SBI Allocation Between Heavy and Singlet Missiles
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Gregory H. Canavan
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ABSTRACT

The optimal allocation of space-based interceptors (SBIs) between fixed, heavy missiles and mobile singlets can be derived from approximate expressions for the boost-phase penetration of each. Singlets can cluster before launch and have shorter burn times, which reduce their availability to SBIs by an order of magnitude. Singlet penetration decreases slowly with the number of SBIs allocated to them; heavy missile penetration falls rapidly. The allocation to the heavy missiles falls linearly with their number. The penetration of heavy and singlet missiles is proportional to their numbers and inversely proportional to their availability.

I. INTRODUCTION

This paper derives and discusses the optimal allocation of SBIs between fixed heavy missiles and mobile single reentry vehicle (RV) missiles. As the fraction of the SBIs allocated to singlets increases, the number of penetrating singlets increases slowly but the numbers of heavy missile RVs falls rapidly. Thus, for large numbers of heavy missiles, all of the SBIs are allocated to them. For small numbers their allocation falls and
the penetrating heavy and singlet RVs is proportional to their numbers and inversely proportional to their availabilities.

II. HEAVY MISSILE ANALYSIS

The geometric fraction of a K SBI constellation within range of the land missile launch area is

\[ f = \frac{z\pi(W+VT)^2}{4\pi R_e^2}, \]

(1)

where \( R_e \approx 6,400 \text{ km} \) is the Earth's radius, \( W \) is the effective radius of the launch area, \( V \) is the SBI divert velocity, \( T \) is the missile burn time, and \( z \approx 2.5/(W+VT) \) is a numerical factor\(^1\) representing the constellation concentration possible over launch areas of modest latitudinal extent.\(^2\) For the current \( T = 600 \text{ s} \) for heavy missiles, the current \( W \approx 2,000 \text{ km} \) gives \( f \approx 0.2 \).

Relocating the missiles in the current SS-18 launch area, which has a radius of \( W \approx 500 \text{ km} \), would give \( f \approx 0.13 \). Thus, the number of SBIs required for one intercept per missile is

\[ K = zK[(W+VT)/2R_e]^2. \]

(2)

The constellations must be oversized by 30-50% to destroy most of the RVs before they are released. The sizes can be calculated exactly for single engagements\(^3\) and approximately for two,\(^4\) but the results are awkward for survey calculations. However, a simple approximation of acceptable accuracy is available.\(^5\)

The approximation is based on the observation that the exact number of RVs killed, \( R \), is linear in \( m_0pK \) for small \( K \) and rolls over at \( K \approx M/f \), where the number of SBIs in range approaches the number of missiles launched, \( M \); \( m_0 \) is the initial number of RVs per missile, and \( p \approx 1 \) is the SBIs' probability of kill. The simplest analytic function with these properties is

\[ R \approx m_0pM[1 - \exp(-fK/M)], \]

(3)

which for \( K \) small is \( \approx m_0pM(fK/M) \approx m_0pK/FK \), i.e., the product of the SBIs available, \( fK \), and the average RVs killed by each in a target-rich environment, \( m_0p \). For large \( K \), \( R \approx m_0pM \), i.e. all of the RVs are killed. The fraction penetrating is in general

\[ P = 1 - R/m_0pM \approx e^{-fK/M}, \]

(4)

which is exact for both small and large \( K \). The intermediate behavior is only approximately correct.\(^6\) The main discrepancy is
near $K \approx M/f$, where the exact result gives 10-20% more kills than
the approximation. The differences are similar in current, START, and midterm force
levels.

The approximation falls off too gently compared to the exact
result; it thus consistently underestimates the number of RV
kills and overestimates the number of RVs penetrating. The exact
formulae could be used, but if the calculations are used to size
boost phases with midcourse underlays, the uncertainties in the
boost phase could at worst double the demands on the midcourse
layer, which is generally a minor part of the total.

III. SINGLET MISSILES

The analysis for mobile singlets largely follows from that
for heavy missiles. For the $S$ singlets $m = 1$. Because they can
cluster at a point before launch, their geometric availability is
that of Eq. (1) for $W = 0$, which is

$$j = \frac{2\pi (VT)^2}{4\pi R_e^2},$$

where $z$ is still $2.5/(W+VT) \approx 1.24$, because SBI constellations
must be optimized for the heavy missiles, so long as they are
dominant. Thus, the number of singlets that would penetrate $K$
SBIs is roughly $S e^{-jK/S}$. For singlets with $T \approx 300$ s, $j \approx 0.03$.

IV. TOTAL PENETRATION

If a fraction $\phi$ of the SBIs were allocated to the heavy
missiles and the rest to the singlets, the total number of
penetrating RVs would be

$$V = S e^{-j(1-\phi)K/S} + m o Me^{-f\phi K/M},$$

which is shown as a function of $\phi$ in Fig. 1 for $m = 10$, $M = 100$,$K = 4,000$ SBIs, and $S = 344$ singlets. The curves are for
$f = 0.13$ and $j = 0.03$, which correspond to concentration of both
the heavy and singlet missiles in the current Soviet heavy
missile launch area in the near-term.\textsuperscript{7}

For $\phi$ small the bottom curve is the number of penetrating
singlets, which increases slowly with $\phi$; the middle curve is the
number of penetrating heavy RVs, which falls rapidly. The top
curve is their total, \( V \), which has a weak minimum at about \( \phi \approx 70\% \) of the SBIs allocated to the heavy missiles.

The defender wishes to minimize \( V \), which he does by differentiating \( V \) with respect to \( \phi \), setting the result to zero, and solving for

\[
\phi_0 = \left[ \ln(\frac{mf}{j}) + \frac{jK}{S} \right] / \left( \frac{j}{S} + \frac{f}{M} \right)K
\]

(7)

as the allocation that minimizes overall penetration. The optimal \( \phi_0 \) is shown as a function of \( M \) by the top curve in the middle in Fig. 2. The lower curves are the total, heavy, and single RV contributions for those \( M \) and \( \phi_0 \). The optimal \( \phi_0 \) is unity for \( M > 150 \); below that it falls linearly with \( M \). The singlet missiles are not attacked for larger \( M \); below \( M = 150 \) they fall slowly. The heavy missiles fall strongly for \( M > 150 \); after that they also fall slowly. For nominal conditions, \( \phi_0 \approx \left[ \ln(\frac{mf}{j}) \right] M/fK \), so the number of penetrating heavy RVs is about \( jM/f \), and the number of penetrating heavy and singlet RVs is in the ratio \( M/f:S/j \).

For the current START \( M \approx 270 \) heavy and \( S \approx 344 \) singlet Soviet missiles \( \phi_0 = 1 \); i.e., all SBIs are allocated to heavy missiles and the singlets get a free ride. Heavy and singlet contributions are about equal at \( M = 250 \). Below \( M = 150 \) they are attrited proportionally, although there is little contribution left from heavy missiles at that point.

V. OBSERVATIONS

It is possible to derive the optimal allocation of SBIs between fixed heavy missiles and mobile single RV missiles if approximate expressions are used for the penetration of each. Singlet missiles are distinguished by their ability to cluster before launch and shorter booster burn times, which gives the SBIs an order of magnitude less availability for them.

The number of penetrating singlets decreases slowly with the allocation of SBIs to them; the number of penetrating heavy RVs falls rapidly. Their sum has a weak minimum at intermediate allocations to heavy missiles. The defender can minimize the total number of penetrating RVs by allocating the SBIs according
to analytic criteria. For large numbers of heavy missiles, all
SBIs are allocated to them. For small numbers their allocation
falls linearly with their numbers, and the penetration of heavy
and singlet RVs is proportional to their numbers and inversely
proportional to their availability.

The proportional allocation applies to two distinct,
relevant cases. The first is restrike. If one side rides out
the other's first strike before launching, given current
accuracies it is likely that the number of missiles in the
restrike will fall below the transition. That appears to be
of academic interest only to the U.S., whose mobile options are
waning. It would, however, be relevant to U.S. allocation of its
SBIs between Soviet singlets and residual heavy missiles, if the
U.S. struck first, which is now, however, thought to be an
unlikely eventuality.

The other case is the postulated build down from heavy
missiles to mobile singlets. At about half the current level of
Soviet heavy missiles, SBI allocation would begin to shift to
singlets. The shift would, however, be too gradual to impact
erlier studies of the stability of the transition.8
REFERENCES


Fig. 1 Penetrating RVs vs allocation

\[ m=10, M=100, f=13\%, K=4k, S=344, J=3\% \]

\begin{figure}[h]
\centering
\begin{tikzpicture}
\begin{axis}[
    title={},
    xlabel={Fract allocated to heavy missiles},
    ylabel={Penetrating RVs (Thousands)},
    xmin=0, xmax=1,
    ymin=0, ymax=1.3,
    xtick={0,0.2,0.4,0.6,0.8,1},
    ytick={0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.1,1.2,1.3},
    legend style={at={(0.5,0.95)},anchor=north west},
    legend cell align={left},
    legend image code/.code={
        $\cdot$
    }
]
\addplot coordinates {\( (0,1.3) \) \( (0.2,1.0) \) \( (0.4,0.8) \) \( (0.6,0.6) \) \( (0.8,0.4) \) \( (1,0.2) \)}; \node at (axis cs:0.5,1.1) {\( \square \) \textbf{Heavy}};
\addplot coordinates {\( (0,1.3) \) \( (0.2,1.0) \) \( (0.4,0.8) \) \( (0.6,0.6) \) \( (0.8,0.4) \) \( (1,0.2) \)}; \node at (axis cs:0.5,1.0) {\( + \) \textbf{Single}};
\addplot coordinates {\( (0,1.3) \) \( (0.2,1.0) \) \( (0.4,0.8) \) \( (0.6,0.6) \) \( (0.8,0.4) \) \( (1,0.2) \)}; \node at (axis cs:0.5,0.9) {\( \circ \) \textbf{Total RVs}};
\end{axis}
\end{tikzpicture}
\end{figure}

Fig. 2 Optimal allocations & pen. RVs

\[ m=10, f=13\%, K=4k, S=344, J=3\% \]

\begin{figure}[h]
\centering
\begin{tikzpicture}
\begin{axis}[
    title={},
    xlabel={heavy missiles},
    ylabel={Penetrating RVs (Thousands)},
    xmin=0, xmax=400,
    ymin=0, ymax=1.5,
    xtick={0,100,200,300,400},
    ytick={0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.1,1.2,1.3,1.4,1.5},
    legend style={at={(0.5,0.95)},anchor=north west},
    legend cell align={left},
    legend image code/.code={
        $\cdot$
    }
]
\addplot coordinates {\( (0,0) \) \( (100,0.1) \) \( (200,0.3) \) \( (300,0.6) \) \( (400,1.0) \)}; \node at (axis cs:0.5,0.9) {\( \square \) \textbf{Heavy RV}};
\addplot coordinates {\( (0,0) \) \( (100,0.1) \) \( (200,0.3) \) \( (300,0.6) \) \( (400,1.0) \)}; \node at (axis cs:0.5,0.8) {\( + \) \textbf{Single RV}};
\addplot coordinates {\( (0,0) \) \( (100,0.1) \) \( (200,0.3) \) \( (300,0.6) \) \( (400,1.0) \)}; \node at (axis cs:0.5,0.7) {\( \circ \) \textbf{Tot RV}};
\addplot coordinates {\( (0,0) \) \( (100,0.1) \) \( (200,0.3) \) \( (300,0.6) \) \( (400,1.0) \)}; \node at (axis cs:0.5,0.6) {\( \triangle \) \textbf{phi}};
\end{axis}
\end{tikzpicture}
\end{figure}
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