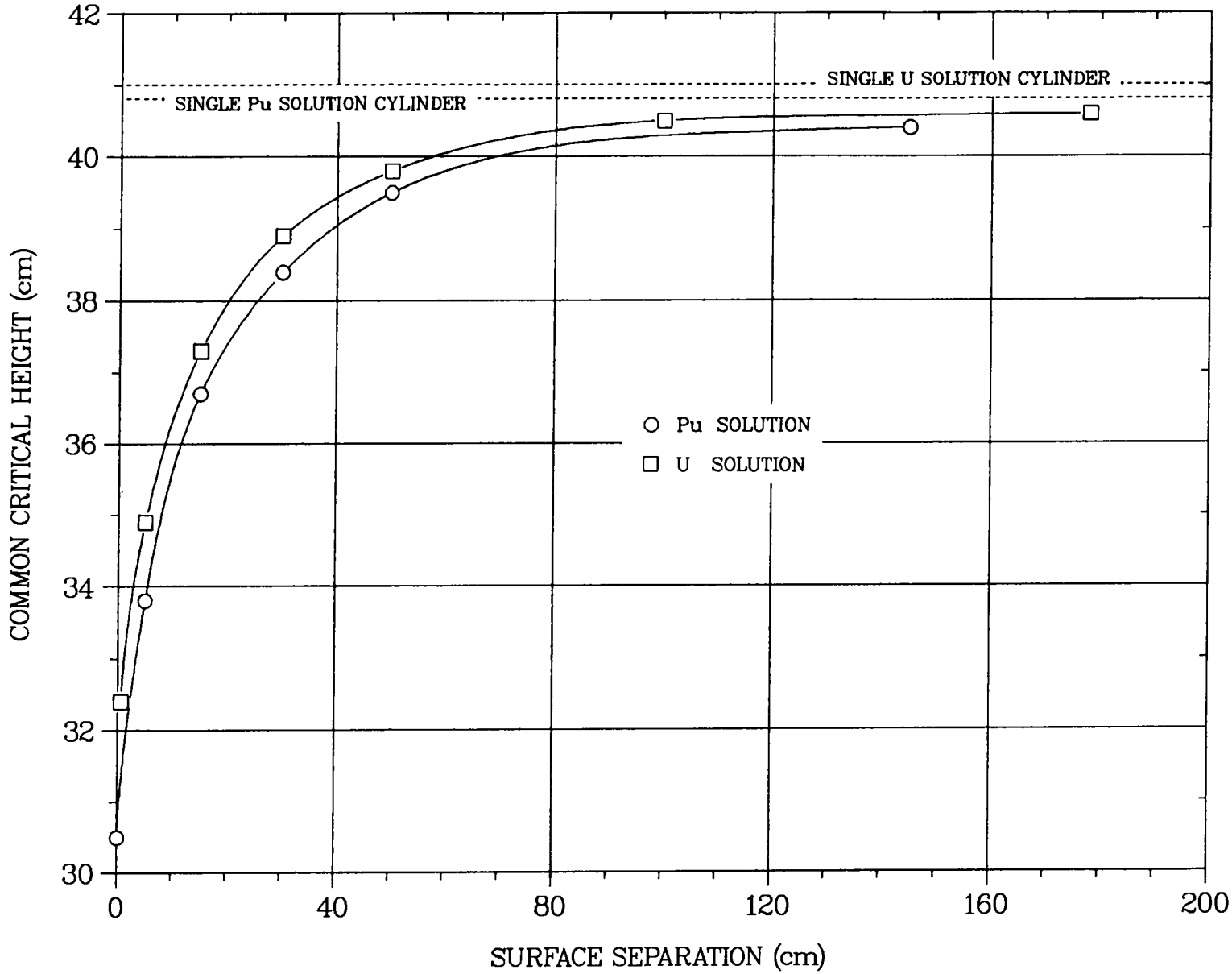


Fig. 72. Common critical height of two unreflected $Pu(NO_3)_4$ solution cylinders at 0.110 g/cm^3 of ^{239}Pu + ^{241}Pu or two $U(90)O_2(NO_3)_2$ solution cylinders at the equivalent concentration of 0.081 g ^{235}U/cm^3. Stainless steel containers 30-cm-o.d. with 0.3-cm-thick walls.

TABLE 55. CRITICAL HEIGHT OF PLUTONIUM SOLUTION AT 110 g ^{239}Pu/L IN VARIOUS CONFIGURATIONS OF 25.6-cm-o.d. CYLINDERS

No. of Cylinders	Surface Spacing (cm): Configuration	Critical Solution Height (cm) 0	1.2	5	10	20
2	—	53.0	—	78.7	103	—
3	equi. triangle	—	—	41.1	53.1	75.6
3	right triangle	36.0	—	48.8	61.0	86.3
4	square	26.7	—	36.3	45.0	63.2
6	2 by 3	24.0	—	31.9	39.2	—
7	hexagonal ("star")	—	21.4	27.1	33.7	—
9	3 by 3	24.0	—	28.3	—	—

Hanford reported critical planar arrays of as many as sixteen 3-L polyethylene bottles containing $Pu(NO_3)_4$ solution.[180,*] The Pu contained 2.8 wt% ^{240}Pu, and the solution had an average concentration of 117 g Pu/L and an excess acid normality of about 6.7. The polyethylene bottles averaged 10.63-cm-i.d. with 0.575-cm-thick walls and 0.635-cm-thick bottom. The top of the vent caps on the bottles extended ~16-cm above the solution and reflector surfaces. The minimum number of bottles in a critical array, reflected sides and bottom by 15.2-cm-thick Plexiglas, was 10.9. Influences of Plexiglas sleeves surrounding the bottles were also given.

* The bottle description was provided by E. D. Clayton, a coauthor of the reference.

Solutions of ^{233}U

A brief study of 2 x 2 x 2 and 3 x 3 x 3 arrays of $^{233}U(97.5\ wt\%)O_2(NO_3)_2$ solution units was conducted at Oak Ridge.[113,181] Each 4.30-liter unit of solution at $H/^{233}U = 73$, 0.325 g $^{233}U/cm^3$, was in a stainless steel container of 18.3-cm o.d., 17.7-cm outside height and 0.025-cm-thick wall. Critical spacings of both unreflected and polyethylene-reflected arrays appear in Table 56.

A report from Hanford describes arrays of polyethylene bottles containing ^{233}U solution, some with Lucite between bottles.[182] Arrays were either unreflected or reflected with Lucite 11.4-cm-thick on top and bottom, and 15.2-cm-thick on the sides. The 3-L bottles were 45.0-cm high, 11.9-cm-o.d., with 0.25-cm-thick walls. The solution was $^{233}U(98.2\%)O_2(NO_3)_2$ with 0.53 $\underline{N}$ HNO_3 at a concentration of 330 g ^{233}U/L. The average 29.2-cm height of solution in a bottle corresponds to 940 g ^{233}U per bottle and varied from array to array. Critical conditions of selected arrays are given in Table 57. Unreflected arrays with Lucite between bottles were also reported.

TABLE 56. CRITICAL ARRAYS OF 4.30-LITER CYLINDRICAL UNITS[a] OF $^{233}U(97.5\ wt\%)O_2(NO_3)_2$ SOLUTION

Solution Concentration	Number of Units[a], Configuration	Polyethylene Reflector Thickness (cm)	Center Spacing (cm)	
			Horizontal	Vertical
$H/^{233}U = 73$				
325 g $^{233}U/L$	8, 2 x 2 x 2	0	20.4	19.1
	27, 3 x 3 x 3	0	25.7	24.6
	8, 2 x 2 x 2	15.2	32.0	30.4
	27, 3 x 3 x 3	15.2	41.0	38.6
$H/^{233}U = 119$				
199 g $^{233}U/L$	8, 2 x 2 x 2	0	20.2	18.8
	27, 3 x 3 x 3	0	25.0	23.9
	8, 2 x 2 x 2	15.2	30.2	25.5
	27, 3 x 3 x 3	15.2	38.0	36.7

[a]Stainless steel containers, outside diameter 18.28-cm, outside height 17.67-cm, wall thickness 0.025-cm.

TABLE 57. CRITICAL ARRAYS OF BOTTLES CONTAINING ^{233}U SOLUTION

Mass of ^{233}U Per Bottle (g)	Configuration	Separation Surface of Bottles (cm)	Moderator Between Bottles
	Reflected Arrays		
917	2 x 2 (4 bottles)	5.5	None
917	2 x 2	6.3	1.27 cm Lucite
917	2 x 2	6.6	1.90 cm Lucite
917	2 x 2	6.8	2.54 cm Lucite
917	2 x 2	6.4	3.81 cm Lucite
930	3 x 3 (9 bottles)	10.1	None
	Unreflected Arrays		
930	3 x 3	1.52	None
944	4 x 4 (16 bottles)	2.95	None

SPATIAL ARRAYS OF ^{235}U SOLUTION

The critical dimensions of rectangular three-dimensional arrays of as many as 125 five-liter units of $U(92.5)O_2(NO_3)_2$ solution were reported from ORNL.[177,183] Containers were 0.64-cm-thick Plexiglas, 20.3-cm-o.d. and 19.05-cm outside height, with a solution volume of 5.00 L. The solution had three different concentrations: 384 g ^{235}U/L or H/^{235}U = 59, 258 g ^{235}U/L or H/^{235}U = 92, and 59 g^{235}U/L or H/^{235}U = 440. Some arrays were reflected with paraffin. Table 58 gives the critical conditions of these arrays.

TABLE 58. SPATIAL ARRAYS OF 5-LITER UNITS OF $U(92.6)O_2(NO_3)_2$ SOLUTION

^{235}U Concentration (g/L)	Configuration	Surface Separation of Units (cm)	Paraffin Reflector Thickness (cm)
384	2 x 2 x 2	1.43	0
384	2 x 2 x 2	3.28	1.3
384	2 x 2 x 2	6.9	3.8
384	2 x 2 x 2	8.5	7.6
384	2 x 2 x 2	9.0	15.2
384	3 x 3 x 3	6.5	0
384	3 x 3 x 3	9.0	1.3
384	3 x 3 x 3	16.5	15.2
384	4 x 4 x 4	10.7	0
384	5 x 5 x 5	14.4	0
258	2 x 2 x 2	1.43	0
258	2 x 2 x 2	8.7	11.4[a]
258	3 x 3 x 3	6.4	0
59	3 x 3 x 3	2.41	0

[a] Paraffin at bottom was 15.2 cm thick.

HETEROGENEOUSLY MODERATED UNITS

PLANAR AND SPATIAL ARRAYS

Interacting Fuel-Rod Lattices

Hanford has reported critical experiments on three in-line clusters of $U(2.35)O_2$ and $U(4.31)O_2$ in fuel rods in water to simulate conditions in fuel shipping casks.[184,185] The $U(2.35)O_2$ in a rod was 1.12-cm-diam by 91-cm long, with a density of 9.2 g/cm^3, and clad with 0.076-cm-thick aluminum. Similarly, the $U(4.31)O_2$ was 1.26-cm diam, 91-cm long, with a density of 10.4 g/cm^3, and clad with 0.066-cm-thick aluminum. In each experiment criticality was attained by adjusting the water separation between clusters, which remained unchanged. Figure 73 shows effects of reflecting walls of 7.6-cm-thick depleted uranium, 10.2-cm-thick lead, and 17.8-cm-thick steel as functions of separation from three 16 x 19 clusters of $U(2.35)O_2$ rods, each arranged at a near-optimum square pitch of 2.03-cm. Figure 74 shows effects of the same reflecting walls beside three 8 x 13 clusters of $U(4.31)O_2$ rods at a near-optimum square pitch of 2.54-cm. Also reported were similar results for undermoderated clusters and effects of various neutron absorbing plates in the water separating clusters. The absorbers included steel, steel containing boron, copper, copper with cadmium, Boral (boron and aluminum), and cadmium.

Units of $U(93)O_2$ - Alcohol

Oak Ridge reported experimental multiplication factors (k_{eff}) for reflected arrays of $U(93.15)O_2$ in thin-wall iron cans and compared them with results of KENO calculations.[58] Generally, the cans were separated by methyl methacrylate or contained ethyl alcohol and water with the UO_2. The array reflector was polyethylene, apparently about 15-cm-thick.

The largest unit contained 20.0 kg UO_2 of 7.65-cm radius and about 19.8-cm height. Two arrays in which moderation was not required to attain criticality were 4 x 4 x 1 units with a 15.7- x 15.7- x 22.3-cm-high cell, and 3 x 3 x 2 units with an 18.2- x 18.2- x 22.3-cm-high cell. Each cell contained 360 g of iron. Other arrays of this unit had methyl methacrylate within the lattice. A precisely critical array contained 3 x 3 x 2 units with a 20.4- x 20.4- x 22.3-cm-high cell in which top and bottom cells were separated by 2.36-cm-thick methyl methacrylate.

Another type unit contained 17.0 kg UO_2 of 6.76-cm radius and 17.9-cm height, in which 712 g C_2H_6O - 5% H_2O was mixed with the oxide and 362 g was in a layer above the oxide. A precisely critical configuration was 3 x 3 x 1 units with an 18.1- x 18.1- x 25.6-cm-high cell containing 340 g of iron. With the same unit except that the C_2H_6O - 5% H_2O layer above the oxide was reduced to 62 g, precise criticality was attained again in an array of 3 x 3 x 1 units with a 17.3- x 17.3- x 25.6-cm-high cell.

Also reported was an array of 196 units of 420 g each, containing a layer of methyl methacrylate, and the same array reduced to 178 units.

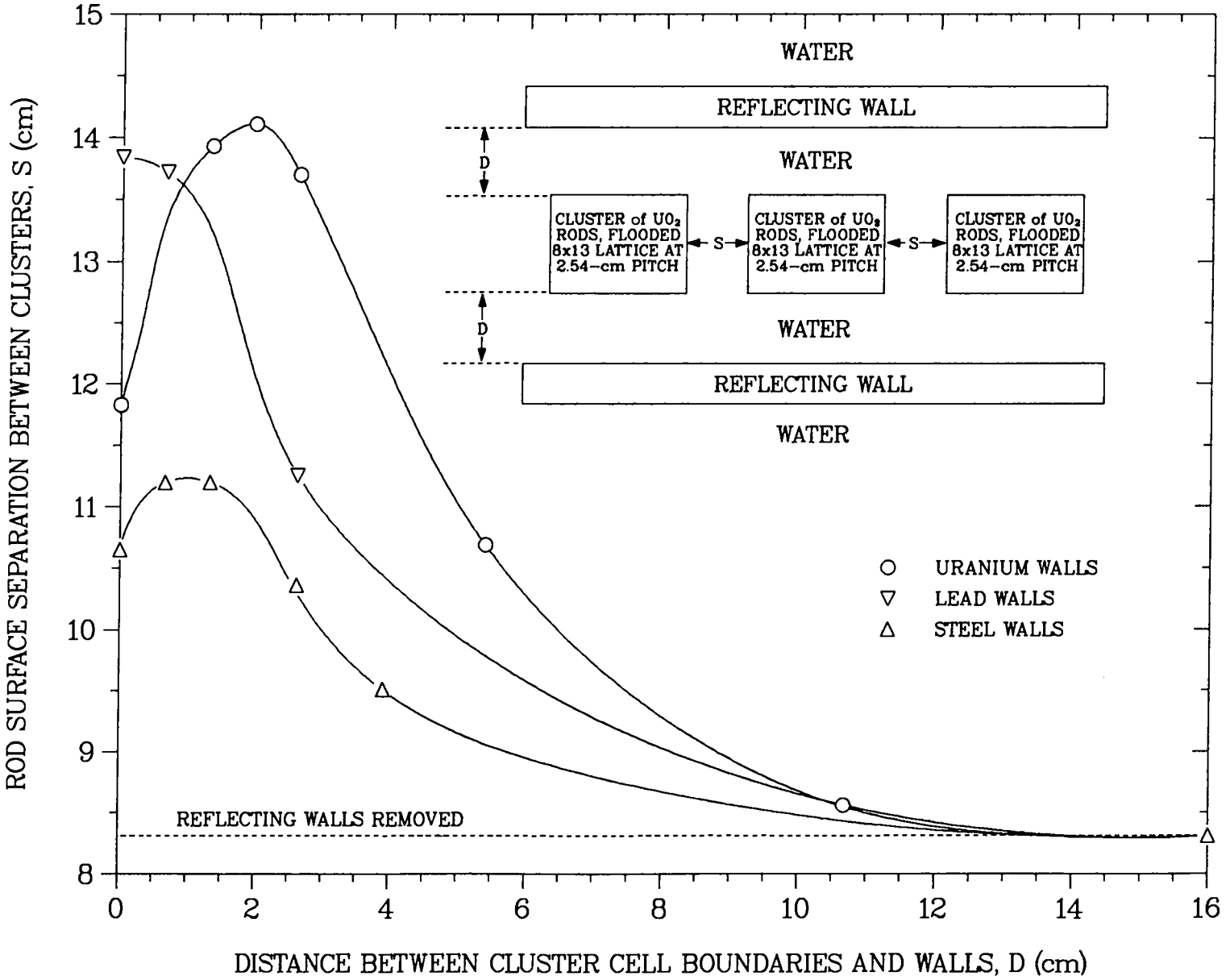


Fig. 73. Critical separations of $U(2.35)O_2$ rod clusters in water as functions of distance to reflecting walls of steel, lead, or depleted uranium; clusters were at near-optimum moderation.

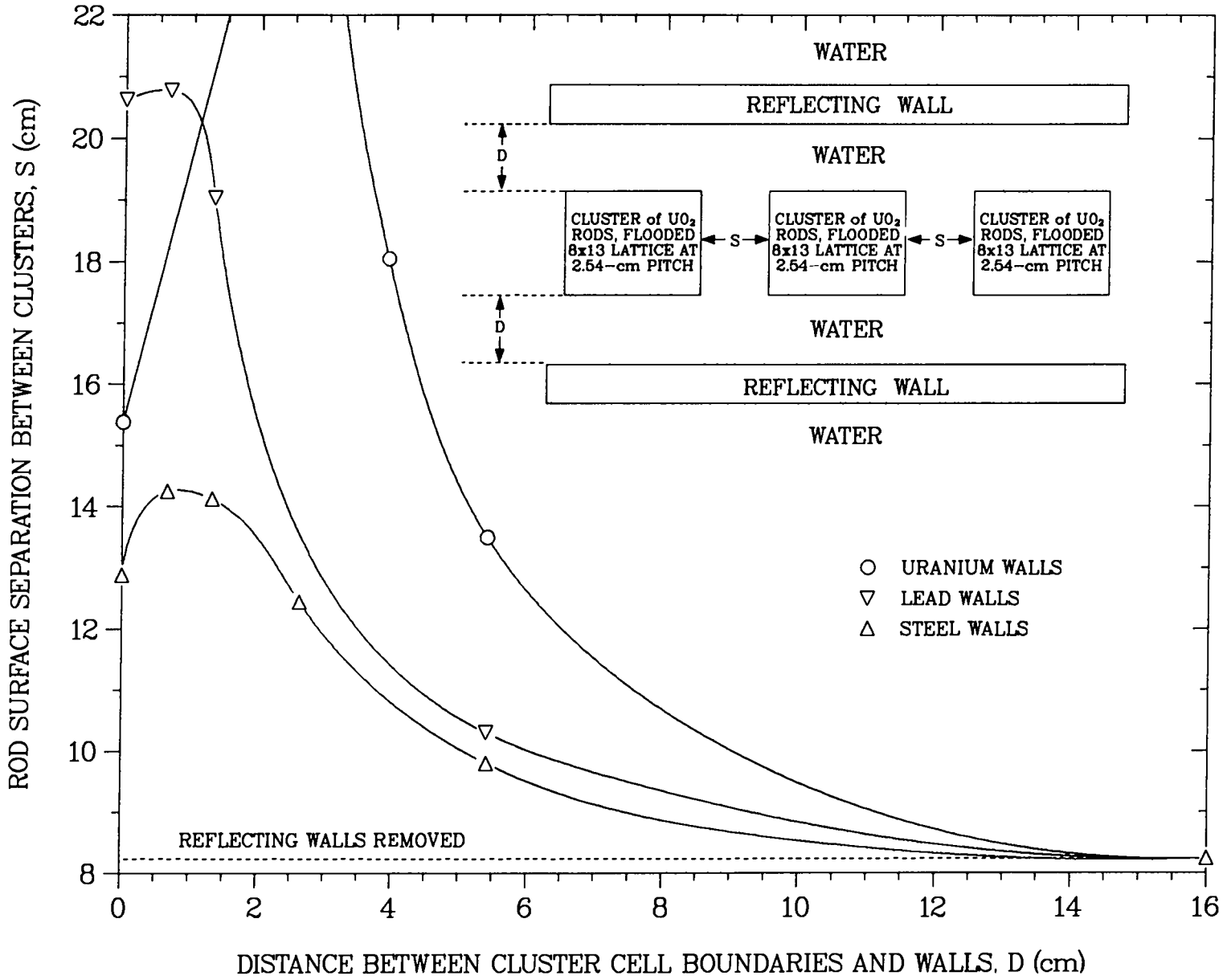


Fig. 74. Critical separations of $U(4.31)O_2$ rod clusters in water as functions of distance to reflecting walls of steel, lead, or depleted uranium; clusters were at near-optimum moderation.

METAL UNITS AND COMPOUNDS

URANIUM

Planar Arrays

Neutron interaction between two[186] and three* component systems was studied at ORNL utilizing uranium metal disks and slabs of varying thicknesses to determine the critical spacing of identical pieces. The units were U(93) metal with a density of 18.7 g/cm^3. The critical separations between the large, parallel, flat surfaces of the units as a function of their geometry and thickness is reported.

Other Oak Ridge experiments involved planar and linear arrays of as many as 16 Model 8A shipping containers each containing an average of 110.8 kg of condensed U(97.66)F_6.[187] The 20.3-cm-i.d., 1.25-m high, containers of 0.48-cm-thick Monel were described in detail. The axis-to-axis spacing of vertical cylinders in contact was 22.52-cm. In addition to the near-critical arrays listed in Table 59, a number of configurations that were critical with gaps separating two portions included a 3 × 3 array with partial polyethylene reflector; a 2 × 2 array with partial polyethylene reflector and Plexiglas between cylinders; 3 × 3, 1 × 3, 1 × 4 and 1 × 12 arrays with partial concrete reflectors; and two 1 × 3 arrays separated by up to 40.8-cm-thick concrete with partial concrete reflectors.

The Oak Ridge experiments with thick concrete between multiplying units were preceded by similar experiments at Los Alamos[188],**and Hanford,[189] for which selected results follow. The Los Alamos measurements showed the effect of 20.3-cm thicknesses of various materials between two 53.3-cm-diam disks of U(93.2) metal. The second disk, 2.4-cm-thick, 20.3-cm away reduced the critical thickness of the first from 7.15-cm to 6.84-cm, the 0.31-cm difference being a measure of interaction. With intervening material, the interaction was the effect of adding the 2.4-cm-thick disk to the material. Intervening polyethylene reduced the interaction to nearly zero, concrete to 0.15-cm, lead to 0.08-cm, wood to 0.25-cm, and beryllium to 0.23-cm.

The Hanford measurements established isolating thicknesses of various materials between two ~30.8-cm-square multiplying plates that were mosaics of PuO_2-polystyrene and Plexiglas cubes. The isolating thickness was that beyond which a further increase had no perceptible effect on criticality of the combination. Reported isolation thicknesses were 16.5±1.3-cm for polyethylene, 20.3±1.3-cm for compressed wood, 22.9±1.3-cm for concrete, and 20.3±1.3-cm for lead.

* J. T. Mihalczo, Oak Ridge National Laboratory, 1963.

** Details were provided by D. R. Smith, a coauthor of the reference.

TABLE 59. ARRAYS OF VERTICAL TYPE 8A SHIPPING CONTAINERS, EACH CONTAINING ABOUT 111 kg OF $U(97.7)F_6$

Configuration	Axial Spacing (cm)	Reflector[a,b]	k_{eff}[a]
4 × 4	29.59	15.2-cm-thick polyethylene, complete	0.9993
2 × 2	22.84	20.3-cm-thick concrete, complete	1.0013
1 × 5	26.66	30.5-cm-thick concrete, 4 sides; 20.3-cm-thick concrete, top and bottom	1.0018
Two parallel 1 × 3 separated by 51-cm concrete[b]	22.52	20.3-cm-thick concrete, 5 sides; 30.5-cm-thick concrete, against one array	1.0007

[a]Polyethylene density = 0.92 g/cm^3.
[b]Concrete density = 2.15 g/cm^3.

Spatial Arrays

A series of source neutron multiplication measurements with spatial arrays of U(93.4) metal slabs 2.54 x 20.3 x 25.4-cm, each containing 22.9 kg of ^{235}U, was also performed at ORNL.[190] Arrays reflected by Plexiglas ($\rho = 1.18$ g/cm^3) were either unmoderated or were moderated by Plexiglas. Table 60 describes the experimental arrays and gives an estimate of the number of units required for critical arrays. Included in this table are the results of two additional experiments in which the void volume of a moderated 27-unit array first contained Styrofoam and then Foamglas.

Extensive studies at ORNL[177,191] utilized U(93.2) metal cylinders arranged in rectangular three-dimensional arrays. Data obtained from critical arrays of 8, 16, 27, 45, and 64 units, both unreflected and reflected by various thicknesses of paraffin, are given in Table 61. The reflector was located at the lattice cell boundary of the peripheral units, where a lattice cell is occupied by a single unit.

The effect of a 15.2-cm-thick paraffin reflector on three sides of an array was also investigated.[191] Two arrays, each constituted of 20.9-kg units whose height-to-diameter ratio (h/d) was 0.94, were constructed, one containing 8 units and the other 27. The results indicated that the thick reflector on three sides of the arrays was slightly less effective than was the 2.5-cm-thick reflector completely surrounding the arrays.

Five arrays of the U(93.2) metal cylinders were constructed[191] with units at equal center spacing, in contrast to the usual equal surface spacing. Two were subcritical and the other three are described in Table 62, which includes for comparison the critical dimensions of arrays of the same units located at equal surface spacing.

TABLE 60. REFLECTED SPATIAL ARRAYS OF 24.5-kg U(93.4) METAL SLABS

Number of Units Assembled	Thickness of Plexiglas Moderator Centered Between Units (cm)	Center Spacing (cm)	Source Neutron Multiplication	Estimated Number of Units in Critical Array
125	0	38.1	2.5	350-480
125	0	30.5	5.2	180-210
125	0	27.9	14.1	140-150
64	2.54	38.1	16.7	73-80
27	2.54	30.5	7.9	34-41
27	2.54	27.9	167	27.5
27	2.54[a]	30.5	3.7	34-41
27	2.54[b]	30.5	1.2	155-215

Note: Units U(93.4) metal slabs 2.5 x 20.3 x 25.4-cm, uranium density 18.7 g/cm^3; reflector 2.5-cm-thick Plexiglas.

[a] Styrofoam ($C_6H_5CH{:}CH_2$; ρ = 0.024 g/cm^3) occupied 96.7% of the air space.

[b] Foamglas (borosilicate glass containing ~2% boron, ρ = 0.141 g/cm^3) occupied 96.7% of the air space.

TABLE 61. CRITICAL CONDITIONS FOR THREE-DIMENSIONAL ARRAYS OF U(93.2) METAL CYLINDERS WITH VARIOUS PARAFFIN REFLECTORS

Unit				Paraffin Array	Surface
Mass (kg)	Diameter (cm)	Height-to-Diam. Ratio	Configuration	Reflector Thickness (cm)	Separation of Units[a] (cm)
10.48	11.51	0.47	2 x 2 x 2		
				1.3	0.23
				3.8	1.98
				7.6	3.42
				15.2	3.70
10.48	11.51	0.47	3 x 3 x 3	0	2.01
				1.3	2.99
				3.8	5.87
				7.6	8.26
				15.2	8.69
10.51	9.12	0.95	2 x 2 x 2		
				1.3	0.60
				3.8	2.36
				7.6	3.97
				15.2	4.31
10.49	9.12	0.95	3 x 3 x 3	0	2.44
				1.3	3.43
				3.8	6.58
				7.6	9.02
				15.2	9.43
10.48	11.51	0.47	2 x 2 x 4	0	1.35
10.46	11.49	0.47	3 x 3 x 5	0	3.44
10.43	11.48	0.47	4 x 4 x 4	0	3.95
				15.2	12.36
15.69	11.49	0.70	2 x 2 x 2	0	0.90
				1.3	1.91
				3.8	4.96
				7.6	7.39
				15.2	7.82

TABLE 61. (cont.)

Unit					
Mass (kg)	Diameter (cm)	Height-to-Diam. Ratio	Configuration	Paraffin Array Reflector Thickness (cm)	Surface Separation of Units[a] (cm)
15.68	11.49	0.70	3 x 3 x 3	0	4.20
				1.3	5.68
				3.8	10.19
				7.6	13.69
				15.2	14.19
20.81	11.46	0.94	2 x 2 x 2	0	2.22
20.96	11.51	0.94	2 x 2 x 2	0	2.25
				1.3	3.68
				2.5	5.71
				3.8	8.21
				7.6	11.51
				15.2	11.99
20.88	11.48	0.94	3 x 3 x 3	0	6.36
				1.3	8.57
				3.8	14.76
				7.6	18.72
				15.2	19.15
26.22	11.51	1.17	2 x 2 x 2	0	3.54
				1.3	5.42
				3.8	11.53
				7.6	15.70
				15.2	16.38
26.11	11.49	1.17	3 x 3 x 3	0	8.49
				1.3	11.32
				3.8	19.61
				7.6	24.50
				15.2	24.99

[a] Errors on all surface separations are ±0.01-cm for unreflected arrays and ± 0.03-cm for reflected arrays.

TABLE 62. COMPARISON OF CELL VOLUMES OF CRITICAL UNREFLECTED CUBIC AND RECTANGULAR PARALLELEPIPED ARRAYS OF U(93.2) METAL CYLINDERS

Configuration	Cubic Array		Rectangular Array		
	Center Spacing (cm)	Cell Volume (L)	Equal Surface Spacing (cm)	Cell Volume (L)	Array Height/(Area)$^{1/2}$
	21.0 kg unit, 11.51-cm diam, 10.82-cm height:				
2 x 2 x 2	13.50	2.46	2.25	2.47	0.95
	20.9 kg unit, 11.48-cm diam, 10.79-cm height:				
3 x 3 x 3	17.60	5.45	6.36	5.46	0.96
	26.2 kg unit, 11.51-cm diam, 13.47-cm height:				
2 x 2 x 2	15.78	3.93	3.54	3.85	1.13

In order to determine the effect of moderation on the critical uranium density of arrays of metal units, cylinders having a mass of 20.9 kg and an h/d of 0.94 were centered in Plexiglas boxes of various thicknesses and sizes; some of the arrays were surrounded with paraffin reflector.[191] Table 63 summarizes the results.

TABLE 63. CRITICAL CONDITIONS FOR MODERATED ARRAYS OF 20.9-kg CYLINDERS OF U(93.2) METAL

Configuration	Plexiglas Moderator Thickness[a] (cm)	Paraffin Reflector Thickness (cm)	Surface Separation of Units[b] (cm)
2 x 2 x 2	1.3	0	4.24
		1.3	5.88
		7.6	12.57
		15.2	12.93
2 x 2 x 2	2.5	0	6.62
		1.3	8.61
		15.2	14.50
2 x 2 x 2	4.8	0	10.24
		15.2	16.45
3 x 3 x 3	4.8	0	16.29

[a]Each unit was centered in approximately cubic Plexiglas containers having wall thicknesses one-half the designated moderator thickness.

[b]The error in the separation of the units in the unreflected arrays is ±0.01-cm; in the reflected arrays it is ±0.03-cm.

In one series of experiments at Oak Ridge, 2 × 2 × 2 arrays of U(93.2) metal cylinders were embedded in graphite[192]. The cylinders were 11.48-cm diameter, 8.08-cm high and averaged 15.96-kg U(93.2). The graphite was Type 873S* at a density of 1.766 g/cm^3. Each of eight units consisted of a cylinder centered in a graphite block with 0.035-cm clearance. In arrays without added polyethylene reflector, blocks were in contact, and in arrays surrounded by 15.2-cm-thick polythene, blocks were spaced 4.13-cm apart. Characteristics of near-critical arrays, including values of k_{eff}, are given in Table 64.

TABLE 64. NEAR-CRITICAL GRAPHITE-MODERATED 2 × 2 × 2 ARRAYS OF 16-kg U(93.2) CYLINDERS

Reflector	Center Spacing of U Cylinders (cm) horizontal	vertical	Outside Dimensions of Graphite Blocks (cm) horizontal	vertical	Experimental k_{eff}[a]
No Polyethylene	37.08±0.02	30.99±0.02	37.08±0.007	30.99±0.04	0.996±0.001
"	32.00±0.02	25.99±0.02	32.00±0.005	25.90±0.004	0.993±0.001
"	26.92±0.02	20.86±0.02	26.92±0.009	20.86±0.019	0.990±0.001
15.2-cm Polyethylene[b]	36.13±0.03	30.03±0.03	32.00±0.005	25.90±0.004	0.9995±0.0004
"	31.05±0.03	24.99±0.03	26.92±0.009	20.86±0.018	0.9995±0.0004

[a] Corrected for structural support and room return.
[b] Inside dimensions of polyethylene cavity were twice the spacing of units.

* Speer Carbon Company

PLUTONIUM

Rocky Flats[193] is the source of subcritical measurements with 2-kg units of plutonium metal (ρ = ~15.8 g/cm^3), each approximately 6.3-cm in diameter and 3.2-cm high, enclosed in thin-walled steel containers 10.2-cm in diameter and 6.8-cm high. These units were arranged with containers in contact on an effectively infinite concrete reflector. Extrapolation of source neutron multiplication data led to the following conclusions:

1. An infinite slab one unit thick, with an array density of 3.38 g Pu/cm^3, will be subcritical.
2. An infinitely high spatial array of units in a square pattern, three units on a side, with an array density of 2.9 g Pu/cm^3, will be subcritical.
3. A spatial array of units in a square pattern with four units on a side will be critical when the array is six units high.
4. A similar array with five units on a side will be critical at a height corresponding to 3.5 units.

The neutron interaction between two facing plutonium disks (ρ = 15.8 g/cm^3) separated by and completely reflected by Plexiglas, was also investigated at Rocky Flats.[194] Both disks, 31.8-cm in diameter, were of equal but variable thickness, and each was completely enclosed in a cylindrical Plexiglas reflector 7.6-cm-thick on the lateral surfaces and 10.2-cm-thick on the ends. Plexiglas also filled the space between the disks, which was varied from 0- to 15.2-cm. The reported critical thickness of the disks as a function of their separation was the result of extrapolations of subcritical measurements.

Livermore reported an extensive study of critical arrays of α-phase plutonium units.[195] Each 3.03 kg unit, 6.52-cm diam by 4.63-cm high, was sealed in a close-fitting aluminum can with an iron lid. Heat sinks at ends of cans, spacers, and tubes containing stacks of units were of various aluminum alloys. Measurements were with either a single unit per cell, or with two units per cell separated by a 0.48-cm-thick aluminum heat sink. Can-surface temperatures of n^3 arrays ranged from 304 to 318 K.

The moderator included in some cells consisted of mock high explosive bonded with epoxy, of composition 30.6 wt% C, 3.14 wt% H, 31.6 wt% N and the remainder mostly oxygen; the density was 1.56 g/cm^3.

In addition to the simple critical n^3 arrays shown in Table 65, some one- and two-dimensional arrays and perturbed three-dimensional arrays were also reported.

TABLE 65. CRITICAL n^3 ARRAYS OF PLUTONIUM-METAL UNITS

Unit Center Spacing (cm)		Mass of Aluminum	Interunit Moderator		
Horizontal	Vertical	Per Cell (g)	Thickness (cm)	Mass Per Cell (g)	Configuration
		3.03-kg Pu Units (8.3 g Fe/cell):			
7.3	5.4	100		none	2 x 2 x 2[a]
7.6	5.7	108		none	2 x 2 x 2[a,b]
9.6	7.7	152		none	3 x 3 x 3[a]
10.2	8.2	164		none	3 x 3 x 3[a,b]
12.5	7.9	153		none	4 x 4 x 4[c]
14.2	9.6	191	1.27	600	4 x 4 x 4[c]
14.6	13.6	279	2.54	1660	4 x 4 x 4[c]
		6.05-kg Pu Units (16.6 g Fe/cell):			
9.8	12.0	224		none	2 x 2 x 2[c]
14.5	13.7	261		none	3 x 3 x 3[c]
11.9	47.1	995		none	4 x 4 x 4[c]
13.1	32.1	666		none	4 x 4 x 4[c]
15.2	22.1	446		none	4 x 4 x 4[c]
17.3	17.1	336		none	4 x 4 x 4[c]
20.2	13.1	249		none	4 x 4 x 4[c]
17.5	25.8	527	1.27	930	4 x 4 x 4[c]
21.2	25.8	527	2.54	2400	4 x 4 x 4[c]
24.5	25.8	527	3.81	4610	4 x 4 x 4[c]

Note: Arrays were centered in the 183-cm space between top and bottom reflectors.

[a]Bottom reflector 2.54-cm-thick by 213-cm^2 aluminum at 88% full density backed by 30-cm-thick region of steel at 7% full density, no top reflector.

[b]Polyethylene reflector, 20.5-cm-thick on one side.

[c]Bottom reflector 2.4-cm-thick by 213-cm^2 aluminum at 88% full density backed by 10.16-cm-thick aluminum at 30% and 43-cm steel at 2% full density below 25-cm-high column at end of each tube containing 856 g aluminum and, in the lower 4.9-cm, 360 g copper, 540 g tin, and 221 g steel.

Top reflector 5.1-cm-thick aluminum at 11.1/h g/cm^3, above 7.9-cm-high column at end of each tube containing 1.1 g aluminum and 422 g steel.

REFERENCES

1. H. C. Paxton, J. T. Thomas, Dixon Callihan, and E. B. Johnson,"Critical Dimensions of Systems Containing U^{235}, Pu^{239}, and U^{233}," Los Alamos Scientific Laboratory and Oak Ridge National Laboratory report TID-7028 (June 1964).
2. J. T. Thomas, Ed., "Nuclear Safety Guide TID-7016," U. S. Nuclear Regulatory Commission report NUREG/CR-0095 (ORNL/NUREG/CSD-6) (June 1978).
3. "Nuclear Criticality Safety in Operations with Fissionable Materials Outside Reactors," ANSI/ANS-8.1-1983 (Revision of ANSI/ N16.1-1975) (American National Standards Institute, Inc., New York, 1983).
4. H. K. Clark, "Subcritical Limits for Plutonium Systems," *Nucl. Sci. Eng.* **79**, 65-84 (1981).
5. H. K. Clark,"Subcritical Limits for Uranium-235 Systems," *Nucl. Sci. Eng.* **81**, 351-378 (1982).
6. H. K. Clark, "Subcritical Limits for Uranium-233 Systems," *Nucl. Sci. Eng.* **81**, 379-395 (1982).
7. H. K. Clark, "Handbook of Nuclear Safety," Savannah River Laboratory report DP-532 (January 1961). F. Abbey, "Manual of Criticality Data, Parts 1, 2, 3," AHSB(s) Handbook 5, UKAEA Health and Safety Branch (1967). "Guide de Criticite, Parts I, II, III," (in French) CEA-R3114, Commissariat a l'Energie Atomique, Saclay (1967). B. G. Dubovskiy et al., "Critical Parameters of Fissionable Materials Systems and Nuclear Safety (A Handbook)," Russian Translation JPRS:42,322, Clearing House for Federal Scientific and Technical Information (1967). R. D. Carter, G. R. Kiel, and K. R. Ridgway, "Criticality Handbook, Vols. I, II, III," Atlantic Richfield Hanford Co. report ARH-600 (1968).
8. H. F. Henry, A. J. Mallett, C. E. Newlon, and W. A. Pryor, "Criticality Data and Nuclear Safety Guide Applicable to the Oak Ridge Gaseous Diffusion Plant," Union Carbide Nuclear Company, K-25 Plant report K-1019 (5^{th} Rev.) (May 1959). E. D. Clayton, "Nuclear Safety in Chemical and Metallurgical Processing of Plutonium," Hanford Atomic Products report HW-68929 (April 1961).
9. D. Callihan, "Experiments for Criticality Control," in *Criticality Control in Chemical and Metallurgical Plant, Karlsruhe Symposium, 1961* (Organisation for Economic Cooperation and Development, European Nuclear Energy Agency, Paris, 1961), pp. 589-614.
10. G. E. Hansen and W. H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," Los Alamos Scientific Laboratory report LAMS-2543 (November 1961).
11. B. G. Carlson, C. Lee, and W. Worlton, "The DSN and TDC Neutron Transport Codes," Los Alamos Scientific Laboratory reports LAMS-2346 and LAMS-2346, Appendix I (October 1959).
12. R. D. O'Dell, F. W. Brinkley, Jr., and D. R. Marr, "User's Manual for ONEDANT: A Code Package for One-Dimensional, Diffusion-Accelerated, Neutral-Particle Transport," Los Alamos National Laboratory report LA-9184-M (February 1982).
13. R. E. Alcouffe, F. W. Brinkley, D. R. Marr, and R. D. O'Dell, "User's Guide for TWODANT: A Code Package for Two-Dimensional, Diffusion-Accelerated, Neutral-

Particle Transport," Los Alamos National Laboratory report LA-10049-M, Rev. 1 (October 1984).
14. W. R. Stratton, "Criticality Data and Factors Affecting Criticality of Single Homogeneous Units," Los Alamos Scientific Laboratory report LA-3612 (September 1967).
15. J. F. Briesmeister, Ed., "MCNP–A General Monte Carlo Code for Neutron and Photon Transport, Version 3A," Los Alamos National Laboratory report LA-7396-M, Rev. 2, Manual (September 1986).
16. H. C. Paxton, "Fast Critical Experiments," in *Progress in Nuclear Energy* (Pergamon Press, Ltd., 1981), Vol. 7, pp. 151-174.
17. Roy Reider, "An Early History of Criticality Safety," Los Alamos Scientific Laboratory report LA-4671 (May 1971).
18. Los Alamos Scientific Laboratory of the University of California, "An Enriched Homogeneous Nuclear Reactor," *Rev. Sci. Inst.***22**, 489-499 (1951); also L. D. P. King, "Water Boilers," Los Alamos Scientific Laboratory report LA-1034, Chapter 4 (December 1947).
19. H. D. Smyth, *Atomic Energy for Military Purposes* (Princeton University Press, Princeton, New Jersey, 1945), pp. 98, 143.
20. H. C. Paxton, "Los Alamos Critical Mass Data," Los Alamos Scientific Laboratory report LA-3067-MS, Rev. (December 1975).
21. A. H. Snell, "Critical Experiments on Fluorinated and Hydrogenated Mixtures Containing Enriched Uranium," Monsanto Clinton Laboratories report MonP-48 (November 1945).
22. C. K. Beck, A. D. Callihan, and R. L. Murray, "Critical Mass Studies, Part I," Carbide and Carbon Chemicals Corporation report A-4716 (June 1947).
23. C. K. Beck, A. D. Callihan, and R. L. Murray, "Critical Mass Studies, Part II," Carbide and Carbon Chemicals Corporation report K-126 (January 1948).
24. R. Caizergues, E. Deilgat, P. Lécorché, L. Maubert, and H. Revol, "Criticality of Liquid Mixtures of Highly ^{235}U-Enriched Uranium Hexafluoride and Hydrofluoric Acid," (translation) Union Carbide Corporation report Y-CDC-9 (May 1971).
25. W. R. Stratton, "A Review of Criticality Accidents," Los Alamos Scientific Laboratory report LA-3611 (1967).
26. F. E. Kruesi, J. O. Erkman, and D. D. Lanning, "Critical Mass Studies of Plutonium Solutions," Hanford Atomic Products Operation report HW-24514 (Del.) (May 1952).
27. R. C. Lloyd, E. D. Clayton, and W. A. Reardon, "Operating Experience in the Hanford Plutonium Critical Mass Facility," *Trans. Am. Nucl. Soc.* **5**, 76,77 (1962).
28. J. G. Walford and A. F. Thomas, "The Equipment and Methods Used in British Criticality Laboratories," in *Criticality Control in Chemical and Metallurgical Plant, Karlsruhe Symposium, 1961* (Organisation for Economic Cooperation and Development, European Nuclear Energy Agency, Paris, 1961), pp. 553-588.
29. J. Carothers, "Hazards Summary Report for the LRL Critical Facility," Lawrence Radiation Laboratory report UCRL-6220 (March 1960).
30. P. R. Lécorché, "Critical Mass Laboratory Program in France," *Trans. Am. Nucl. Soc.***16**, 158,159 (1973).

31. C. L. Schuske, "Experimental Programs at the Dow Rocky Flats Nuclear Safety Laboratory," *Trans. Am. Nucl. Soc.***16**, 157,158 (1973).
32. C. K. Beck, A. D. Callihan, J. W. Morfitt, and R. L. Murray, "Critical Mass Studies, Part III," Carbide and Carbon Chemicals Corporation report K-343 (April 1949).
33. J. K. Fox, L. W. Gilley, and D. Callihan, "Critical Mass Studies, Part IX. Aqueous U^{235} Solutions," Oak Ridge National Laboratoty report ORNL-2367 (March 1958).
34. J. K. Fox, L. W. Gilley, and J. H. Marable, "Critical Parameters of a Proton-Moderated and Proton-Reflected Slab of U^{235}," *Nucl. Sci. Eng.***3**, 694-697 (1958).
35. J. T. Mihalczo and J. J. Lynn, "Neutron Multiplication Experiments with Enriched Uranium Metal in Slab Geometry," Oak Ridge National Laboratory report ORNL-CF-61-4-33 (April 1961).
36. J. T. Mihalczo, "Graphite and Polyethylene Reflected Uranium Metal Cylinders and Annuli," Union Carbide Corporation, Nuclear Division report Y-DR-81 (April 1972).
37. G. E. Hansen, H. C. Paxton, and D. P. Wood, "Critical Plutonium and Enriched-Uranium-Metal Cylinders of Extreme Shape," *Nucl. Sci. Eng.***8**, 570-577 (1960).
38. E. C. Mallary, "Oralloy Cylindrical Shape Factor and Critical Mass Measurements in Graphite, Paraffin, and Water Tampers," Los Alamos Scientific Laboratory report LA-1305 (October 1951).
39. G. E. Hansen, D. P. Wood, and B. Pena, "Reflector Savings of Moderating Materials on Large-Diameter U(93.2) Slabs," Los Alamos Scientific Laboratory report LAMS-2744 (October 1962).
40. J. K. Fox, L. W. Gilley, and E. R. Rohrer, "Critical Mass Studies, Part VIII. Aqueous Solutions of U^{233}," Oak Ridge National Laboratory report ORNL-2143 (September 1959).
41. F. A. Kloverstrom, "Spherical and Cylindrical Plutonium Critical Masses," University of California Radiation Laboratory report UCRL-4957 (September 1957).
42. C. L. Schuske, M. G. Arthur, and D. F. Smith, "Criticality Measurements on Plutonium Metal Preliminary to the Design of a Melting Crucible," Dow Chemical Co., Rocky Flats Plant report RFP-63 (June 1956).
43. R. C. Lane, "Measurements of the Critical Parameters of Under-Moderated Uranium-Hydrogen Mixtures at Intermediate Enrichment," in *Proceedings of the Symposium Criticality Control of Fissile Materials, Stockholm, 1-5 November 1965* (International Atomic Energy Agency, Vienna, 1966) pp. 177-191.
44. A. D. Callihan, D. F. Cronin, J. K. Fox, and J. W. Morfitt, "Critical Mass Studies, Part V," Carbide and Carbon Chemicals Corp., K-25 Plant report K-643 (June 1950).
45. G. A. Linenberger, J. D. Orndoff, and H. C. Paxton, "Enriched-Uranium Hydride Critical Assemblies," *Nucl. Sci. Eng.***7**, 44-57 (1960).
46. L. E. Hansen and E. D. Clayton, "Criticality of Plutonium Compounds in the Undermoderated Range, $H/Pu \leq 20$," *Nucl. Appl.***3**, 481-487 (1967).
47. J. S. Johnson and K. A. Kraus, "Density and Refractive Index of Uranyl Fluoride Solutions," *J. Am. Chem. Soc.***75**, 4594-4595 (1953).
48. J. T. Thomas, J. K. Fox, and Dixon Callihan, "A Direct Comparison of Some Nuclear Properties of U-233 and U-235," *Nucl. Sci. Eng.***1**, 20-32 (1956).
49. J. T. Thomas, "Parameters for Two Group Analysis of Critical Experiments with Wa-

ter Reflected Spheres of UO_2F_2 Aqueous Solutions," Oak Ridge National Laboratory report ORNL-CF-56-8-201 (August 1956).

50. J. K. Fox, L. W. Gilley, R. Gwin, and J. T. Thomas, "Critical Parameters of Uranium Solutions in Simple Geometry," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1958," Oak Ridge National Laboratory report ORNL-2609 (October 1958), p.42.
51. R. Gwin and D. W. Magnuson, "The Measurement of Eta and Other Nuclear Properties of U^{233} and U^{235} in Critical Aqueous Solutions," *Nucl. Sci. Eng.***12**, 364-380 (1962).
52. R. H. Masterson, J. D. White, and T. J. Powell, "The Limiting Critical Concentrations for Pu^{239} and U^{235} in Aqueous Solutions," Hanford Atomic Products Operation report HW-77089 (March 1963).
53. R. E. Peterson and G. A. Newby, "An Unreflected U-235 Critical Assembly," *Nucl. Sci. Eng.***1**, 112-125 (1956).
54. C. C. Byers, J. J. Koelling, G. E. Hansen, D. R. Smith, and H. R. Dyer, "Critical Measurements of a Water-Reflected Enriched Uranium Sphere," *Trans. Am. Nucl. Soc.***27**, 412-413 (1977).
55. J. T. Mihalczo and J. J. Lynn, "Critical Parameters of Bare and Reflected 93.4 wt% U^{235}-Enriched Uranium Metal Slabs," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1960," Oak Ridge National Laboratory report ORNL-3016 (December 1960), pp. 73-76.
56. H. R. Ralston, "Critical Masses of Spherical Systems of Oralloy Reflected in Beryllium," University of California Radiation Laboratory report UCRL-4975 (October 1957).
57. R. E. Donaldson and W. K. Brown, "Critical-Mass Determinations of Lead-Reflected Systems," University of California Radiation Laboratory report UCRL-5255 (June 1958).
58. D. W. Magnuson, "Critical Experiments with Enriched Uranium Dioxide," Union Carbide Corporation, Y-12 Plant report Y-DR-120 (November 1973).
59. J. G. Bruna, J. P. Brunet, R. Caizergues, C. Clouet d'Orval, and P. Verriere, "Results of Homogeneous Critical Experiments Carried Out with ^{239}Pu, ^{235}U, and ^{233}U," (in French) in *Proceedings of the Symposium Criticality Control of Fissile Materials, Stockholm, 1-5 November 1965* (International Atomic Energy Agency, Vienna, 1966), pp. 235-248.
60. P. Lécorché and R. L. Seale, "A Review of the Experiments Performed to Determine the Radiological Consequences of a Criticality Accident," Oak Ridge Criticality Data Center report Y-CDC-12 (November 1973).
61. B. G. Dubovskii, A. V. Kamaev, V. V. Orlov, G. M. Vladykov, V. N. Gurin, F. M. Kuznetsov, V. P. Kochergin, I. P. Markelov, G. A. Popov, and V. J. Sviridenko, "The Critical Parameters of Aqueous Solutions of $UO_2(NO_3)_2$ and Nuclear Safety," in *Proceedings of the Third International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1964* (United Nations, New York, 1965), Vol. 13, pp. 254-263.
62. D. F. Cronin, "Critical Mass Studies, Part X, Uranium of Intermediate Enrichment," Oak Ridge National Laboratory report ORNL-2968 (October 1960).

63. S. J. Raffety and J. T. Mihalczo, "Homogeneous Critical Assemblies of 2 and 3% Uranium-235-Enriched Uranium in Paraffin," *Nucl. Sci. Eng.***48**, 433-443 (1972).
64. J. C. Smith, A. V. Parker, J. G. Walford, and C. White, "Criticality of 30% Enriched Uranium Solutions in Cylindrical Geometry," Dounreay Experimental Reactor Establishment report DEG-Memo-663 (March 1960).
65. R. E. Carter, J. C. Hinton, L. D. P. King, and R. E. Schreiber, "Water Tamper Measurements," Los Alamos Scientific Laboratory report LA-241 (March 1945).
66. E. B. Johnson, "Criticality of a Sphere of U(4.98)UO_2F_2 Solution," in "Neutron Physics Division Annual Progress Report for Period Ending May 31, 1966," Oak Ridge National Laboratory report ORNL-3973 (September 1966), pp. 14, 15.
67. E. B. Johnson, "Critical Lattices of U(4.89) Rods in Water and in Aqueous Boron Solution," *Trans. Am. Nucl. Soc.***11**, 675 (1968).
68. C. G. Chezem and R. G. Steinke, "Low-Enrichment Uranium-Metal Exponential Experiments," *Nucl. Sci. Eng.***31**, 549,550 (1968).
69. J. J. Neuer, "Critical Assembly of Uranium Metal at an Average U^{235} Concentration of 16-1/4%," Los Alamos Scientific Laboratory report LA-2085 (January 1957).
70. R. H. White, "Topsy, A Remotely Controlled Critical Assembly Machine," *Nucl. Sci. Eng.***1**, 53-61 (1956).
71. H. C. Paxton, "Bare Critical Assemblies of Oralloy at Intermediate Concentrations of U^{235}," Los Alamos Scientific Laboratory report LA-1671 (July 1954).
72. Darrouzet, J. P. Chandat, E. A. Fischer, G. Ingram, J. E. Sanders, and W. Scholtyssek, "Studies of Unit k Lattices in Metallic Uranium Assemblies Zebra 8H, Sneak 8, Ermine and Harmonie," in *Proc. Int. Symposium on Physics of Fast Reactors, Tokyo, October 16-19, 1973* (Power Reactor and Nuclear Fuel Development Corporation, Tokyo), Vol. I, pp. 537-570.
73. E. B. Johnson, "Critical Parameters of U(1.95) Metal Cylindrical Annuli," *Trans. Am. Nucl. Soc.***9**, 185,186 (1966). E. B. Johnson, "Criticality of U(3.85) Rods and Cylindrical Annuli in Water," *Trans. Am. Nucl. Soc.***13,** 379 (1970).
74. E. B. Johnson and L. M. Petrie, "The Criticality of Large Uranium Metal Units of Low Enrichment in ^{235}U," Oak Ridge National Laboratory report ORNL-6310 (1987).
75. J. C. Hoogterp, "Unreflected Plexiglas-Graphite-Uranium Critical Measurements," *Trans. Am. Nucl. Soc.***11**, 389,390 (1968).
76. H. Kouts, G. Price, K. Downes, R. Sher, and V. Walsh, "Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water," in *Proceedings of the First International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 5, pp. 183-202.
77. H. Kouts, R. Sher, J. R. Brown, D. Klein, S. Stein, R. L. Hellens, H. Arnold, R. M. Ball, and P. W. Davison, "Physics of Slightly Enriched, Normal Water Lattices (Theory and Experiment)," in *Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1957* (United Nations, New York, 1958), Vol. 12, pp. 446-482.
78. E. B. Johnson, "Criticality of Uranium of Low Enrichment in Water," *Trans Am. Nucl. Soc.***12,** 336 (1969).
79. C. R. Richey, R. C. Lloyd, and E. D. Clayton, Criticality of Slightly Enriched Uranium

in Water-Moderated Lattices," *Nucl. Sci. Eng.* **21**, 217-226 (1965).
80. W. B. Rogers, Jr., and F. E. Kinard, "Material Buckling and Critical Masses of Uranium Rods Containing 3 wt% U^{235} in H_20," *Nucl. Sci. Eng.* **20**, 266-271 (1964).
81. J. C. Manaranche, D. Mangin, L. Maubert, G. Colomb, and G. Poullot, "Critical Experiments with Lattices of 4.75 wt% ^{235}U-Enriched UO_2 Rods in Water," *Nucl. Sci. Eng.***71**, 154-163 (1979).
82. J. C. Hoogterp, "Critical Masses of Oralloy Lattices Immersed in Water," Los Alamos Scientific Laboratory report LA-2026 (March 1957).
83. J. K. Fox and L. W. Gilley, "Critical Experiments with Arrays of ORR and BSR Fuel Elements," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1958," Oak Ridge National Laboratory report ORNL-2609 (October 1958), pp. 34-36.
84. E. B. Johnson and R. K. Reedy, Jr., "Critical Experiments with SPERT-D Fuel Elements," Oak Ridge National Laboratory report ORNL-TM-1207 (July 1965).
85. A. Goodwin, Jr., G. H. Bidinger, and C. L. Schuske, "Criticality Studies of Enriched Uranium Metal in $UO_2(NO_3)_2$ Solutions," Dow Chemical Co., Rocky Flats Plant report RFP-182 (July 1960).
86. C. L. Schuske, M. G. Arthur, and D. F. Smith, "Neutron Multiplication Measurements on Oralloy Slabs Immersed in Solutions," Dow Chemical Co., Rocky Flats Plant report RFP-66 (August 1956).
87. C. L. Schuske, M. G. Arthur, and D. F. Smith, "Neutron Multiplication Measurements on Oralloy Slabs Immersed in Solution, Part II," Dow Chemical Co., Rocky Flats Plant report RFP-69 (October 1956).
88. W. A. Reardon and J. D. White, "Calculations of Criticality Properties of Plutonium Nitrate Systems," Hanford Atomic Products Operation report HW-72586 (January 1962), pp. 66-78.
89. C. C. Horton and J. D. McCullen, "Plutonium-Water Critical Assemblies," in *Proceedings of the First International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 5, pp. 156-161.
90. J. Bruna, J. P. Brunet, R. Caizergues, C. Clouet d'Orval, J. Kremser, J. LeClerc, and P. Verriere, "Criticality Experiment on a Plutonium Solution," (in French) Comissariat a l'Energie Atomique report CEA-2274, Centre d'Etudes Nucleaires, Saclay (1963).
91. M. F. Ithurralde, J. Kremser, J. LeClerc, C. Lombard, J. Moreau, and C. Robin, "Interpretation of Criticality Experiments on Homogeneous Solutions of Plutonium and Uranium," (in French) Commissariat a l'Energie Atomique report CEA-R-2488 (1964).
92. D. Breton, P. Lécorché, and C. Clouet d'Orval,"Criticality Studies," (in French) in *Proceedings of the Third United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1964* (United Nations, New York, 1965), Vol. 13, pp. 234-242.
93. R. C. Lloyd, C. R. Richey, E. D. Clayton, and D. R. Skeen, "Criticality Studies with Plutonium Solutions," *Nucl. Sci. Eng.***25**, 165-173 (1966).
94. R. C. Lloyd, E. D. Clayton, L. E. Hansen, and S. R. Bierman, "Criticality of Plutonium Nitrate Solutions in Slab Geometry," *Nucl. Technol.***18**, 225-230 (1973).

95. D. R. Smith and W. U. Geer, "Critical Mass of a Water-Reflected Plutonium Sphere," *Nucl. Appl. Technol.***7**, 405-408 (1969).
96. R. C. Lloyd, R. A. Libby, and E. D. Clayton, "The Measurement of Eta and the Limiting Critical Concentration of ^{239}Pu, in Critical Aqueous Solutions," *Nucl. Sci. Eng.* **82**, 325-331 (1982).
97. G. Colomb, D. Mangin, and L. Maubert, "Criticality of Plutonium Nitrate Solutions (19% ^{240}Pu)," (in French) Commissariat a l'Energie Atomique report CEA-N-1898 (September 1976).
98. R. C. Lloyd and E. D. Clayton, "The Criticality of High Burnup Plutonium," *Nucl. Sci. Eng.***52**, 73-75 (1973).
99. C. R. Richey, J. D. White, E. D. Clayton, and R. C. Lloyd, "Criticality of Homogeneous Plutonium Oxide-Plastic Compacts at H:Pu = 15," *Nucl. Sci. Eng.***23,** 150-158 (1965).
100. S. R. Bierman, L. E. Hansen, R. C. Lloyd, and E. D. Clayton, "Critical Experiments with Homogeneous PuO_2-Polystyrene at 5 H/Pu," *Nucl. Appl.***6**, 23-26 (1969).
101. S. R. Bierman and E. D. Clayton, "Critical Experiments with Homogeneous PuO_2-Polystyrene at 50 H/Pu," *Nucl. Technol.***15**, 5-13 (1972).
102. S. R. Bierman and E. D. Clayton, "Critical Experiments with Unmoderated Plutonium Oxide," *Nucl. Technol.***11**, 185-190 (1971).
103. S. R. Bierman, E. D. Clayton, and L. E. Hansen, "Critical Experiments with Homogeneous Mixtures of Plutonium and Uranium Oxides Containing 8, 15, and 30 wt% Plutonium," *Nucl. Sci. Eng.***50,** 115-126 (1973).
104. R. C. Lloyd, S. R. Bierman, and E. D. Clayton, "Criticality of Plutonium-Uranium Mixtures Containing 5 to 8 wt% Plutonium," *Nucl. Sci. Eng.***55**, 51-57 (1974).
105. S. R. Bierman and E. D. Clayton, "Critical Experiments with Low-Moderated Homogeneous Mixtures of Plutonium and Uranium Oxides Containing 8, 15, and 30 wt% Plutonium," *Nucl. Sci. Eng.***61**, 370-376 (1976).
106. R. C. Lloyd and E. D. Clayton, "Criticality of Plutonium- Uranium Nitrate Solutions," *Nucl. Sci. Eng.***60**, 143-146 (1976).
107. R. C. Lane and O. J. E. Perkins, "The Measurement of the Critical Size of a Homogeneous Mixture of Plutonium and Natural Uranium Oxides with Polythene," United Kingdom Atomic Energy Authority report AWRE 0 32/68, Aldermaston (July 1968).
108. R. C. Lane and C. Parker, "Measurement of the Critical Size of Solutions of Plutonium and Natural Uranium Nitrates with Pu/U = 0.3," United Kingdom Ministry of Defense report AWRE 0 58/73, Aldermaston (December 1973).
109. E. D. Clayton, H. K. Clark, D. W. Magnuson, J. H. Chalmers, Gordon Walker, N. Ketzlach, Ryohei Kiyose, C. L. Brown, D. R. Smith, and R. Artigas, "Basis for Subcritical Limits in Proposed Criticality Safety Standard for Mixed Oxides," *Nucl. Technol.***35**, 97-111 (1977). E. D. Clayton, H. K. Clark, Gordon Walker, and R. A. Libby, "Basis for Extending Limits in ANSI Standard for Mixed Oxides to Heterogeneous Systems," *Nucl Technol.***75,** 225-229 (November 1986).
110. "Nuclear Criticality Control and Safety of Plutonium-Uranium Fuel Mixtures Outside Reactors," ANSI/ANS-8.12-1987 (American National Standards Institute, Inc., New York, 1987).
111. V. I. Neeley, R. C. Lloyd, and E. D. Clayton, "Neutron Multiplication Measurements

with Pu-Al Alloy Rods in Light Water," Hanford Atomic Products Operation report HW-70944 (August 1961).

112. S. R. Bierman, B. M. Durst, E. D. Clayton, R. I. Scherpelz, and H. T. Kerr, "Critical Experiment with Fast Test Reactor Fuel Pins in Water," *Nucl. Technol.***44**, 141-151 (1979).

113. J. T. Thomas, "Critical Experiments with Aqueous Solutions of $^{233}UO_2(NO_3)_2$," in "Neutron Physics Division Annual Progress Report for Period Ending May 31, 1968," Oak Ridge National Laboratory report ORNL-4280 (October 1968), pp. 53-55.

114. W. E. Converse, R. C. Lloyd, E. D. Clayton, and W. A. Yuill, "Critical Experiments Using High-Enriched Uranyl Nitrate with Cadmium Absorber," *Trans. Am. Nucl. Soc.***32**, 328-330 (1979).

115. E. B. Johnson, "The Criticality of Heterogeneous Lattices of Experimental Beryllium Oxide Reactor Fuel Pins in Water and in Aqueous Solutions Containing Boron and Uranyl Nitrate," Oak Ridge National Laboratory report ORNL/ENG-2 (July 1976).

116. V. I. Neeley, J. A. Berberet, and R. H. Masterson, "k_∞ of Three Weight Percent U^{235} Enriched UO_3 and $UO_2(NO_3)_2$ Hydrogenous Systems," Hanford Atomic Products Operation report HW-66882 (September 1961).

117. J. T. Mihalczo and V. I. Neeley, "The Infinite Neutron Multiplication Constant of Homogeneous Hydrogen-Moderated 2.0 wt% U^{235}-Enriched Uranium," *Nucl. Sci. Eng.***13**, 6-11 (1962).

118. H. E. Handler, "Measurement of Multiplication Constant for Slightly Enriched Homogeneous UO_3-Water Mixtures and Minimum Enrichment for Criticality," Hanford Atomic Products Operation report HW-70310 (August 1961).

119. R. C. Lloyd, E. D. Clayton, and L. E. Hansen, "Criticality of Plutonium Nitrate Solution Containing Soluble Gadolinium," *Nucl. Sci. Eng.***48,** 300-304 (1972).

120. R. C. Lloyd and E. D. Clayton, "Criticality of Plutonium-Uranium Nitrate Solutions," *Nucl. Sci. Eng.***60**, 143-146 (1976).

121. R. C. Lloyd, S. R. Bierman, and E. D. Clayton, "Criticality of Plutonium Nitrate Solutions Containing Borated Raschig Rings," *Nucl. Sci. Eng.***50**, 127-134 (1973).

122. S. R. Bierman and E. D. Clayton, "Critical Experiments to Measure the Neutron Poisoning Effects of Copper and Copper-Cadmium Plates," *Nucl. Sci. Eng.***55,** 58-66 (1974).

123. S. R. Bierman, B. M. Durst, and E. D. Clayton, "Critical Experiments Measuring the Reactivity Worths of Materials Commonly Encountered as Fixed Neutron Absorbers," *Nucl. Sci. Eng.***65**, 41-48 (1978).

124. J. T. Thomas, J. K. Fox, and E. B. Johnson, "Critical Mass Studies, Part XIII. Borosilicate Glass Raschig Rings in Aqueous Uranyl Nitrate Solutions," Oak Ridge National Laboratory report ORNL-TM-499 (February 1963).

125. G. H. Bidinger, C. L. Schuske, and D. F. Smith, "Nuclear Safety Experiments on Plutonium and Enriched Uranium Hydrogen Moderated Assemblies Containing Boron," Dow Chemical Co., Rocky Flats Plant report RFP-201 (October 1960).

126. C. L. Schuske and G. H. Bidinger, "Nuclear Safety Measurements on Systems Containing Boron and Enriched Uranium," Dow Chemical Co., Rocky Flats Plant report RFP-246 (October 1961).

127. J. K. Fox and L. W. Gilley, "The Poisoning Effect of Copper Lattices in Aqueous Solutions of Enriched Uranyl Oxyfluoride," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1959," Oak Ridge National Laboratory report ORNL-2842 (November 1959), pp. 73-76.
128. L. W. Gilley, D. F. Cronin, and V. G. Harness, "Boron Poisoning in Critical Slabs," in "Physics Division Semiannual Progress Report for Period Ending March 10, 1954," Oak Ridge National Laboratory report ORNL- 1715 (July 1954), pp. 12,13.
129. L. W. Gilley and Dixon Callihan, "Nuclear Safety Tests on a Proposed Ball Mill," Oak Ridge National Laboratory report ORNL-CF-54-9-89 (September 1954).
130. J. K. Fox and L. W. Gilley, "Critical Parameters for 20-in.-diam Stainless Steel Cylinders Containing Aqueous Solutions of U^{235} Poisoned with Pyrex Glass," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1959," Oak Ridge National Laboratory report ORNL-2842 (November 1959), pp. 78-81.
131. R. C. Lloyd, "Summary Listing of Subcritical Measurements of Heterogeneous Water-Uranium Lattices Made at Hanford," Hanford Atomic Products Operation report HW-65552 (June 1960).
132. H. Kouts and R. Sher, "Experimental Studies of Slightly Enriched Uranium, Water-Moderated Lattices," Brookhaven National Laboratory report BNL-486 (September 1957).
133. W. H. Arnold, Jr., "Critical Masses and Lattice Parameters of H_2O-UO_2 Critical Experiments. A Comparison of Theory and Experiment," Westinghouse Atomic Power Department report YAEC-152 (November 1959).
134. R. C. Lloyd, B. M. Durst, and E. D. Clayton, "Effect of Soluble Neutron Absorbers on the Criticality of Low-Uranium-235-Enriched UO_2 Lattices," *Nucl. Sci. Eng.***71,** 164-169 (1979).
135. G. E. Hansen and D. P. Wood, "Precision Critical Mass Determinations for Oralloy and Plutonium in Spherical Tuballoy Tampers," Los Alamos Scientific Laboratory report LA-1356 (Del.) (February 1952).
136. G. E. Hansen, H. C. Paxton, and D. P. Wood, "Critical Masses of Oralloy in Thin Reflectors," Los Alamos Scientific Laboratory report LA-2203 (July 1958).
137. D. C. Coonfield, Grover Tuck, H. E. Clark, and B. B. Ernst, "Critical Mass Irregularity of Steel-Moderated Enriched Uranium Metal Assemblies with Composite Steel-Oil Reflectors," *Nucl. Sci. Eng.***39**, 320-328 (1970).
138. J. R. Dominey, R. C. Lane, and A. F. Thomas, "Critical Mass Measurements with Thin Discs of 45.5% Enriched Uranium," United Kingdom Atomic Energy Authority report AWRE NR/A-1/62, Aldermaston (January 1962).
139. J. J. McEnhill and J. W. Weale, "Integral Experiments on Fast Systems of Plutonium, Uranium, and Thorium," in *Proc. of Conf. on Physics of Fast and Intermediate Reactors, Vienna* (International Atomic Energy Agency, Vienna, 1962), Vol. I, pp. 253-262.
140. E. A. Plassman and D. P. Wood, "Critical Reflector Thicknesses for Spherical U^{233} and Pu^{239} Systems," *Nucl. Sci. Eng.***8**, 615-620 (1960).
141. G. E. Hansen and H. C. Paxton, "Reevaluated Critical Specifications of Some Los Alamos Fast-Neutron Systems," Los Alamos Scientific Laboratory report LA-4208

(September 1969).
142. H. R. Ralston, "Critical Parameters of Spherical Systems of Alpha-Phase Plutonium Reflected by Beryllium," University of California Radiation Laboratory report UCRL-5349 (September 1958).
143. R. E. Rothe and I. Oh, "Benchmark Critical Experiments on High-Enriched Uranyl Nitrate Solution Systems," *Nucl. Technol.***41**, 207-225 (1978).
144. E. B. Johnson and C. E. Newlon, "The Effect of Steel-Water Reflectors on the Criticality of Low-Enriched Uranyl Fluoride Solution," *Trans. Am. Nucl. Soc.***11**, 383,384 (1968).
145. L. B. Engle, G. E. Hansen, and H. C. Paxton, "Reactivity Contributions of Various Materials in Topsy, Godiva, and Jezebel," *Nucl. Sci. Eng.***8**, 543-569 (1960).
146. D. P. Wood, C. C. Byers, and L. C. Osborn, "Critical Masses of Cylinders of Plutonium Diluted with Other Metals," *Nucl. Sci. Eng.***8**, 578-587 (1960).
147. A. J. Kirschbaum, "Studies of Enriched Uranium Graphite Reactor Systems," University of California Radiation Laboratory report UCRL-4983-T (November 1957).
148. J. E. Schwager, F. A. Kloverstrom, and W. S. Gilbert, "Critical Measurements on Intermediate-Energy Graphite-U^{235} Systems," University of California Radiation Laboratory report UCRL-5006 (November 1957).
149. H. L. Reynolds, "Critical Measurements and Calculations for Enriched-Uranium Graphite-Moderated Systems," in *Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, New York, 1958), Vol. 12, pp. 632-642.
150. F. A. Kloverstrom, R. M. R. Deck, and A. J. Reyenga, "Critical Measurements on Near-Homogeneous BeO-Moderated, Enriched-Uranium Fueled Systems," *Nucl. Sci. Eng.***8**, 221-225 (1960).
151. E. L. Zimmerman, "Two Beryllium-Moderated Critical Assemblies," Oak Ridge National Laboratory report ORNL-2201 (October 1958).
152. R. N. Olcott, "Homogeneous Heavy Water Moderated Critical Assemblies, Part I, Experimental," *Nucl. Sci. Eng.***1**, 327-341 (1956).
153. A. Goodwin, Jr., and C. L. Schuske, "Plutonium Graphite Assemblies," Dow Chemical Company Rocky Flats Plant report RFP-158 (August 1959).
154. A. Goodwin, Jr., and C. L. Schuske, "Plutonium Graphite Assemblies," Dow Chemical Co., Rocky Flats Plant report RFP-123 (September 1958).
155. J. K. Fox and L. W. Gilley, "Critical Parameters for Poisoned Annular Cylinders Containing Aqueous Solutions of U^{235}," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1958," Oak Ridge National Laboratory report ORNL-2609 (October 1958), pp. 31-33.
156. J. E. Tanner and H. M. Forehand, "Critical Experiments for Large Scale Enriched Uranium Solution Handling," in *Proc. of Topical Meeting on Criticality Safety in the Storage of Fissile Material, September 8-11, 1985, Jackson, Wyoming* (American Nuclear Society report ISBN:89448-119-3, 1985), pp. 65-79.
157. C. Clouet d'Orval, E. Deilgat, M. Houelle, and P. Lécorché, "Experimental Research in France on Criticality Problems," (in French) in *Proceedings of the Symposium Criticality Control of Fissile Materials, Stockholm, 1-5 November 1965* (International Atomic

Energy Agency, Vienna, 1966), pp. 193-213.
158. J. T. Mihalczo, "Critical Experiments and Calculations with Annular Cylinders of U(93.2) Metal," in "Neutron Physics Division Annual Progress Report for Period Ending August 1, 1963," Oak Ridge National Laboratory report ORNL-3499 (October 1963), Vol. 1, pp. 62-63.
159. E. B. Johnson, "The Nuclear Criticality of Intersecting Cylinders of Aqueous Uranyl Fluoride Solutions," Union Carbide Corp., Y-12 Plant report Y-DR-129 (October 1974).
160. Bruce B. Ernst and C. L. Schuske, "Empirical Method for Calculating Pipe Intersections Containing Fissile Solutions," Dow Chemical Co., Rocky Flats Plant report RFP-1197 (September 1968).
161. C. E. Newlon, "The Uses of Criticality Codes in Nuclear Safety Considerations of the Oak Ridge Gaseous Diffusion Plant," Oak Ridge Gaseous Diffusion Plant report AECU-4173 (April 1959).
162. C. L. Schuske and J. W. Morfitt, "An Empirical Study of Some Critical Mass Data," Carbide and Carbon Chemicals Corp., Y-12 Plant report Y-533 (December 1949).
163. Deanne Dickinson and C. L. Schuske, "An Empirical Model for Safe Pipe Intersections Containing Fissile Solution," *Nucl. Technol.***10**, 179-187 (1971).
164. N. F. Cross, G. E. Whitesides, and R. J. Hinton, "Monte Carlo Analysis of Experimentally Critical Pipe Intersections," *Trans. Am. Nucl. Soc.***17**, 268 (1973).
165. Deanne Dickinson, "Nominally Reflected Pipe Intersections Containing Fissile Solution," *Nucl. Technol.***26**, 265-277 (1975).
166. "Nuclear Criticality Safety Criteria for Steel-Pipe Intersections Containing Aqueous Solutions of Fissile Materials," ANSI/ANS-8.9-1987 (American National Standards Institute, Inc., New York, 1987).
167. J. T. Thomas, "Reflectors, Infinite Cylinders, and Nuclear Criticality," *Nucl. Sci. Eng.***67**, 279-295 (1978).
168. Dixon Callihan, D. F. Cronin, J. K. Fox, R. L. Macklin, and J. W. Morfitt, "Critical Mass Studies, Part IV," Carbide and Carbon Chemicals Corp., K-25 Plant report K-406 (November 1949).
169. L. W. Gilley, D. F. Cronin, J. K. Fox, and J. T. Thomas, "Critical Arrays of Neutron-Interacting Units," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1961," Oak Ridge National Laboratory report ORNL-3193 (October 1961), pp. 159-167.
170. J. K. Fox and L. W. Gilley, "Critical Parameters of Unreflected Arrays of Interacting Cylinders Containing Aqueous Solutions of U^{235}," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1959," Oak Ridge National Laboratory report ORNL-2842 (November 1959), pp. 82-84.
171. J. K. Fox and L. W. Gilley, "Critical Parameters of Aqueous Solutions of U^{235}," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1957," Oak Ridge National Laboratory report ORNL-2389 (October 1957), pp. 71-83.
172. A. V. Kamaev, B. G. Dubovskii, V. V. Vavilov, G. A. Popov, Yu. D. Palamarchuk, and S. P. Ivanov, "Experimental Investigation of Effects of Interaction of Two Subcritical Reactors," translated from a publication of the State Committee of the Council

of Ministers of the USSR on the Utilization of Atomic Energy, Moscow, 1960, U. S. Atomic Energy Commission report AEC-tr-4708 (U. S. Department of Energy, Technical Information Center, Oak Ridge, Tennessee).

173. B. G. Dubovskii, A. V. Kamaev, G. M. Vladykov, F. M. Kuznetsov, V. Z. Nozik, Yu. D. Palamarchuk, G. A. Popov, and V. V. Vavilov, "Interaction of Subcritical Reactors," *J. Nucl. Energy,* Parts A/B **19**, 271-277 (1965).

174. J. K. Fox and L. W. Gilley, "Some Studies of Water, Styrofoam, and Plexiglas Reflectors," in "Neutron Physics Division Annual Progress Report for Period Ending September 1, 1958," Oak Ridge National Laboratory report ORNL-2609 (October 1958), pp. 38-40.

175. Grover Tuck and H. E. Clark, "Critical Parameters of a Uranium Solution Slab-Cylinder System," *Nucl. Sci. Eng.***40**, 407-413 (1970).

176. E. B. Johnson, "Critical Dimensions of Arrays of Aqueous Uranyl Fluoride Solution Containing Uranium Enriched to 5% in ^{235}U," in "Neutron Physics Division Annual Progress Report for Period Ending August 1, 1965," Oak Ridge National Laboratory report ORNL-3858 (November 1965), pp. 15,16.

177. J. T. Thomas, "Experimental and Calculated System Criticality," in *Proceedings of the Symposium Criticality Control of Fissile Materials, Stockholm, 1-5 November 1965* (International Atomic Energy Agency, Vienna, 1966), pp. 149-175.

178. C. L. Schuske, "Two Experimental Subcritical Arrays of $Pu(NO_3)_4$ Solution," Dow Chemical Co., Rocky Flats Plant report RFP-325 (July 1963).

179. J. C. Bouly, R. Caizergues, E. Deilgat, M. Houelle, and L. Maubert, "Neutron Interaction in Air Between Cylindrical Containers Holding Either Uranium or Plutonium Solutions," (in French) Commissariat a l'Energie Atomique report CEA-R-3946 (March 1970).

180. B. M. Durst, E. D. Clayton, and J. H. Smith, "Critical Experiments with Arrays of Three-Liter Bottles Filled with $Pu(2.8)(NO_3)_4$," *Trans. Am. Nucl. Soc.***41**, 356-358 (1982).

181. J. T. Thomas, "Criticality of ^{233}U Aqueous Nitrate Solution in Reflected and Unreflected Arrays," *Trans. Am. Nucl. Soc.***10,** 538,539 (1967).

182. R. C. Lloyd, E. D. Clayton, and J. H. Chalmers, "Criticality of Arrays of ^{233}U Solution," *Nucl. Appl.***4**, 136-141 (1968).

183. J. T. Thomas, "Critical Three-Dimensional Arrays of Neutron-Interacting Units," Oak Ridge National Laboratory report ORNL-TM-719 (October 1963).

184. S. R. Bierman, B. M. Durst, and E. D. Clayton, "Criticality Experiments with Subcritical Clusters of Low Enriched UO_2 Rods in Water with Uranium or Lead Reflecting Walls," *Nucl. Technol.***47**, 51-58 (1980).

185. S. R. Bierman and E. D. Clayton, "Criticality Experiments with Subcritical Clusters of 2.35 and 4.31 wt% ^{235}U-Enriched Rods in Water with Steel Reflecting Walls," *Nucl. Technol.***54**, 131-144 (1981).

186. J. T. Mihalczo, "Prompt-Neutron Decay in a Two Component Enriched- Uranium-Metal Critical Assembly," *Trans. Am. Nucl. Soc.***6**, 60,61 (1963).

187. J. T. Thomas, "Critical Experiments with UF_6 Cylinder Model 8A Containers," Union Carbide Corp., Y-12 Plant report Y-DR-128 (September 1974).

188. T. G. McCreless, D. R. Smith, G. A. Jarvis, and Dick Duffy, "Neutronic Isolation Characteristics of Concrete, Lead, Wood, Polyethylene, and Beryllium," *Trans. Am. Nucl. Soc.***8**, 441 (1965).
189. J. D. White and C. R. Richey, "Neutron Interaction Between Multiplying Media Separated by Various Materials," *Trans. Am. Nucl. Soc.***8**, 441,442 (1965).
190. J. T. Mihalczo and J. J. Lynn, "Multiplication Measurements with Highly Enriched Uranium Metal Slabs," Oak Ridge National Laboratory report ORNL-CF-59-7-87 (July 1959).
191. J. T. Thomas, "Critical Three-Dimensional Arrays of U(93.2)-Metal Cylinders," *Nucl. Sci. Eng.***52**, 350-359 (1973).
192. E. C. Crume and J. T. Thomas, "Critical and Near-Critical Graphite-Moderated Arrays of U(93.2) Cylinders," *Trans. Am. Nucl. Soc.* **12,** 336,337 (1969).
193. C. L. Schuske, C. L. Bell, G. H. Bidinger, and D. F. Smith, "Industrial Criticality Measurements on Enriched Uranium and Plutonium. Part II," Dow Chemical Co., Rocky Flats Plant report RFP-248 (January 1962).
194. C. L. Schuske, A. Goodwin, Jr., G. H. Bidinger, and D. F. Smith, "Interaction of Two Metal Slabs of Plutonium in Plexiglas," Dow Chemical Co., Rocky Flats Plant report RFP-174 (December 1959).
195. O. C. Kolar, H. F. Finn, and N. L. Pruvost, "Livermore Plutonium Program: Experiments and Calculations," *Nucl. Technol.***29**, 57-72 (1976).

APPENDIX

Compositions and densities of the more significant materials in this document appear in the following table.

COMPOSITIONS AND DENSITIES OF SELECTED MATERIALS

Material	Density (g/cm^3)	Element, Wt Fraction	
Aluminum, 2S,1100	2.71	Al	0.992
		remainder impurities	
Beryllium, QMV	1.84	Be	0.98min
(now S-200-E Be		BeO	0.02max
Inconel (Wrought)	8.51	Ni	0.77
		Cr	0.15
		Fe	0.07
Paraffin, $C_{25}H_{52}$	0.93	H	0.149
		C	0.851
Polymethyl methacrylate	1.18	H	0.080
(Lucite, Perspex, Plexiglas)		C	0.600
$(C_5H_8O_2)_n$		O	0.320
Polyethylene	0.92	H	0.144
$(CH_2)_n$		C	0.856
Polystyrene	1.04-1.08	H	0.077
$(CH)_n$		C	0.923
Stainless steel, 304	7.9	Fe	0.665-0.74
		Cr	0.18-0.20
		Ni	0.08-0.16
Sterotex	0.862	H	0.124
$(C_{17}H_{35}CO_2)_3C_3H_5$		C	0.768
		O	0.108
Graphite, CS-312	1.67	C	0.995
		Fe	0.0034
		S	0.0014
Graphite, ideal	2.25[a]	C	1.00
Zircaloy 2	6.5	Zr	0.982
		Sn	0.015
		Fe	0.0015
		Cr	0.001
		Ni	0.0005
Uranium metal	19.05[a]	U	1.00

COMPOSITIONS AND DENSITIES OF SELECTED MATERIALS (cont.)

Material	Density (g/cm^3)	Element, Wt Fraction	
Uranium tetrafluoride	6.70[a]	U	0.758
UF_4		F	0.242
Uranium hexafluoride	4.68[b]	U	0.676
UF_6		F	0.324
Uranium dioxide	10.96[a]	U	0.832
UO_2		O	0.168
Uranyl fluoride	6.37[a]	U	0.773
UO_2F_2		F	0.123
		O	0.104
Plutonium metal (alpha)	19.86[a]	Pu	1.00
Plutonium metal (delta)	15.76	Pu	0.99
		Ga	0.01
Plutonium dioxide	11.46[a]	Pu	0.882
PuO_2		O	0.118

[a] Theoretical density.
[b] Density of liquid at elevated pressure.