Background Issues
for Defensive Interceptors
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for Defensive Interceptors

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BACKGROUND ISSUES FOR DEFENSIVE INTERCEPTORS

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Gregory H. Canavan

ABSTRACT

Mean nuclear backgrounds are large, but are arguably amenable to frame-to-frame subtraction. Striated backgrounds on the sensors for defensive interceptors could, however, cause clutter leak-through, which could make detection and track difficult. Nominal motions and backgrounds give signal to clutter ratios too low to be useful. Clutter leakage due to line-of-sight drift can be reduced by stabilizing the line of sight around the background clutter itself. Current interceptors have detector arrays large enough for operation independent of nuclear backgrounds in their fields of view.

I. INTRODUCTION

This note discusses the impact of cluttered nuclear backgrounds on the sensors for autonomous defensive interceptors. While mean backgrounds are arguably tolerable, the apparent fluctuations caused by the interceptor's motion could cause large clutter leak-through, which could make detection and track difficult. It appears, however, that advanced correlation techniques could make interceptors' sensors relatively insensitive to nuclear clutter.
II. ANALYSIS

The mean and fluctuating backgrounds from nuclear detonations both present problems for sensors. Calculations predict background brightnesses in the MWIR-LWIR (3-12 μm) region of the spectrum of $B'' = 10^{-6}-10^{-7}$ watt/cm$^2$-sr over significant spatial regions.\(^1\) About 10 bursts could apparently produce an IR "fence" about 2,000 km wide and 800 km high with $B'' \approx 10^{-7}$ watt/cm$^2$-sr which most of the interceptors would have to view.

A. Background

Each pixel of a LWIR sensor operating at a wavelength of $\lambda \approx 10$ μm with a $D \approx 10$ cm aperture would have a footprint on a threat cloud at $R = 1,000$ km of

$$L \approx Rw/D \approx 1,000 \text{ km} \cdot 10 \text{ μm} / 10 \text{ cm} \approx 100 \text{ m},$$

so the total background IR signal would be

$$B''L^2 \approx 10^{-7} \text{ watt/cm}^2\text{-sr} \cdot (10^2 \text{ m})^2 \approx 10 \text{ watt/sr},$$

which is about 10 times larger than the $J \approx 1$ watt/sr from a 300° RV or decoy. The signatures for discrimination are actually $\approx 10\%$ of $J$, so they could be about 1% of $B''L^2$.

The usual approach to eliminating this background is to subtract successive frames to eliminate the background and keep the target signature, which shows up as a time variation when the target moves from one pixel to the next.\(^2\) That differencing can be done easily from large satellites that are carefully stabilized.

Stabilization is more difficult for small interceptors. They go through rotations of $\theta \approx 1$ rad over $\approx 100$ s, or $\theta' \approx 10$ mrad/s, during flyout, and they are guided by simple star trackers, which have residual drift rates of $\theta' \approx 1$ mrad/s.\(^3\) For a frame time of $t \approx 10$ ms that gives a frame-to-frame displacement of $\theta \approx 1$ mrad/s·$10$ ms $\approx 10$ μrad, which is about 10% of the pixel diameter of $w/D = 10^{-5}$ m/0.1 m $\approx 100$ μrad. That angular displacement would give a frame-to-frame miss-registration of the footprint on the target cloud of

$$\delta \approx \theta \cdot R \approx 10 \text{ μrad} \cdot 1,000 \text{ km} \approx 10 \text{ m}.$$
Larger $\theta'$, $t$, or acquisition ranges could give values 10-100 times larger, but even $\delta = 10$ m gives $\delta/L \approx 10\%$, which could cause significant clutter to leak through differencing filters.

B. Clutter Leakage

If the background in the pixel at time 0 is $f(x,t=0)$, where $x$ is the coordinate transverse to the striations which form rapidly in the background, $f$ can be represented in terms of its Fourier transform as

$$f(x,0) = \sum dk \, F(k,0)e^{ikx},$$

where the root mean square value of $f$ is

$$<f(x,0)^2> = \sum dk \, 1/L \int_{-\infty}^{\infty} |F(k,0)|^2$$

$$\approx \sum dk \, 1/L \int_{-\infty}^{\infty} B^2 nk^{-2} \approx B^2 L^2$$

for a striated background with power spectral density $B^2 L k^{-2}$. If the line of sight drifts a distance $\delta$ between frames, the background seen at time $t$ is $f(x,t) = f(x-\delta,0)$, so that

$$f(x,t) = \sum F(k,0)e^{ik\delta}e^{ikx}.$$  \hspace{1cm} (6)

Thus, the signal through a first differencing filter is

$$f(x,t) - f(x,0) = \sum F(k,0)e^{ik\delta}(e^{ik\delta} - 1)$$  \hspace{1cm} (7)

and the average clutter power leaking through it is

$$C = <[f(x,t)-f(x,0)]^2> = \sum dk |F(k,0)|^2[1-\cos(k\delta)]$$

$$\approx \sum 1/L \pi/\delta |F(k,0)|^2(k\delta)^2.$$  \hspace{1cm} (8)

For a striation spectrum $B^2 L k^{-2}$

$$C \approx \sum 1/L \pi/\delta B^2 L^2 = B^2 L^2 (\pi/\delta - 1/L) \approx \pi B^2 L^2 \delta/L.$$  \hspace{1cm} (9)

Thus, the clutter leakage is a fraction $\delta/L$ of the total emission subtended. The signal to clutter ratio is

$$S/C = J/\pi B^2 L^2 (\delta/L),$$  \hspace{1cm} (10)

which for $J = 1$ W/sr, $B^2 L^2 = 10$ W/sr, and $\delta/L \approx 10/100 \approx 0.1$, as above, gives $S/C \approx 1/\pi$. Such a value would not support reliable detection at the 1,000 km used, let alone the longer ranges needed.

Equation (9) tends to overestimate the clutter leak-through in that it is estimated that the clutter variance is only about 10% of the mean value, the power spectral density of the clutter actually falls off more rapidly than $k^{-2}$, and the integral should actually be truncated at an outer scale of 3 to 10 km.
Offsetting those factors is that it is often necessary to see very dim targets, the nuclear background is evolving, and the non-uniformity of real detector arrays. It is not clear that these corrections would significantly alter any of the arguments above, as they are factor-of two corrections to a mean background that is probably not known that accurately.

Figure 1 shows $\frac{dC}{dk}$ from Eq. (8) as well as the approximation from Eq. (9). $\frac{dC}{dk}$ is relatively constant at $\approx 0.37$ out to $k = 0.2$, where it starts to fall as $\approx k^{-2}$, leading to an effective cutoff at $k \approx \pi/\delta$. The approximation is useful out to about $k \approx 0.3$, where the numerical solution rolls over to $C \approx 2.9$. The shape of the curves holds approximately for other $L$ and $\delta$. This integral covers only the first cycle of $\cos(k\delta)$. Further cycles could add as powers of $(k^{-2})^n$, $n = 1, 2, 3$, etc., but they are further suppressed at about this length scale by the finite mean free path of atoms and ions at altitudes of interest, which truncates the clutter spectrum.

Figure 2 shows signal to clutter ratios from Eq. (10) as functions of range and line of sight angular drift rate for a pixel diameter, i.e. wavelength to mirror diameter, of $\theta \approx w/D \approx 10 \:\mu m/10 \:cm \approx 10^{-4}$ rad, for which Eq. (10) can be rewritten as

$$S/C = J/\pi B''(rw/D)(\theta'rt) = J/\pi B''(w/D)(\theta't)r^2.$$  (11)

The figure is for $B'' = 1 \:mW/sr$, $w/D \approx 0.1$ mrad, and a frame time of $t = 0.01$ s; $\theta'$ is varied a factor of 3 either way from 1 mrad/s. The top curve is for $\theta' = 0.3$ mrad/s. It gives a $S/C \approx 3$ at $r = 500$ km, which falls to about unity at 1,000 km. At 2,000 km it is about 0.25. The next curve is for 1 mrad/s, which is about 0.3 at 1,000 km as shown by the earlier point estimate. The bottom curve for 3 mrad/s is about 0.4 at 500 km and falls to $\approx 0.1$ at 1,000 km. None of the curves would support a handoff from another long-range sensor. Nor would they be adequate for short-range track, which requires $S/C \approx 10$ at a range of 500-1,000 km. The results could be improved by increasing the sensor's mirror, but the diameters required would appear to be $\approx 1$ m, which is large for individual interceptors.
III. LINE OF SIGHT STABILIZATION

It has been shown that clutter leakage due to line of sight drift can be greatly reduced by stabilizing the sensor's line of sight around the background clutter itself.\textsuperscript{5} That is accomplished by estimating the distance the background drifts between scenes with

\[
<\delta> \approx d \cdot \Sigma_i \left[ f_i(t) - f_{i+1}(0) \right]^2 + 2N\Sigma_i \left[ f_i(0) - f_{i+1}(0) \right]^2,
\]

where \( i \) is the index corresponding to one of \( N \) measurements at intervals \( d \), and then correcting for the motion by correcting the line of sight of the sensor mechanically or by shifting the information on the second frame to correct for the motion computationally. With \( \approx 100 \) detector arrays it has been possible to reduce the error to \( \approx 1 \) millipixel.\textsuperscript{6}

The impact of an error of \( \varepsilon \) pixels that can be seen by modifying Eq. (9) to

\[
C_{\text{stab}} = \pi B'' L^2 (\delta/L) = \pi B'' L^2 \varepsilon = \pi B'' (rw/D)^2 \varepsilon,
\]

whose ratio to the unstabilized clutter of Eq. (11) is

\[
\frac{C_{\text{stab}}}{C_{\text{unstab}}} = \frac{\pi B'' (rw/D)^2 \varepsilon + B'' (rw/D) (\theta' \tau t)}{\varepsilon w/D + \theta' t},
\]

so that stabilization decreases the clutter leaking through by a factor of \( \varepsilon w/D + \theta' t \). For the above parameters and 1 mrad/s,

\[
C_{\text{stab}}/C_{\text{unstab}} = 0.001 \cdot 0.1 \text{ mrad/s} + 1 \text{ mrad/s} \cdot 10 \text{ ms}
\]

so stabilization to that level would reduce the clutter leaking through by about a factor of 100. The impact is shown by the right ordinate of Fig. 2. Increasing the signal to clutter ratio would put \( S/C \) in the range of 10-100 at 1,000 km and 2.5-25 at 2,000 km, which is more than adequate for handover or track. With clutter leakage eliminated, the interceptor's detectors would only have to deal with the shot noise in or physical motion of the uniform background, which is much smaller.

Current interceptors have detector arrays larger than the 100 or so needed for millipixel registration, so they could apparently be made to operate in a mode that was independent of nuclear backgrounds in their fields of view.
IV. CONCLUSIONS

This note has discussed the impact of striated nuclear backgrounds on the sensors for autonomous defensive interceptors. Mean backgrounds are large, but are arguably amenable to frame-to-frame subtraction. The fluctuations caused by interceptor motion could, however, cause large clutter leak-through, which could make detection and track difficult. Nominal motions and backgrounds give signal to clutter ratios too low to be useful.

Clutter leakage due to line of sight drift can be greatly reduced by stabilizing the line of sight around the background clutter itself. With $\approx 100$ detectors it has been possible to reduce the error to $\approx 1$ millipixel. Current interceptors have detector arrays large enough for millipixel registration, and hence for operation in a mode independent of nuclear backgrounds in their fields of view.

ACKNOWLEDGMENT

The author would like to acknowledge stimulating discussions with Drs. Bill White and Dave Sowle and useful discussions of uncertainties in nuclear backgrounds with Drs. Roy Hendricks and Leon Wittwer.
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Fig. 1 Clutter vs wavenumber

$B'' = .001, L = 100, d = 10$

- $dC/dk$
- $C(k)$
- Approx $C(k)$

Fig. 2 Signal to clutter vs range

$B'' = .001, v = 10$ micron, $D = 0.1, t = .01$s

- $r'$ = 0.3
- 1
- 3 m/s
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